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Undergraduate Admission Test 2022: Mathematics Honours

Full Marks : 150 **Time : 2hours**

Each question has only one correct answer.
Each question carries 3 marks for correct answer and (–1) mark for wrong answer.

(The symbols have their usual meanings)

1. The value of $\lim_{x\to\infty} x\cos\left(\frac{\pi}{4x}\right)\sin\left(\frac{\pi}{4x}\right)$ is
   a. $\frac{\pi}{2}$
   b. $\frac{\pi}{4}$
   c. 1
   d. π

2. The sum of infinite series $\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\ldots+\infty$ is
   a) $\frac{1}{3}$
   b) 3
   c) $\frac{1}{4}$
   d) ∞

3. The function $f(x)=\max\{1-x,1+x,2\}, x\in(-\infty,\infty)$ is
   a) Differentiable at all points
   b) Differentiable at all points at $x=1$ and $x=-1$.
   c) at all points except at $x=1$ and $x=-1$, where it is discontinuous
   d) None of these.

4. $\left(\sqrt{3}+1\right)^4+\left(\sqrt{3}-1\right)^4$ is equal to
   a) A rational number
   b) An irrational number
   c) A transcendental number
   d) None of the above

5. If $f(x)=\begin{cases}\frac{\sin[x]}{[x]}, & \text{for } [x]\neq 0\\ 0, & \text{for } [x]=0\end{cases}$, where [x] denotes the greatest integer less than or equal to x, then $\lim_{x\to 0} f(x)$ *equals*
   a) 1
   b) 0
   c) -1
   d) doesn't exist

6. The function $2\tan^3 x - 3\tan^2 x + 12\tan x + 3,\ x \in \left(0, \frac{\pi}{2}\right)$ is

   a) Increasing
   b) Decreasing
   c) Increasing in $\left(0, \frac{\pi}{4}\right)$ and decreasing in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$
   d) None of these

7. The function $y = x - \cot^{-1} x - \log\left(x + \sqrt{x^2 + 1}\right)$ is increasing on

   a) $(-\infty, 0)$
   b) $(-\infty, \infty)$
   c) $(0, \infty)$
   d) $\mathbf{R} - \{0\}$

8. If $2a + 3b + 6c = 0$, then equation $ax^2 + bx + c = 0$ has roots in the interval
   a) (0,1)
   b) $(2,3)$
   c) $(1,2)$
   d) $(0,2)$

9. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$, then $x^3 + y^3 + z^3$ is equal to
   a) -3
   b) 3
   c) 0
   d) None of these

10. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$, $\cos^{-1} x + \cos^{-1} y = \frac{\pi}{3}$, then the number of ordered pairs (x, y) is
    a) 0
    b) 1
    c) 2
    d) None of these

11. If $A = 2\tan^{-1}\left(2\sqrt{2} - 1\right)$ and $B = 3\sin^{-1}(1/3) + \sin^{-1}(3/5)$, then
    a) $A = B$
    b) $A < B$
    c) $A > B$
    d) None of these

12. $\log_{\sqrt{3}} x + \log_{\sqrt[4]{3}} x + \log_{\sqrt[8]{3}} x + \ldots + \log_{\sqrt[16]{3}} x = 36$ gives
    a) $x = 3$
    b) $x = 4\sqrt{3}$
    c) $x = 9$
    d) $x = \sqrt{3}$

13. If coefficient of $x^2 y^3 z^4$ in $(x + y + z)^n$ is A, then coefficient of $x^4 y^4 z$ is
    a) $2A$
    b) $\frac{nA}{2}$
    c) $\frac{A}{2}$
    d) None of these

14. If the second, third and fourth terms in the expression of $(a+b)^n$ are 135, 39 and 10/3 respectively, then
    a) $a = 3$
    b) $b = 1/3$
    c) $n = 5$
    d) All the above

15. $\vec{a} \times \vec{b} = \vec{c}$, $\vec{b} \times \vec{c} = \vec{a}$, then
    a) $|\vec{a}| = 1, |\vec{b}| = |\vec{c}|$
    b) $|\vec{c}| = 1, |\vec{a}| = 1$
    c) $|\vec{b}| = 2, |\vec{b}| = 2|\vec{a}|$
    d) $|\vec{b}| = 1, |\vec{b}| = |\vec{a}|$

16. The value of derivative of $f(x) = |x-1| + |x-3|$ at $x = 2$ is
    a) -2
    b) 0
    c) 2
    d) Not defined

17. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has
    a) Exactly one real root
    b) Exactly four real roots
    c) Infinite number of real roots
    d) No real roots

18. If $z^2 + z + 1 = 0$, where z is a complex number, the value of $\sum_{r=1}^{2022} \left(z^r + \frac{1}{z^r}\right)^2$ is
    a) 2022
    b) 4044
    c) 3033
    d) 1011

19. The total number of integral solutions of $xyz = 24$ is
    a) 120
    b) 90
    c) 36
    d) 30

20. The remainder , if $1 + 2 + 2^2 + 2^3 + \cdots + 2^{2021}$ is divided by 5 , is
    a) 0
    b) 1
    c) 2
    d) 3

21. If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and A adj A = AA^T, then 5a + b is equal to
    a) 13
    b) 5
    c) 4
    d) -1

22. If p is the length of the perpendicular drawn from the origin to any normal to the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$, then the maximum value of p is
    a) 5
    b) 4
    c) 2
    d) 1

23. If $f(x)$ satisfies the relation $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$ and $f(1) = 5$, then the value of $\sum_{n=1}^{2022} f(n)$ is
    a) 10110×2023
    b) 10110×2022
    c) 5055×2023
    d) 5055×2022

24. Let $g(x) = \int_0^x f(t)dt$, where f is such that $^1/_2 \leq f(t) \leq 1,\ for\ t \in [0,1]\ and\ 0 \leq f(t) \leq {}^1/_2\ ,\ for\ t \in [1,2]$, then $g(2)$ lies
    a) $[{}^1/_2, {}^3/_2]$
    b) $[{}^3/_2, 2]$
    c) $[0, {}^1/_2]$
    d) $[2, {}^5/_2]$

25. The probability that a man who is 52 years now is alive till he is 77 years is $\frac{1}{4}$ and the probability that a second man who is 63 years old now will be alive till he is 88 years is $\frac{3}{8}$. Find the probability that at least one of them is alive at the end of 25 years.
    a) $\frac{3}{32}$
    b) $\frac{15}{32}$
    c) $\frac{17}{32}$
    d) $\frac{5}{8}$

26. The value of $\lim_{m\to\infty}(\cos\frac{x}{m})^m$ is
    a) 1
    b) e
    c) e^{-1}
    d) None of these

27. The value of $(\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ)$ is
    a) 1
    b) 4
    c) -1
    d) -4

28. The area of the region bounded by the curve $y = x^2$ and $x = y^2$ is

a) $\frac{1}{2}$

b) $\frac{1}{3}$

c) $\frac{1}{4}$

d) None of these

29. If α, β are roots of the equation $x^2 + x + 1 = 0$, then the equation whose roots are $\alpha^2 + \beta^2 \; and \; \alpha^{-2} + \beta^{-2}$ will be

a) $x^2 - x + 1 = 0$

b) $x^2 - x - 1 = 0$

c) $x^2 - 2x + 1 = 0$

d) $(x + 1)^2 = 0$

30. If $|z_1 - 2000| < 2000, |z_2 - 2021| < 2021, |z_3 - 2022| < 2022$, then $|z_1 + z_2 + z_3|$

a) is greater than 6043
b) is less than 6043
c) is less than 12086
d) lies between 6043 and 12086

31. If the points whose position vectors be $-2\hat{\imath} + \hat{\jmath} + \hat{k}, \; \hat{\imath} + \hat{\jmath} + \hat{k}, \; \hat{\jmath} - \hat{k} \; and \; \alpha\hat{\jmath} + \hat{k}$ be coplanar, then the value of α is

a) 1
b) 2
c) -1
d) 0

32. If $\frac{x^2+1}{(2+x)(2-x)(x-1)} = \frac{a}{3(x-1)} + \frac{b}{4(2-x)} - \frac{c}{12(x+2)} \; and \; A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$, then $|A|$ is equal to

a) -34

b) 34

c) -24

d) -108

33. If $x = \sqrt{1+\sqrt{1+\sqrt{1+\cdots to\ \infty}}}$, then $x =$

a) $\frac{1+\sqrt{5}}{2}$

b) $\frac{1-2\sqrt{5}}{2}$

c) $\frac{1+2\sqrt{5}}{2}$

d) None of these

34. Which of the following function is differentiable at $x = 0$?

a) $\cos(|x|) + |x|$
b) $\cos(|x|) - |x|$
c) $\sin(|x|) + |x|$
d) $\sin(|x|) - |x|$

35. If $I = \int_{-1}^{2} |x \sin \pi x|\, dx$, then I equals to

a) $\frac{1}{\pi}$

b) $\frac{2}{\pi}$

c) $\frac{4}{\pi}$

d) $\frac{5}{\pi}$

36. Let A be any $2x2$ matrix and B be its Adjoint. Then the determinant of the product of matrix AB is

a. 1
b. $\det A$
c. $(\det A)^2$
d. None of the above

37. The modulus of $\sqrt{2i} - \sqrt{-2i}$ is

a) 2
b) $\sqrt{2}$
c) 0
d) $2\sqrt{2}$

38. Let $|X| = 4, |Y| = 3$. The number of surjective maps from X to Y is

a) 24
b) 30
c) 36
d) None of the above

39. The maximum value of $(\frac{1}{x})^x$ $(x>0)$ is

a) $(\frac{1}{e})^e$
b) e^e
c) $e^{\frac{1}{e}}$
d) None of these

40. The determinant of the matrix $A = \begin{pmatrix} x^2+x & x^3+x^2 & x^4+x^3 \\ x^2 & x^3 & x^4 \\ x^2-x & x^3-x^2 & x^4-x^3 \end{pmatrix}, x \in R$ is

a) A polynomial in x of degree 7
b) 0
c) A polynomial of degree 9
d) None of these

41. $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ *and* $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two sets then

a) $P \subset Q$ *and* $Q - p \neq \emptyset$
b) $Q \not\subseteq P$
c) $P \not\subset Q$
d) $P = Q$

42. The value of $\lim_{x\to 0} \frac{\int_0^{x^2} \cos t^2 dt}{x \sin x}$ is

a) $\frac{3}{2}$
b) 1
c) -1
d) None of these

43. If $P(A \cap B) = \frac{5}{13}$, then the value of $P(A^c \cup B^c)$ is

a) $\frac{4}{13}$
b) $\frac{5}{13}$
c) $\frac{7}{13}$
d) $\frac{8}{13}$

44. A determinant is chosen at random from the determinant of order 2 with elements 0 or 1 only. The probability that the determinant chosen is non-zero is

a) $3/16$
b) $3/8$
c) $1/4$
d) $1/8$

45. Let $a, b \in \mathbb{R}$ and $a\sqrt{a} + b\sqrt{b} = 183$ and $b\sqrt{a} + a\sqrt{b} = 182$ then $\frac{9}{5}(a+b) =$

a) 91

b) 61

c) 85

d) 73

46. If $f(x)$ is a polynomial satisfying $f(x).f(1/x) = f(x) + f(1/x)$ and $f(3) = 28$.

Then $f(4)$ is

a) 63

b) 65

c) 67

d) 68

47. If $f(0) = 0, f'(0) = 2$, then the derivative of $y = f(f(f(f(x))))$ at $x = 0$ is

a) 2

b) 8

c) 4

d) 16

48. The integral $\int_0^{1.5}[x^2]dx$, where $[x]$ denotes the greatest integer function, equals

a) $2 - \sqrt{2}$
b) $2 + \sqrt{2}$
c) $1 - \sqrt{2}$
d) 2

49. Projection of the vector $(-2\hat{\imath} + 3\hat{\jmath} + 3\hat{k})$ on the vector $\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ is

a) $\frac{2}{\sqrt{14}}$
b) $\frac{1}{\sqrt{14}}$
c) $\frac{3}{\sqrt{14}}$
d) None of these

50. In the following L.P.P.,

Minimize $z = -2x + y$
Subject to $x + y \geq 6$,
$3x + 2y \geq 16$,
$y \leq 9$

and $x, y \geq 0$, in which point the z_{min} attain?

a) (6,0)
b) (4,2)
c) (0,8)
d) None of these