LAPLACE'S EQUATION

The project submitted, in partial fulfilment of the requirement for the assignments in PHSA CC-11,CC-12,DSE-1,DSE-2 Paper (Semester-5) in the Department of Physics

Submitted by

Anjan Jana

Registration No: A01-1112-111-001-2019

Supervisor Teacher: Asok Kumar Pal

RAMAKRISHNA MISSION
VIVEKANANDA CENTENARY
COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

RAMAKRISHNA MISSION VIVEKANANDA CENTENARY COLLEGE

Name :- Anjan Jana

Dept. :- Physics

Roll :- 102

Sem :- 5th

Reg. No :- A01-1112-111-001-2019

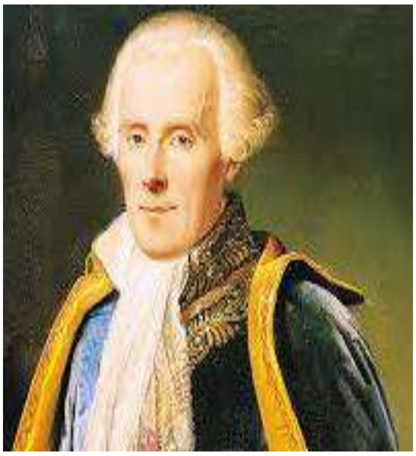
Exam roll no :- 2022151001

Project :- Laplace's equation

INTRODUCTION :-

In this following section we are gonna discuss about LAPLACE'S EQUATION. First we will write Laplace equation in Cartesian,spherical polar and as well as cylindrical co-ordinate. Then we are gonna discuss about first uniquenesa theorem, second uniqueness theorem. At last we are gonna write Laplace equation in Schwartz child matric format.

LAPLACE'S EQUATION



The primary task of electrostatics is to find the electric field of a given stationary charge distribution. In principle, this purpose is accomplished by Coulomb's law, in the form of following figure:

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}\rho(\mathbf{r}')\,d\tau'.$$

Unfortunately, integrals of this type can be difficult to calculate for any but the simplest charge configurations. Occasionally we can get around this by exploiting symmetry and using Gauss's law, but ordinarily the best strategy is first to calculate the potential, V, which is given by the somewhat more tractable Equation:

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{1}{\mathscr{r}}\rho(\mathbf{r}')\,d\tau'.$$

Still, even this integral is often too tough to handle analytically. Moreover, in problems involving conductors ρ itself may not be known in advance; since charge is free to move around, the only thing we control directly is the total charge (or perhaps the potential) of each conductor. In such cases, it is fruitful to recast the problem in differential form, using Poisson's equation.

which, together with appropriate boundary conditions, is equivalent to the following equation. Very often, in fact, we are interested in finding the potential in a region where $\rho = 0$. (If $\rho = 0$ everywhere, of course, then $V = 0$, and there is nothing further to say—that's not what I mean. There may be plenty of charge elsewhere, but we're confining our attention to places where there is no charge.) In this case, Poisson's equation reduces to Laplace's equation:

$$\nabla^2 V = 0,$$

or, written out in Cartesian coordinates,

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0.$$

This formula is so fundamental to the subject that one might almost say electrostatics is the study of Laplace's equation. At the same time, it is a ubiquitous equation, appearing in such diverse branches of physics as gravitation and magnetism, the theory of heat, and the study of soap bubbles. In mathematics, it plays a major role in analytic function theory. To get a feel for Laplace's equation and its solutions (which are called harmonic functions), we shall begin with the one and two-dimensional versions, which are easier to picture, and illustrate all the essential properties of the three-dimensional case.

Laplace's Equation in One Dimension:

Suppose V depends on only one variable, x. Then Laplace's equation becomes

$$\frac{d^2V}{dx^2} = 0.$$

The general solution is

$$V(x) = mx + b,$$

The exact value of potential depends on boundary condition of the problem.

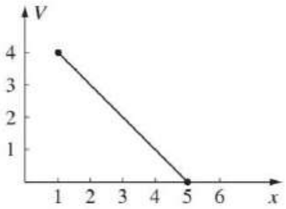


FIGURE 3.1

- V(x) is the average of V(x + a) and V(x − a), for any a:

$$V(x) = \frac{1}{2}[V(x+a) + V(x-a)].$$

  Laplace's equation is a kind of averaging instruction; it tells you to assign to the point x the average of the values to the left and to the right of x.

- Laplace's equation tolerates no local maxima or minima; extreme values of V must occur at the end points. Actually, this is a consequence of (1), for if there were a local maximum, V would be greater at that point than on either side, and therefore could not be the average. (Ordinarily, you expect the second derivative to be negative at a maximum and positive at a minimum.

Laplace's Equation in Two Dimensions:

If V depends on two variables, Laplace's equation becomes

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0.$$

- The value of V at a point (x, y) is the average of those around the point. More precisely, if you draw a circle of any radius R about the point (x, y), the average value of V on the circle is equal to the value at the center:

$$V(x, y) = \frac{1}{2\pi R} \oint_{\text{circle}} V\, dl.$$

- V has no local maxima or minima; all extrema occur at the boundaries. (As before, this follows from the equation). Again, Laplace's equation picks the most featureless function possible, consistent with the boundary conditions: no hills, no valleys, just the smoothest conceivable surface. For instance, if you put a ping-pong ball on the stretched rubber sheet of the following Figure shown below, it will roll over to one side and fall off—it will not find a "pocket" somewhere to settle into, for Laplace's equation allows no such dents in the surface. From a geometrical point of view, just as a straight line is the shortest distance between two points, so a harmonic function in two dimensions minimizes the surface area spanning the given boundary line.

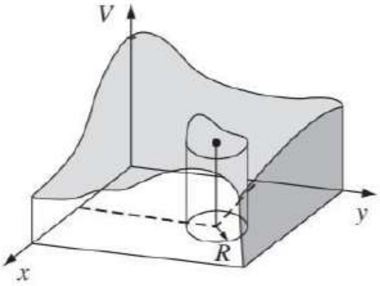


FIGURE 3.2

Laplace's Equation in Three Dimensions:

- The value of V at point r is the average value of V over a spherical surface of radius R centered at r:

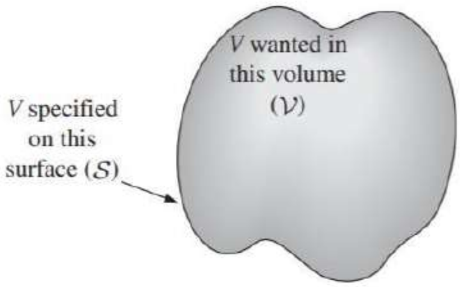
$$V(\mathbf{r}) = \frac{1}{4\pi R^2} \oint_{\text{sphere}} V\, da.$$

- As a consequence, V can have no local maxima or minima; the extreme values of V must occur at the boundaries. (For if V had a local maximum at r, then by the very nature of maximum I could draw a sphere around r over which all values of V—and a fortiori the average—would be less than at r.)

First Uniqueness Theorem:

The solution of the Laplace's Equation in some volume V is uniquely determined if V is specified on the boundary surface S.

- The following figure shown below has drawn such a region and its boundary. (There could also be "islands" inside, so long as V is given on all their surfaces; also, the outer



boundary could be at infinity, where V is ordinarily taken to be zero.) Suppose there were two solutions to Laplace's equation:

$$\nabla^2 V_1 = 0 \quad \text{and} \quad \nabla^2 V_2 = 0,$$

both of which assume the specified value on the surface. I want to prove that they must be equal. The trick is look at their difference:

$$\nabla^2 V_1 = 0 \quad \text{and} \quad \nabla^2 V_2 = 0,$$

This obeys Laplace's equation,

$$\nabla^2 V_3 = \nabla^2 V_1 - \nabla^2 V_2 = 0,$$

and it takes the value zero on all boundaries (since V1 and V2 are equal there). But Laplace's equation allows no local maxima or minima—all extrema occur on the boundaries. So the maximum and minimum of V3 are both zero. Therefore V3 must be zero everywhere, and hence

$V_1 = V_2$

We may as well as throw some charges which obeys poissons equation the argument will be the same

$$\nabla^2 V_1 = -\frac{1}{\epsilon_0}\rho, \qquad \nabla^2 V_2 = -\frac{1}{\epsilon_0}\rho,$$

$$\nabla^2 V_3 = \nabla^2 V_1 - \nabla^2 V_2 = -\frac{1}{\epsilon_0}\rho + \frac{1}{\epsilon_0}\rho = 0.$$

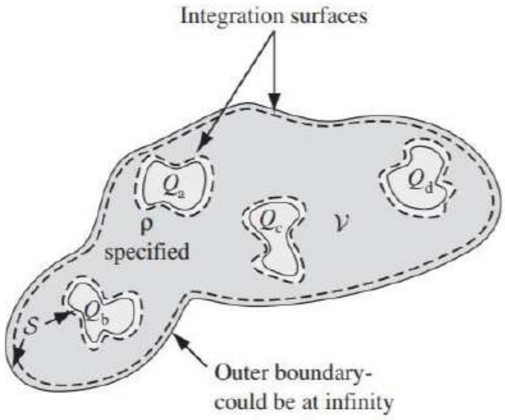
once again the difference V3=V2-V1 satisfies laplaces equation and has the value zero on the boundary.

Second uniqueness theorm:

In a volume V surrounded by conductors and containing a specified charge density ρ, the electric field is uniquely determined if the total charge on each conductor is given figure. (The region as a whole can be bounded by another conductor or else bounded)

Proof:

Suppose the fields satisfying the conditions of the problem



$$\nabla \cdot \mathbf{E}_1 = \frac{1}{\epsilon_0}\rho, \qquad \nabla \cdot \mathbf{E}_2 = \frac{1}{\epsilon_0}\rho.$$

And both obey Gauss's law in integral form for a Gaussian surface enclosing each conductor:

conductor:

$$\oint_{\substack{i\text{th conducting}\\ \text{surface}}} \mathbf{E}_1 \cdot d\mathbf{a} = \frac{1}{\epsilon_0} Q_i, \qquad \oint_{\substack{i\text{th conducting}\\ \text{surface}}} \mathbf{E}_2 \cdot d\mathbf{a} = \frac{1}{\epsilon_0} Q_i.$$

Likewise, for the outer boundary (whether this is just inside an enclosing conductor or at infinity),

$$\oint_{\substack{\text{outer}\\ \text{boundary}}} \mathbf{E}_1 \cdot d\mathbf{a} = \frac{1}{\epsilon_0} Q_{\text{tot}}, \qquad \oint_{\substack{\text{outer}\\ \text{boundary}}} \mathbf{E}_2 \cdot d\mathbf{a} = \frac{1}{\epsilon_0} Q_{\text{tot}}.$$

As before, we examine the difference

$$\mathbf{E}_3 \equiv \mathbf{E}_1 - \mathbf{E}_2,$$

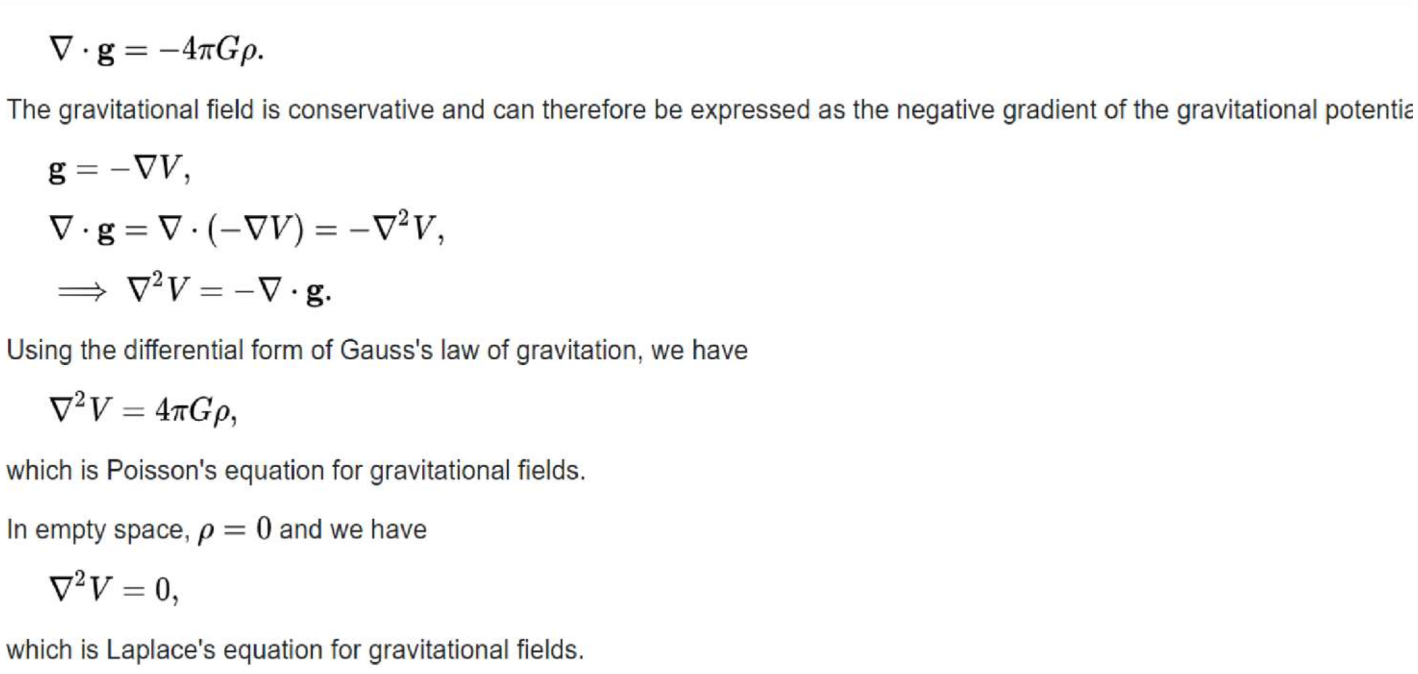
which obeys

$$\nabla \cdot \mathbf{E}_3 = 0 \tag{3.7}$$

$$\int_{\mathcal{V}} (E_3)^2 \, d\tau = 0.$$

But this integrand is never negative; the only way the integral can vanish is if $E_3 = 0$ everywhere. Consequently, $\mathbf{E}_1 = \mathbf{E}_2$, and the theorem is proved. ☐

Laplace equation in spherical polar co-ordinate:



$$\nabla \cdot \mathbf{g} = -4\pi G\rho.$$

The gravitational field is conservative and can therefore be expressed as the negative gradient of the gravitational potentia

$$\mathbf{g} = -\nabla V,$$

$$\nabla \cdot \mathbf{g} = \nabla \cdot (-\nabla V) = -\nabla^2 V,$$

$$\implies \nabla^2 V = -\nabla \cdot \mathbf{g}.$$

Using the differential form of Gauss's law of gravitation, we have

$$\nabla^2 V = 4\pi G\rho,$$

which is Poisson's equation for gravitational fields.

In empty space, $\rho = 0$ and we have

$$\nabla^2 V = 0,$$

which is Laplace's equation for gravitational fields.

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2} = 0.$$

$$V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta).$$

Laplace equation in cylindrical co-ordinate:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} + \frac{\partial^2\phi}{\partial z^2} = 0.$$

$$\phi(r,\theta,z) = \sum_{m=0,\infty}\sum_{n=1,\infty}\sinh(j_{mn}z/a)\,J_m(j_{mn}r/a)\left[A_{mn}\cos(m\theta) + B_{mn}\sin(m\theta)\right].$$

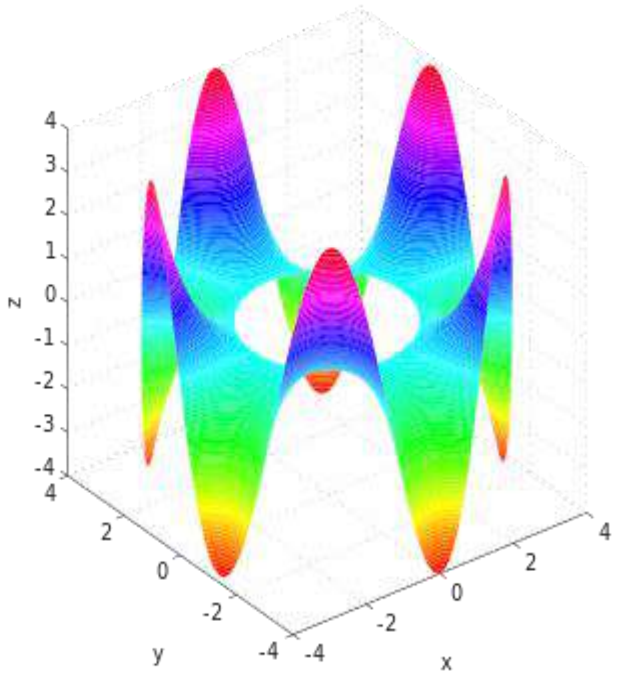
In the Schwarzschild metric:

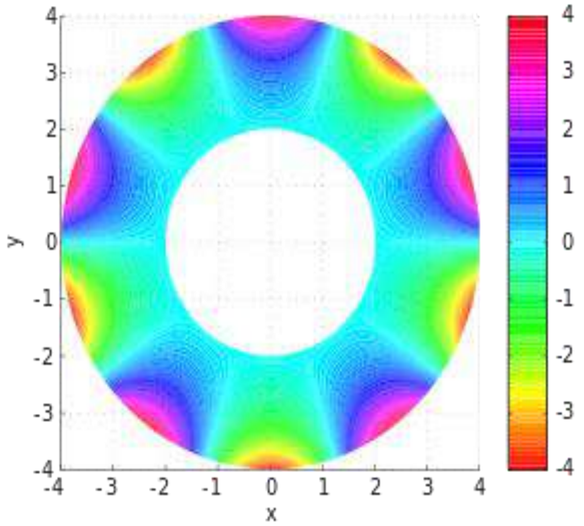
S. Persides solved the Laplace equation in Schwarzschild spacetime on hypersurfaces of constant t. Using the canonical variables r, θ, φ the solution is

where Yl(θ, φ) is a spherical harmonic function, and

$$R(r) = (-1)^l\frac{(l!)^2 r_s^l}{(2l)!}P_l\left(1 - \frac{2r}{r_s}\right) + (-1)^{l+1}\frac{2(2l+1)!}{(l)!^2 r_s^{l+1}}Q_l\left(1 - \frac{2r}{r_s}\right).$$

Here **Pl** and **Ql** are Legendre functions of the first and second kind, respectively, while **rs** is the Schwarzschild radius. The parameter l is an arbitrary non-negative integer.(references:Wikipedia and David J Grifith).





ACKNOWLEDGEMENT

The success and the final outcome of the project required a lot of guidance and assistance of many people and I extremely privileged to have got this all along the completion of my project. All that I have done is only due to such supervision and Assistance and I would not forget to thank them. I respect and thank our Principal, Swami Kamalasthananda and Vice Principal, Swami Vedanuragananda for providing me an opportunity to do the project work. I am extremely thankful to the Head of the Department of Physics, Prof. Asok Kumar Pal for providing such a nice support and guidance although he had a very busy schedule.

I owe my deep gratitude to our project guide Prof. Asok kumar pal , who took keen interest on my project work and guided me all along, till the completion of the project work by providing all the necessary information for developing a good system. I heartily thank all my friends and classmates for their encouragement and more over for their timely support and guidance till the completion of my project work. I am thankful to and fortunate enough to get constant encouragement, support and guidance from all Teaching staffs of RKMVCC Physics Department which helped me in successfully completing my project work.

Anjan Jana, Physics Department

REFERENCES

1.INTRODUCTION TO ELECTRODYNAMICS- by DAVID J GRIFITH

2.BASIC ELECTRODYNAMICS- by RK BHATYACHARYA

3.CLASSICAL ELECTRODYNAMICS - by JOHN DAVID JACKSON

Faraday's Law: A Revolution in our day-to-day life

The project submitted, in partial fulfilment of the requirement for the assignments in CC-XI, CC-XII, DSE-I, DSE-II Paper (Semester V) in the Department of Physics

Submitted by

SAYAN JANA

Registration No:

A01-1112-111-002-2019

.

Supervisor Teacher:

Prof. CHANDAN KUMAR DAS

RAMAKRISHNA MISSION VIVEKANANDA CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

ACKNOWLEDGEMENT

In the accomplishment of this project successfully, many people have given me their heart pledged support. I would like to thank all the people who have helped me in this project. I would like to thank our Principal, Swami KAMALASTHANANDA MAHARAJ. I would like to thank our Physics faculty Dr. CHANDAN KUMAR DAS, whose valuable guidance has helped me to complete this project successfully. His instructions and suggestions helped me very much. Besides I would like to thank my parents and friends who have helped me with their valuable suggestions and guidance. These all helped me in completion of this project.

FARADAY'S LAW: A REVOLUTION IN OUR DAY-TO-DAY LIFE

For whom modern civilization gets so much benefits & luxury, he is none but Michael Faraday because the modern instruments are based on electricity. The electricity is produced mainly by motional emf and induced emf .we will discuss it later on elaborately.

HISTORY

Faraday's law was originally an experimental law based upon observations. Later it was formalized, and along with the other laws of electromagnetism a partial time derivative restricted version of it was incorporated into the modern Heaviside versions of Maxwell's equations. Faraday's law of induction is based on Michael Faraday's experiments in 1831.The effect was also discovered by Joseph Henry at about the same time, but Faraday published first.

Lenz's law, formulated by Baltic German physicist Heinrich Lenz in 1834, gives the direction of the induced electromotive force and current resulting from electromagnetic induction.

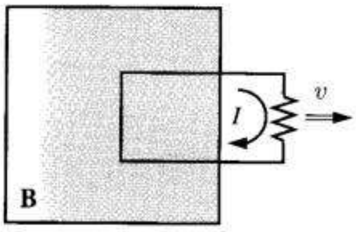
EXPERIMENTS & OBSERVATION:

In 1831 Michael Faraday reported on a series of experiments ,including 3 that (withsome violence to history) can be characterised as follows.

For this experiments Faraday used a magnetic field & a closed loop

EXPERIMENT 1:

He pulled the loop of wire to the right through a magnetic field & a current flowed in the indicated direction like the diagram (a).



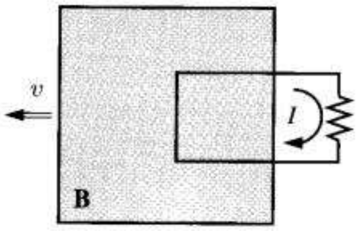
(a)

EXPERIMENT 2:

At this time he changed his experimental procedure slightly .

He now pulled that magnetic field with some velocity keeping the loop still at rest.

at this time also he observed there is a current in that loop.

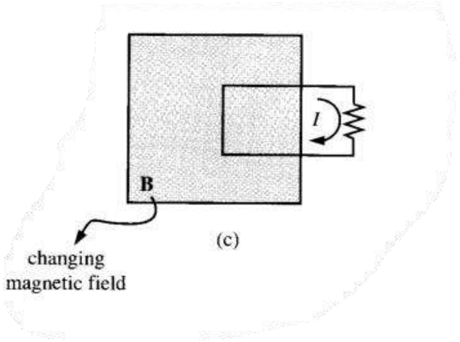


(b)

EXPERIMENT 3:

When he kept both the magnet and the loop fixed but he change the magnetic field line by changing the current in a electromagnet.

He noticed same result which is some sort of unexpected. He observed that there was a current flowing through the loop.



CONCLUSION:

Here basically from experiment 1 the obtained result is expected .When he pulled the loop a charge on the side of wire ,which is under the magnetic field ,feel a force (Lorentz force).

$\vec{F}$=q($\vec{V}$×$\vec{B}$)

The charges on the other two sides will not effectively move due to Lorentz force .The emf is the integral of force per unit charge .hence, $e = \oint f.dl$

=vBl

But the next 2 experiments gives some unexpected result .though the experiment 2 was really a maater of relative motion of the magnet and the loop but that time special theory of relativity was not discovered hence it was some sort of puzzle .

For next two experiments if the loop moves then the magnetic force will work but here no kinds of motion of the loop is observed hence no kinds of such Lorentz force will work.Here the stationary charges will not experience of Lorentz force. In that case what is responsible? What sort of field exert a force on charges at rest ? well, electric field do ,of course, but in this case there does not seem to ne any electric field inside ,here Faraday comes and he told that :

A CHANGING MAGNETIC FIELD INDUCES AN ELECTRIC FIELD

$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt},$$

Then E is related to the change in B

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\int \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{a}.$$

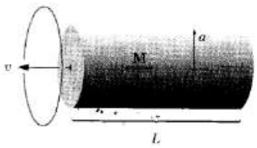
This is Faraday law an integral form .we can convert it into a differential form by stokes theorem

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.$$

Note that Faraday's law reduces to the old rule $\oint E$.dl =0 (or, in differential form, ∇x $\vec{E}$ = 0 in the static case (constant B) as, of course, it should, In Experiment 3, the magnetic field changes for entirely different reasons, but according to Faraday's law an electric field will again be induced, giving rise to an emf -d/dt. Indeed, one can subsume all three cases (and for that matter anu combination of them) into a kind of universal flux rule: Whenever (and for whatever reason) the magnetic flux through a loop changes, an emf $\varepsilon = -\frac{d\phi}{dt}$ will appear in the loop.

LENZ’S LAW

Now we know the cause of induction from faraday’s law but the million dollar question is what will be the direction of the induced emf. Now Lenz came and gives us the right way to our problem.



Let’s take an example like a long cylinder magnet of length L and radius a .it passes at constant velocity v through a circular wire of slightly greater diameter like the diagram.

Experimentally we will observe that when the magnet is very close to the ring there will be an induced emf and when the magnet is coming out of that ring there will also some emf. The induced current will flow in such a direction that the flux it produces tends to cancel the change. Notice that it is the change in flux, not the flux it- self, that nature abhors (when the tail end of the magnet exits the ring, the flux drops, so the induced current flows counter clockwise, in an effort to restore it). Faraday induction is a kind of "inertial" phenomenon: A conducting loop "likes" to maintain a constant flux through it; if I try to change the flux, the loop responds by sending a current around in such a direction as to frustrate my efforts. (It doesn't succeed completely; the flux produced by the induced current is typically only a tiny fraction of the original. All Lenz's law tells you is the direction of the flow. Lenz’s law states that the direction of the induced current in the loop will be such that it will oppose its own cause creation

Lets take a beautiful example of a jumping coil. The " jumping ring " demonstration . If I wind a solenoid around an iron core (the iron is there to beef up the magnetic field , place a metal ring on top , and plug it in , the ring will jump several feet in the air.

Solution: Before I turned on the current , the flux through the ring was zero . Afterward flux appeared (upward , in the diagram) , and the emf generated in the ring led current (in the ring) which , according to Lenz's law , was in such a direction its field tended to cancel this new flux . This means that the current in the loop opposite to the current in the solenoid . And opposite currents repel ,so it flies off .

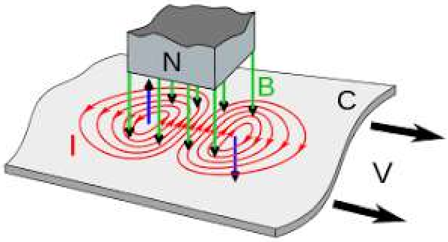
to the resistivity of the conductor.

EDDY CURRENT:

HISTORY:

Eddy currents were first observed in 1824 by scientist and then Prime Minister of France, François Arago. He realized that it was possible to magnetize most conductive objects and was the first to witness rotary magnetism. Ten years later, Lenz’s Law was postulated by Heinrich Lenz, but it wasn’t until 1855 that the French physicist Léon Foucault officially discovered eddy currents.

A BRIEF DISCUSSION ON EDDY CURRENT:



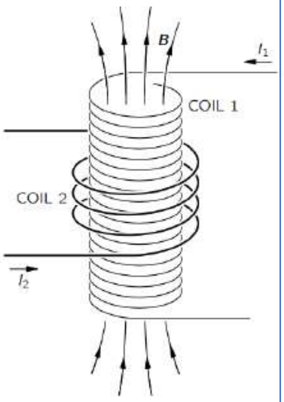
Eddy currents are currents which circulate in conductors like swirling eddies in a stream. They are induced by changing magnetic fields and flow in closed loops, perpendicular to the plane of the magnetic field. According to **Faraday’s law ,**They can be created when a conductor is moving through a magnetic field, or when the magnetic field surrounding a stationary conductor is varying i.e. anything which results in the conductor experiencing a change in the intensity or direction of a magnetic field can produce eddy currents. The size of the eddy current is proportional to the size of the magnetic field, the area of the loop and the rate of change of magnetic flux, and inversely proportional to the size of the magnetic field, the area of the loop and the rate of change of magnetic flux, and inversely proportional to the resistivity of the conductor.

There are numerous applications of eddy currents in our day-to-day life. We will discuss it later on .

MUTUAL INDUCTION:

Suppose two closed circuits are placed close to each other and a current i is passed in one. It produces a magnetic field and this field has a flux Φ through the area bounded by the other circuit. As the magnetic field at a point is proportional to the current producing it, we can write

$$\Phi = Mi$$



where M is a constant depending on the geometrical shapes of the two circuits and their placing. This constant is called mutual inductance of the given pair of circuits. If the same current i is passed in the second circuit and the flux is calculated through the area bounded by the first circuit, the same proportionality constant M appears. If there are more than one turns in a circuit, one has to add the flux through each turn before applying equation . If the current i in one circuit changes with time, the flux through the area bounded by the second circuit also changes.Hence from **Faraday's law** we can say, an emf is induced in the second circuit . This phenomenon is called mutual induction. From equation, the induced emf is do E =-dΦ/dt

$$= -M\, di/dt$$

In the diagram, shown on the right side of this page, an arrangement of two coils which demonstrates the basic effects responsible for the operation of a transformer. Coil 1 consists of a conducting wire wound in the form of a long solenoid. Around this coil—and insulated from it—is wound coil 2, consisting of a few turns of wire. If now a current is passed through coil 1, we know that a magnetic field will appear inside it. This magnetic field also passes through coil 2. As the current in coil 1 is varied, the magnetic flux will also vary, and there will be an induced emf in coil 2.

We will now calculate the mutual inductance.
magnetic field inside a long solenoid is uniform & is given below

B2=$\mu_0 n_2 I_2$

Where
I_2 =current in coil 1

, n_2=no of turn per unit length in coil 2

Due to that, flux through each loop will be in coil 1 , ϕ_1=B2*A1.so,total flux through total l1 length of coil 1, ϕ=n1*l1*ϕ_1.we know that

M12 =ϕ/I_2=$n_2 n1 \mu_0 \pi a^2 l1$

Here,n1=no of turns per unit length in coil 1

And a, l1 are the radius and the length of coil 1 respectively.

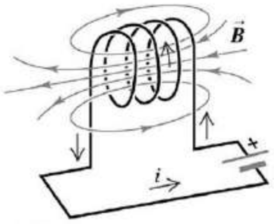
And m12 is the mutual inductance of the coils

SELF INDUCTION:

Self-induction is the property of the current-carrying coil that resists or opposes the change of current flowing through it. This occurs mainly due to the self-induced emf produced in the coil itself. In simple terms, we can also say that self-inductance is a phenomenon where there is the induction of a voltage in a current-carrying wire.

The self-induced emf present in the coil will resist the rise of current when the current increases and it will also resist the fall of current if the current decreases. In essence, the direction of the induced emf is opposite to the applied voltage if the current is increasing and the direction of the induced emf is in the same direction as the applied voltage if the current is falling.

The above property of the coil exists only for the changing current which is the alternating current and not for the direct or steady current. Self-inductance is always opposing the changing current and is measured in Henry (SI unit).



Induced current always opposes the change in current in the circuit, whether the change in the current is an increase or a decrease. Self-inductance is a type of electromagnetic induction.

APPLICATION OF ELELCTROMAGNETIC INDUCTION:

There are several application of electromagnetic induction in our day to day life .I will discuss here some of them.

The principles of electromagnetic induction are applied in many devices and systems, including:

Induction Sealing

Induction motors

Electrical generators

Transformers

Contactless charging of rechargeable batteries

The 6.6kW Magne Charge system for Battery electric vehicles

Induction cookers

Induction welding

Inductors

Electromagnetic forming

Magnetic flow meters

Transcranial magnetic stimulation

Faraday Flashlight

Graphics tablet

Wireless energy transfer

Electric Guitar Pickups

Hall effect meters

Current transformer meters

Clamp meter

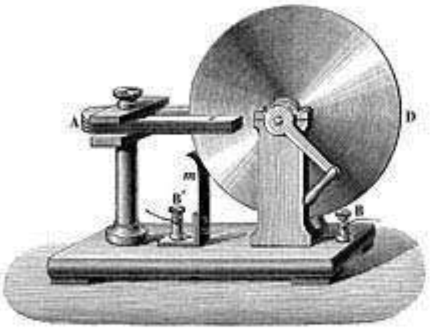
Audio and video tapes

Now I will discuss some of them in a describetic way.

ELELCTRICAL GENERATOR:

Faraday's disc electric generator. The disc rotates with angular rate ω, sweeping the conducting radius circularly in the static magnetic field B. The magnetic Lorentz force v × B drives the current along the conducting radius to the conducting rim, and from there the circuit completes through the lower brush and the axle supporting the disc. Thus, current is generated from mechanical motion.

The EMF generated by Faraday's law of induction due to relative movement of a circuit and a magnetic field is the phenomenon underlying electrical generators. When a permanent magnet is moved relative to a conductor, or vice versa, an electromotive force is created. If the wire is connected through an electrical load, current will flow, and thus electrical energy is generated, converting the mechanical energy of motion to electrical energy. For example, the drum generator is based upon . A different implementation of this idea is the Faraday's disc, shown in simplified form in Figure 8. Note that either the analysis of Figure 5, or direct application of the Lorentz force law, shows that a solid conducting disc works the same way.



In the Faraday's disc example, the disc is rotated in a uniform magnetic field perpendicular to the disc, causing a current to flow in the radial arm due to the Lorentz force. It is interesting to understand how it arises that mechanical work is necessary to drive this current. When the generated current flows through the conducting rim, a magnetic field is generated by this current through Ampere's circuital law .The rim thus becomes an electromagnet that resists rotation of the disc (an example of Lenz's law). On the far side of the figure, the return current flows from the rotating arm through the far side of the rim to the bottom brush. The B-field induced by this return current opposes the applied B-field, tending to decrease the flux through that side of the circuit, opposing the increase in flux due to rotation. On the near side of the figure, the return current flows from the rotating arm through the near side of the rim to the bottom brush. The induced B-field increases the flux on this side of the circuit, opposing the decrease in flux due to rotation. Thus, both sides of the circuit generate an emf opposing the rotation. The energy required to keep the disc moving, despite this reactive force, is exactly equal to the electrical energy generated (plus energy wasted due to friction, Joule heating, and other inefficiencies). This behavior is common to all generators converting mechanical energy to electrical energy.

Although Faraday's law always describes the working of electrical generators, the detailed mechanism can differ in different cases. When the magnet is rotated around a stationary conductor, the changing magnetic field creates an electric field, as described by the Maxwell-Faraday equation, and that electric field pushes the charges through the wire. This case is called an induced EMF. On the other hand, when the magnet is stationary and the conductor is rotated, the moving charges experience a magnetic force (as described by the Lorentz force law), and this magnetic force pushes the charges through the wire. This case is called motional EMF. (For more information on motional EMF, induced EMF, Faraday’s law, and the Lorentz force.

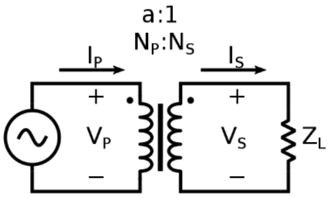
ELELCTRIC MOTOR:

An electrical generator can be run “backwards” to become a motor. For example, with the Faraday disc, suppose a DC current is driven through the conducting radial arm by a voltage. Then by the Lorentz force law, this traveling charge experiences a force in the magnetic field B that will turn the disc in a direction given by Fleming’s left hand rule.

ELELCTRIC TRANSFORMER:

An ideal transformer is a theoretical linear transformer that is lossless and perfectly coupled. Perfect coupling implies infinitely high core magnetic permeability and winding inductance and zero net magnetomotive force Ideal transformer connected with source VP on primary and load impedance ZL on secondary, where 0 < ZL < ∞.

Ideal transformer and induction law



A varying current in the transformer's primary winding attempts to create a varying magnetic flux in the transformer core, which is also encircled by the secondary winding. This varying flux at the secondary winding induces a varying electromotive force (EMF, voltage) in the secondary winding due to electromagnetic induction and the secondary current so produced creates a flux equal and opposite to that produced by the primary winding, in accordance with Lenz's law.

The windings are wound around a core of infinitely high magnetic permeability so that all of the magnetic flux passes through both the primary and secondary windings. With a voltage source connected to the primary winding and a load connected to the secondary winding, the transformer currents flow in the indicated directions and the core magnetomotive force cancels to zero.

According to Faraday's law, since the same magnetic flux passes through both the primary and secondary windings in an ideal transformer, a voltage is induced in each winding proportional to its number of windings. The transformer winding voltage ratio is directly proportional to the winding turns ratio.

The ideal transformer identity is a reasonable approximation for the typical commercial transformer, with voltage ratio and winding turns ratio both being inversely proportional to the corresponding current ratio.

The load impedance referred to the primary circuit is equal to the turns ratio squared times the secondary circuit load impedance.

MAGNETIC FLOW METER:

Faraday’s law is used for measuring the flow of electrically conductive liquids and slurries. Such instruments are called magnetic flow meters. The induced voltage E

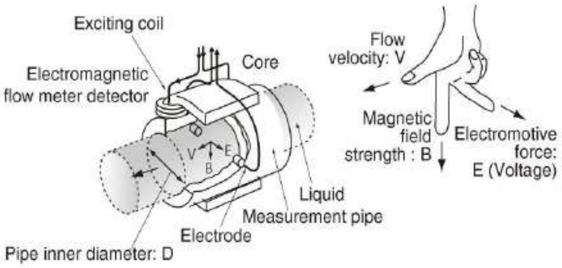
E is proportional to V x B x D where:

E = The voltage generated in a conductor

V = The velocity of the conductor

B = The magnetic field strength

D = The length of the conductor



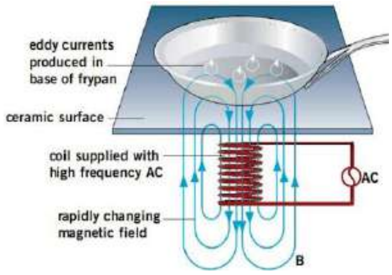
<u>WORK PROCEDURE</u>:

Magnetic flow meters use a magnetic field to generate and channel liquid flow through a pipe. A voltage signal is created when a conductive liquid flows through the flowmeter’s magnetic field. The faster the flow of the fluid, the greater the voltage signal generated. Electrode sensors located on the flow tube walls pick up the voltage signal and send it to the electronic transmitter, which processes the signal to determine liquid flow.

INDUCTION COOKER:

Induction heating is the process of heating electrically conductive materials like metals by electromagnetic induction, through heat transfer passing through an induction coil that creates an electromagnetic field within the coil to melt down steel, copper, brass, graphite, gold, silver, aluminium, and carbide. An induction heater consists of an electromagnet and an electronic oscillator that passes a high-frequency alternating current (AC) through the electromagnet. The rapidly alternating magnetic field penetrates the object, generating electric currents inside the conductor, called eddy currents. The eddy currents flow through the resistance of the material, and heat it by Joule heating. In ferromagnetic and ferrimagnetic materials, such as iron, heat also is generated by magnetic hysteresis losses. The frequency of the electrical current used for induction heating depends on the object size, material type, coupling (between the work coil and the object to be heated) and the penetration depth.

An important feature of the induction heating process is that the heat is generated inside the object itself, instead of by an external heat source via heat conduction. Thus objects can be heated very rapidly. In addition, there need not be any external contact, which can be important where contamination is an issue. Induction heating is used in many industrial processes, such as heat treatment in metallurgy, Czochralski crystal growth and zone refining used in the semiconductor industry, and to melt refractory metals that



require very high temperatures. It is also used in induction cook-tops for heating containers of food; this is called induction cooking.

AC Generator

An electrical generator is a device that converts mechanical energy to electrical energy.

The generation of alternating currents (ac) is based on the phenomenon of electromagnetic induction.

Whenever the magnetic flux changes an emf will be induced in the coil.

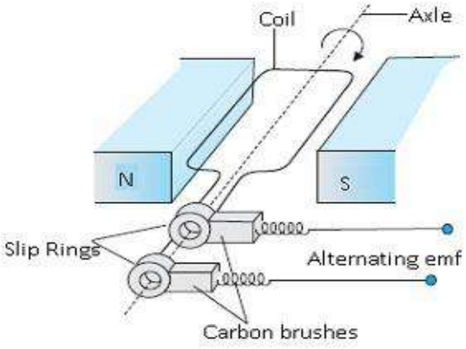
AC generator is a device that converts mechanical energy into electrical energy(alternating currents).

AC generator is an example of inducing an emf or current in a loop by varying the magnetic field and area vector.

Principle:-

Current is induced in a loop through a change in its orientation or a change in its effective area.

Induced emf is produced either by changing θ or by changing the area vector.Direction of current is given by Fleming's right hand rule.



Direction of the current in the circuit changes by the up and down movement of the loops.

Construction:-

It consists of 2 poles(north and south) of a magnet in order to have uniform magnetic field.

There is a coil of rectangular shape also known as armature.

The armature is connected to 2 slip rings.

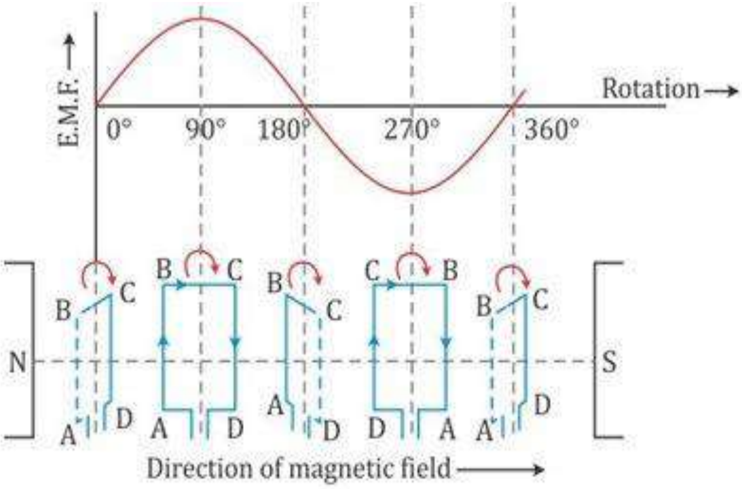
Slip rings helps in electrical contact with the brushes.It does not change current direction.

The slip rings provide a continuous connectionwith the wire around the armature.

These slip rings are attached to carbon brushes.

The rectangular coil is capable of rotating about an axis which is perpendicular to the magnetic field. In this way,we generate AC sinusoidal E.M.F.

The axis of rotation is known as axle.



These all are the important uses of Faraday’s law in our modern day-to-day life.

Discussion

1.We all know that we got ac supply in our homes and we got this supply by transmitting ac over long distances. AC can be transmitted using step up transformers but direct current or dc can not be transmitted by this method.

2.The ac is easy to generate than dc.

3.It is cheaper to generate ac than dc.

4.The ac generators have higher efficiency than dc.

5.The loss of energy during transmission is negligible for ac.

6.The ac can be easily converted into dc.

7.The variation of ac can easily be done using transformers either step up or step down.

8.The value or magnitude of ac can be decreased easily without loss of excess of energy. This can be done by using choke coil.

These are the benefits of ac current (Basiclly comes from faraday's law) over dc current.

As every thing in this world has a good and a bad side like that ac current also has also some bad aspects., There is a radiation from ac current whereas from dc it is negligible. That radiation is very harmful to human body. Besides that, when ac current flows through the human body it effects immensely than dc. However we can't ever neglect the diverge advantages of ac in our modern day to day life to make our life more pleasant and luxury. In this way Faraday really make a revolution for the future generations of human beings ever!

BIBLIOGRAPHY

The Feynman lectures on physics vol_2

Introduction to electrodynamics (Fourth edition) by David J Griffith)

Wikipedia

Classical Electrodynamics (3rd edition) by John David Jackson)

https://www.elprocus.com/what-is-self-inductance-theory-factors-its-applications/

https://university.listenlights.com/2018/02/07/faraday-law-of-electromagnetic-induction/

BASIC MATRIX FORMULATION OF GEOMETRICAL OPTICS

The project submitted, in partial fulfilment of the requirement for the assignments in CC-XI, CC-XII,DSE-I,DSE-II Paper (Semester V) in the Department of Physics

Submitted by

Rishav Saha

Registration No: A01-1112-111-003-2019

Supervisor Teacher: Atisdipankar Chakrabarti

RAMAKRISHNA MISSION VIVEKANANDA CENTENARY COLLEGE

P.O. RAHARA, KOLKATA-700118

WEST BENGAL, INDIA

PROJECT WORK

Basic Matrix Formulation of Geometrical Optics

Rishav Saha ,BSc. Physics
Department (5th Semester),
Reg. No.-A01-1112-111-003-2019
Roll no.-105

HISTORY OF THE SUBJECT

The optics of paraxial imaging is often referred to as Gaussian optics, since it was *Carl Frederick Gauss* who in 1840 laid the foundations of the subject. In his classical memoir, *Dioptrische Untersuchung,* Gauss showed that the behaviour of any lens system can be determined from knowledge of its six cardinal points - namely two focal points, two nodal points of unit angular magnification and two principal points of unit linear magnification. Included in the paper were recipes for experimentally determining the positions of these points and iterative methods for calculating them in terms of the surface curvatures, separations and refractive indices of the lens system. In formulating the latter, Gauss wrote down explicitly the two linear simultaneous equations whereby the ray height and ray angle of an output ray are linked to the corresponding quantities for an input ray. But matrix formalism was at that time unknown, and Gauss contented himself by using an algorithm which he had learnt from Euler to express the four coefficients of these equations in convenient computational form. (The expressions that he used, a shorthand form of continued fraction, are now known as 'Gaussian brackets'; they are by no means obsolete since very nearly the same economical course of calculation is used in the modern *'ynv'* method.) As we will see, matrices provide an alternative method for performing this type of calculation. It would seem that they were first used in optics by Sampson about sixty years ago, but it is only recently that they have been widely adopted. The first books on matrix methods were E.L. *O'Neill's, Introduction to Statistical Optics,* Addison-Wesley, Reading, Mass., 1963 and *W. Brouwer's, Matrix Methods in Optical Instrument Design, W.A*. Benjamin Inc., New York, 1964. During the next two years, discussions of the methods appeared in papers by *Halbach* and by *Richards* in the *American Journal of Physics,* but there was some dis26 agreement over the order in which the calculations should be arranged. During 1965 *Kogelnik* published an important extension of the method whereby a ray-transfer matrix could be used to describe not only the geometric optics of paraxial rays but also the propagation of a diffraction limited laser beam.

Here we will be focusing on the matrix methods in paraxial optics only.

Matrix Methods

In Paraxial Optics

Matrices can be useful to describe the geometric formation of images by a centred lens system - a succession of spherical refracting surfaces all centred on the same optical axis and reflection of light ray from a reflecting surface only within the limits of two main approximations.

- *The first is the basic assumption of all geometric optics* - that the wavelength of light is negligibly small compared to the dimension of the optical instruments (lens, mirror etc.) and that propagation of light can be described not in terms of wave fronts but in terms of individual rays. As can be shown by means of Huygens' construction, if light waves are allowed to travel without encountering any obstacles they are propagated along a direction which is normal to the wave front. The concept of a geometric ray is an idealization of this wave normal; in vector terms it can be thought of as the Poynting vector of the electromagnetic field, or as the gradient of the scalar function which describes the phase of the wave disturbance. A consequence of this is that each ray obeys Fermat's principle of least time; if we consider the neighbourhood of any short section of the ray path , the path which the ray chooses between a given entry point and a given exit point is that which minimizes the time taken.
- *Second approximation is that we shall consider only paraxial rays - those that remain close to the axis and almost parallel to it* so that we can use first-order approximations for the sins or tangents of any angles that are involved. Our treatment will therefore give no information about third-order effects such as spherical aberration or the oblique aberrations coma, astigmatism, field curvature and distortion.

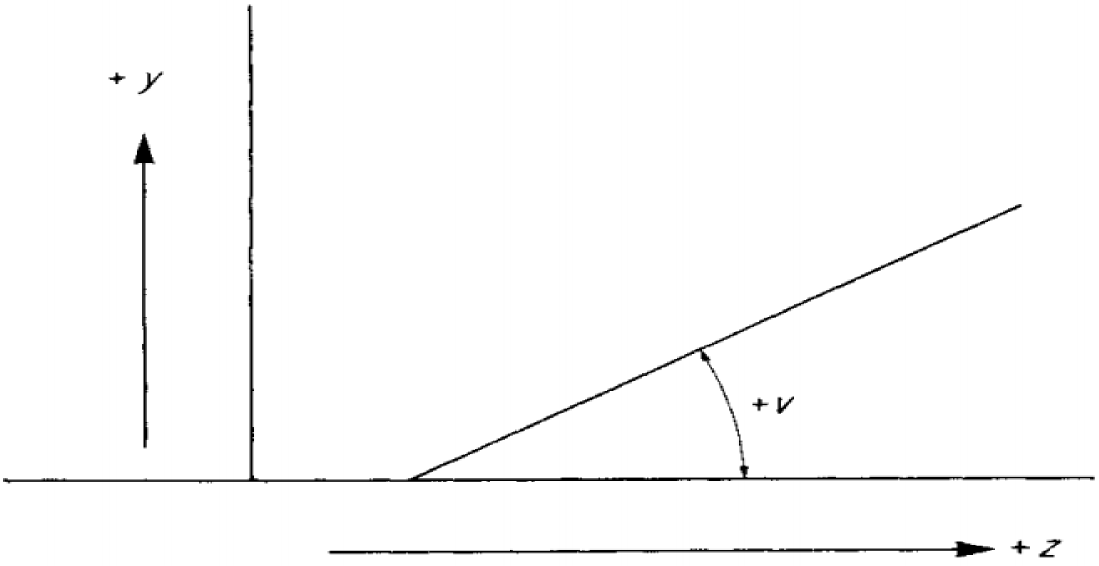
RAY-TRANSFER MATRICES

Let us now consider the propagation of a paraxial ray through a system of centred lenses. In conformity with modern practice we shall use a Cartesian coordinate system drawn so that *Oz*, which points from left to right, represents the optical axis of the system and also the general direction in which the rays travel. Of the transverse axes, *OY* will be taken to point upwards in the plane of the diagram and *Ox* will be assumed to point normal to the diagram and away from the reader.

The trajectory of a ray as it passes through the various refracting surfaces of the system will consist of a series of straight lines, each of which can be specified by stating

- the coordinates of one point of it
- Angle it makes with the z-axis.

If we choose in advance any plane perpendicular to the z-axis, a plane of constant a, we can call it a Reference Plane (RP). In terms of any particular reference plane we can then specify a ray by the height *y* at which it intersects the reference plane, and the angle *V* which it makes with the z-direction; *V* is measured in radians and is considered positive if an anticlockwise rotation turns a line from the positive direction of the z-axis to the direction in which light travels along the ray



Although we could attempt to describe all the rays encountered in a calculation by referring them all to a *single* reference plane (for example the RP, $z = 0$), it is in practice much more convenient to choose a new RP for each stage of the initial calculation. However, once this initial calculation has been done right through the system, we emerge with an overall ray-transfer matrix which will convert all the ray data we wish to consider from our chosen input

RP directly to our chosen output RP. It has already been pointed out that at any RP the specification of a ray can be made in terms of its ray height *y* and its ray angle *V.* It will, however, make the calculations more convenient if we replace the ray angle *V* by the corresponding optical direction-cosine *nv* (or, strictly speaking, *nsinv)* where *n is* the refractive index of the medium in which the ray *is* travelling. This optical direction cosine, which we shall denote by *V,* has the property that, by Snell's law, it will remain unchanged as it crosses a plane boundary between two different media.

We shall investigate the effect which each of these two basic elements has on the y-value and V-value of a ray passing between the two reference planes, one on either side of the element. The ray first passes through RP_1, with values

Y_1 and V_1, then through the element and then through RP_2, with values Y_2 and V_2. We seek equations giving Y_2 and V_2 terms of Y_1 and V_1 and the properties of the element between the reference planes. For both types of element the equations are linear and can therefore be written in the matrix form

$$\begin{pmatrix} Y2 \\ V2 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \times \begin{pmatrix} Y1 \\ V1 \end{pmatrix}$$

The matrix elements being such that the determinant

(AD - BC) equals unity. Alternatively, if we wish to trace a ray backwards, the matrix equation can be inverted to yield

$$\begin{pmatrix} Y1 \\ V1 \end{pmatrix} = \begin{pmatrix} D & -B \\ -C & A \end{pmatrix} \begin{pmatrix} Y2 \\ V2 \end{pmatrix}$$

Finally, *having verified that each element in a system can be represented by a ray-transfer matrix of this unimodular form, we shall multiply together in correct sequence all the elementary translation and refraction matrices to obtain the single ray-transfer matrix that represents the complete assembly; this may be anything from a single thin lens to a complicated optical system. This matrix is known as the system matrix for the optical system given*.

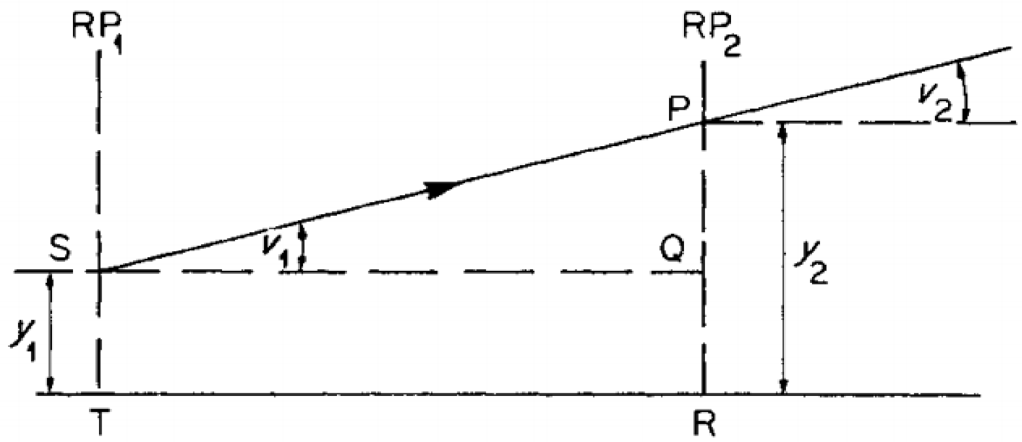
Now when a ray propagates through an optical system it undergoes only major two operations *(a) translation*

(b) Refraction.

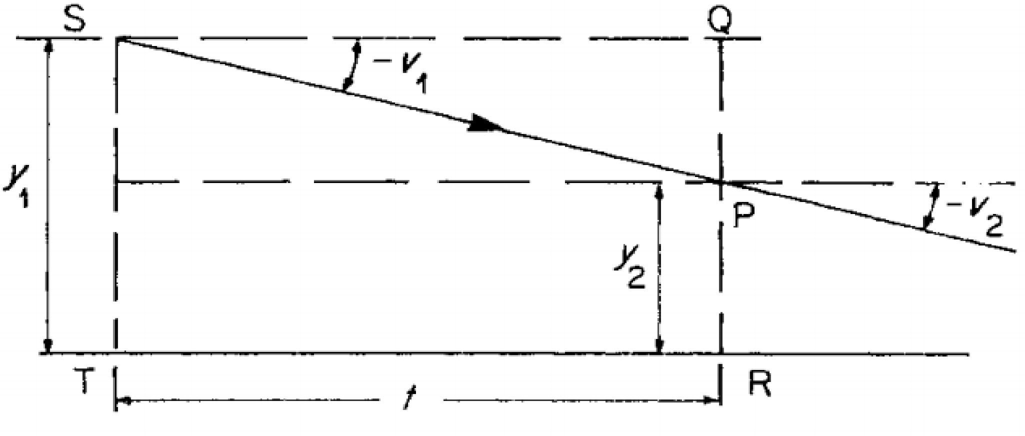
The rays undergo translation when they travel through a homogeneous medium however when it strikes an interface of two media it undergoes refraction. So, we can find out the system matrix for trajectory of a light ray by just finding out the translation and refraction matrices.

THE TRANSLATION MATRIX

The figures below show two examples of a ray which travels a distance t to the right between the two reference planes. Clearly the angle at which the ray *is* travelling will remain the same throughout the translation, but not so its distance from the axis. In both figures the angles *V* are shown exaggerated for clarity - in practice the maximum V-value for a paraxial ray will be less than 0·1 (one-tenth of a radian or about 6 degrees)



(a)



(b)

Referring to Figure (a) and figure (b)

Y2= RP= RQ + QP	*Y2* = RP= RQ - PQ
= TS + SQ tan PSQ	= TS+ SQ tan PSQ
= Y1 + t tan (v1)	= Y1 - t tan (-v1)
= Y1 + *tV1*	= Y1 + *tV1*

It has already been pointed out that the ray quantities upon which our translation matrix is going to operate are the height of a ray and its optical direction cosine or V-value, rather than just its angle *V*. So if *n* is the refractive index of the medium between RP1 andRP2, the above equation needs to be rewritten *Y2 = Y1 + (t/n) (nV1) = Y1+ TV1* where *T = (t/n)* is the 'reduced thickness' of the gap.

It is immediately apparent from both diagrams that *V1* and *V2* are equal, so the equation for the new optical direction cosine *V2* can be written

V2 = nV2 = nV1 = 0Y1 + nV1

The pair of equations that we have obtained can now be rewritten in matrix form

$$\begin{pmatrix} Y2 \\ V2 \end{pmatrix} = \begin{pmatrix} 1 & T \\ 0 & 1 \end{pmatrix} \begin{pmatrix} Y1 \\ V1 \end{pmatrix}$$

So the matrix representing a translation to the right through a reduced distance *T* is

$$\mathrm{T}=\begin{pmatrix} 1 & T \\ 0 & 1 \end{pmatrix}$$

Its determinant is obviously unity

Compound layers and plane- parallel plates

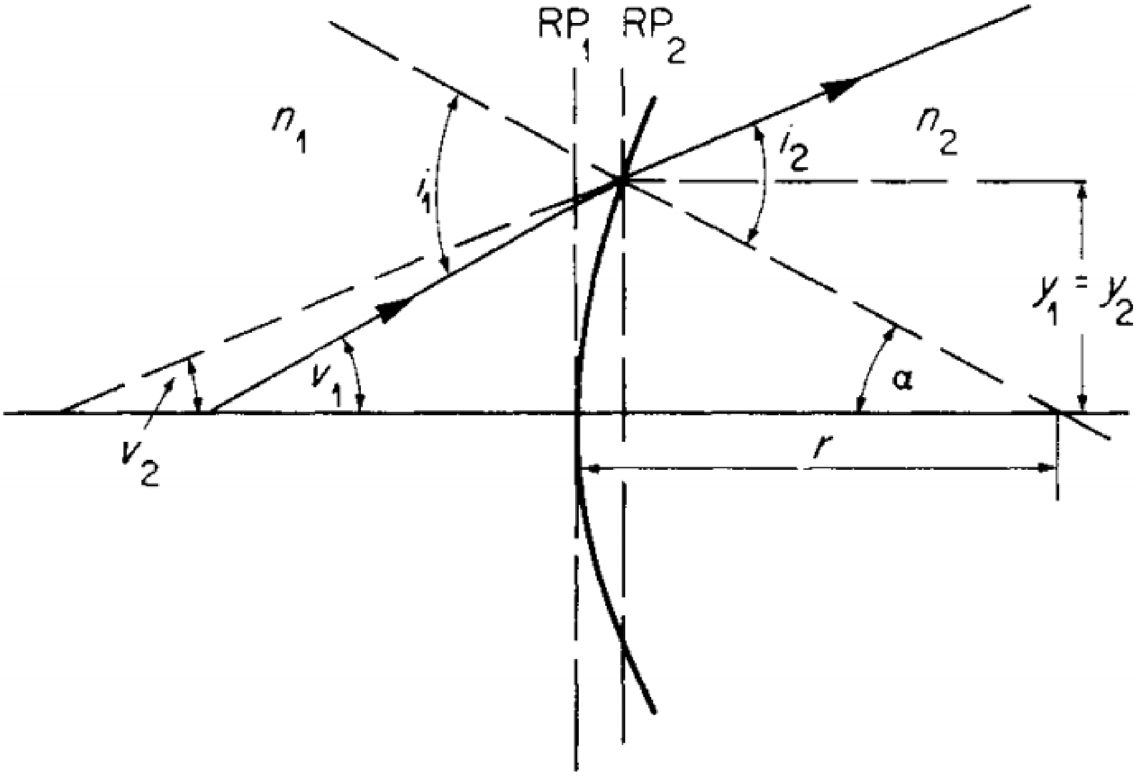
A similar situation applies where a given gap of total thickness t consists of several separate layers such that each ith layer has its own thickness t_i and its own refractive index n_i. Provided that all the boundary surfaces are flat and perpendicular to the axis, the ray height and the optical direction *cosine* will remain unchanged at each boundary. Then all the individual translation matrices can be multiplied together to produce a single matrix for the effect of the whole gap. These translation matrices have the useful property that the order in which they are multiplied together does not matter. The T-value appearing in the product matrix *is* just the sum of the T-values *in* the individual

$$\begin{pmatrix} 1 & T1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & T2 \\ 0 & 1 \end{pmatrix} \cdots \begin{pmatrix} 1 & Tn \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & \sum Ti \\ 0 & 1 \end{pmatrix}$$

Having seen that translation matrices produce the same product *in* whatever order they are taken, let us consider the optical corollary to this situation. If we are looking perpendicularly through a whole series of plane-parallel plates, then moving the plates or even interchanging them may affect the amount of light that is reflected, but the geometry of the transmitted images will remain exactly the same.

THE REFRACTION MATRIX

We now investigate the action of a curved surface separating two regions of refractive index n1 and *n2.* The radius of curvature of the surface will be taken as positive if the centre of curvature lies to the right of the surface.



As in Figure, for paraxial rays the separation between these two planes will be r (l - cosα) and therefore negligible since α is assumed to be a small angle like V1 and V2. We therefore have *Y2* = Y1·
Applying Snell's law in the diagram, we have

n1sin i1= n2 sin i2 or, n1.i1=n2.i2

But, by the exterior angle theorem,

i1 =v1+α   i2 =v2+α

=v1+ (Y1/r)   = v2+ (Y1/r)

Hence

n1 (V1 + *y1/r) = n2 (v2 + y1/r)*

Or, V1 + n1 (y1/r) = *V2* + *n2 (y1/r)*

So, rearranging the equations in matrix form, we obtain finally

$$\begin{pmatrix} Y2 \\ V2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -(n2-n1)/r & 1 \end{pmatrix} \begin{pmatrix} Y1 \\ V1 \end{pmatrix}$$

The quantity *(n2* – n1)/r is usually termed the refracting power of the surface. We have obtained the refraction matrix by considering one special case in which all the quantities are positive. But if a thorough analysis is made, investigating the other cases where the change of refractive index or the curvature is reversed, or where the *y*- or V-values are negative, the same refraction matrix will be found to give consistent and correct results for the change in ray direction that is produced.

Thin lens approximation

A similar telescoping of refraction matrices is possible when several refracting surfaces are so close together that the intervening gaps are negligible (in this case it is the translation matrix that degenerates into a unit matrix). If each ith refracting surface possesses a curvature r_i and refractive indices n_i and n_{i+1} we shall be able to represent its refracting power by Pi = $(n_{i+l} - n_i)/r_i$ and the telescoped matrix for the thin lens combination will be $\begin{pmatrix} 1 & 0 \\ -\sum Pi & 1 \end{pmatrix}$ independent of the order in which the individual powers are added.

The refraction matrix of a single thin lens is the same whichever way round it is mounted.

Its refracting power will be

$$P = P1 + P2 = (n-1)/r_1 + (1-n)/r_2$$
$$= (n-1).\ (1/r_1 - 1/r_2) = 1/f$$

$$f = \{(n-1)\ (1/r1 - 1/r2)\}^{-1}$$

f represents the focal length of the lens. The equation is well known as thin lens formula. Thus the system matrix for a thin lens is given by

$$S=\begin{pmatrix}1 & 0\\ -(\frac{1}{f}) & 1\end{pmatrix}$$

For a thick lens of thickness t the translation matrix does not become unit matrix and the system matrix takes the form

$$\begin{pmatrix}Y2\\V2\end{pmatrix}=\begin{pmatrix}1 & 0\\ -P2 & 1\end{pmatrix}\begin{pmatrix}1 & T\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ -P1 & 1\end{pmatrix}\begin{pmatrix}Y1\\V1\end{pmatrix}$$

Where P2= (n-1)/r_1 and P1= (1-n)/r_2= -P1 and T= (t/n)

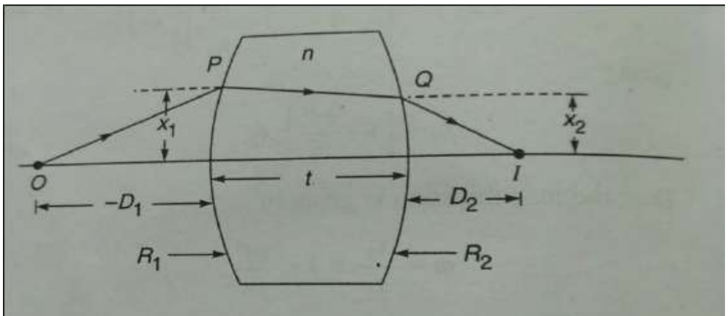
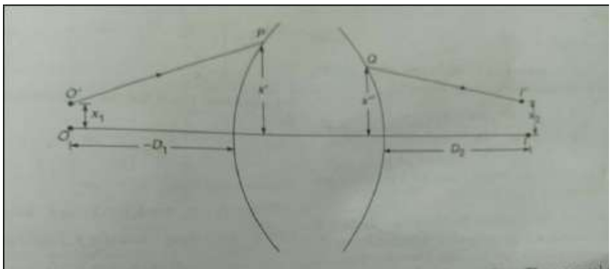


Fig. 5.6 A paraxial ray passing through a thick lens of thickness *t*.

IMAGING BY A COAXIAL OPTICAL SYSTEM



The object point *O* is at a distance ($-D_1$) from the first refracting surface. The paraxial
assumed to be formed at a distance D_2 from the last refracting surface.

We can derive the position of image plane for an object plane which is at distance –D1 as shown in the figure considering the sign convention the distance is negative for points on the left of the refracting surface. We just have to find the system matrix which would be of the form

$$\begin{matrix}Y2\\V2\end{matrix}=\begin{pmatrix}1 & D2\\ 0 & 1\end{pmatrix}\begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}1 & -D1\\ 0 & 1\end{pmatrix}$$

$$\begin{pmatrix} Y2 \\ V2 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} Y1 \\ V1 \end{pmatrix}$$

From considering B=0 as discussed in detail in the next section we will find the relationship between D2 and D1 in terms of a, b, c, d.

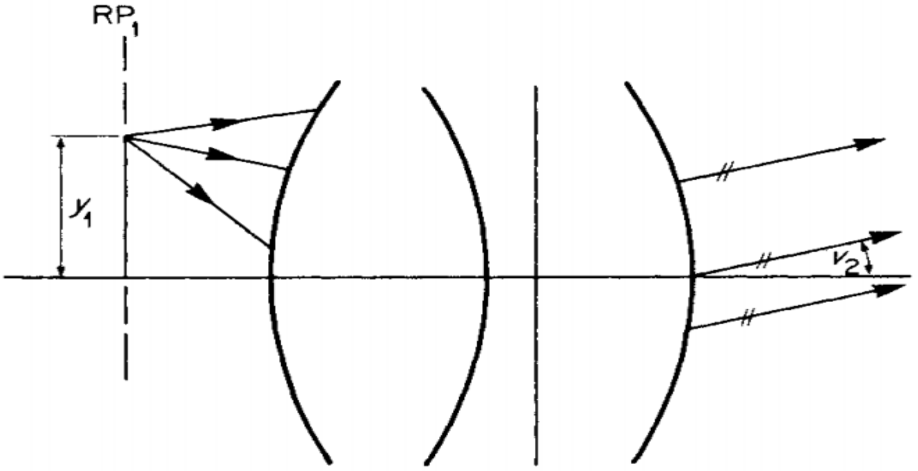
DERIVATION OF PROPERTIES OF A SYSTEM FROM ITS MATRIX

If the matrix M for a complete optical system is numerically evaluated and the equation can be written as

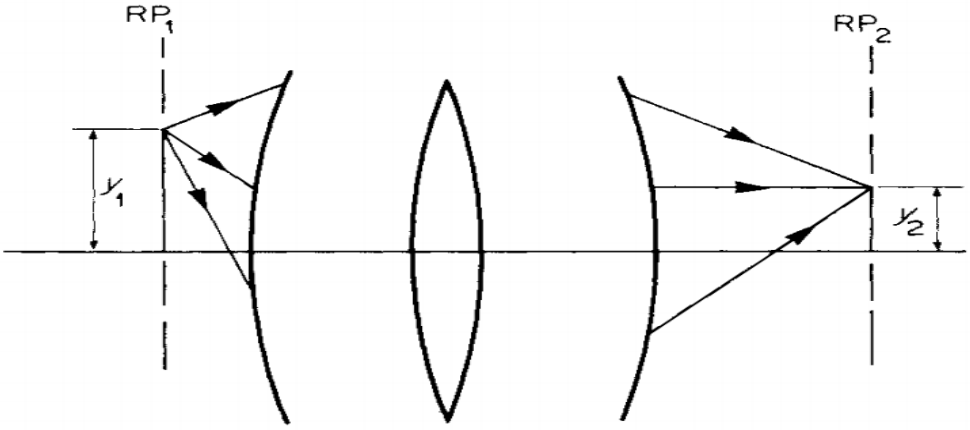
$$\begin{pmatrix} Y2 \\ V2 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} Y1 \\ V1 \end{pmatrix}$$

Where $(AD - BC) = 1$. In order to understand better the significance of these four quantities A, B, C and D, let us consider what happens if one of them vanishes.

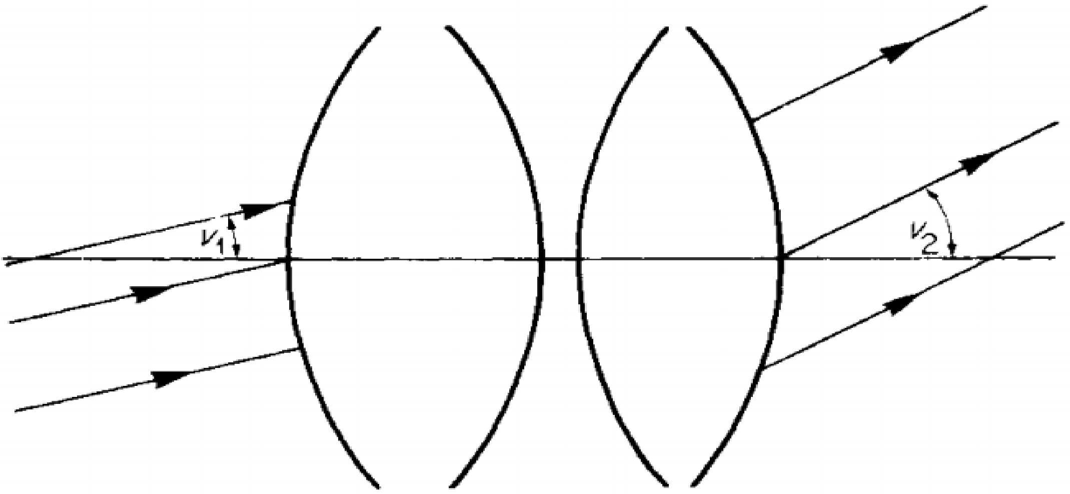
(a) If $D = 0$, the equation for V2 reads $V2 = CY1 + 0V1 = CY1$. This means that all rays entering the input plane from the same point (characterized by Y1) emerge at the output plane making the same angle $V2 = CY1$ with the axis, no matter at what angle V1 they entered the system. It follows that the input plane RP1 must be the first focal plane of the system



(b) If $B = 0$, the equation for Y2 reads $Y2 = AY1 + 0V1 = AY1$. This means that all rays leaving the point O (characterized by Y1) in RP1 will pass through the same point I (characterized by Y2) in RP2. Thus O and I are object and image points, so that RP1and *RP2* are now conjugate plane; In addition, A = Y2/Y1 gives the magnification produced by the system in these circumstances.

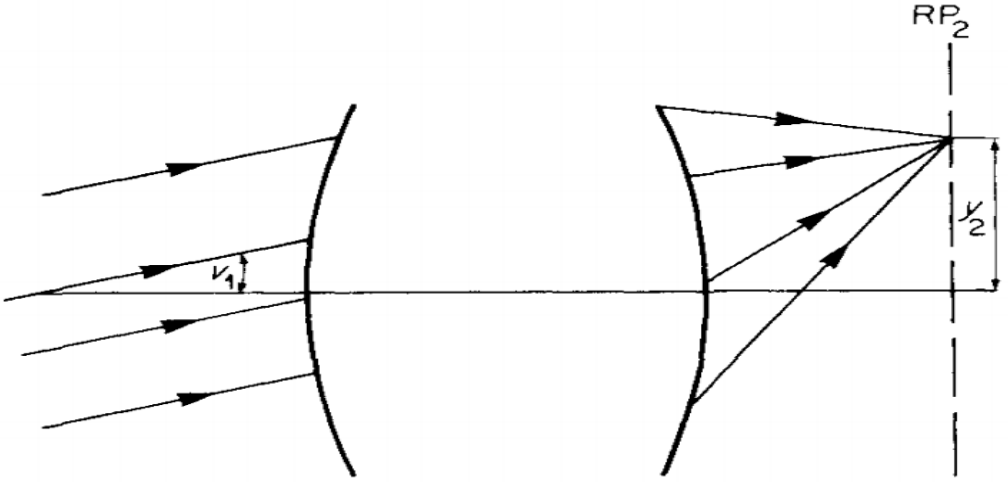


(c) If $C = 0$, then V2 = DV1. This means that all rays which enter the system parallel to one another (V1 gives their direction on entry) will emerge parallel to one another in a new direction V2. A lens system like this, which transforms a parallel beam into another parallel beam in a different direction, is called an afocal or telescopic system. In this case (n1D/n2) = (V2/V1) is the angular magnification produced by the system



(d) If A = 0, the equation for Y2 reads Y2 = BV1. This means that all rays entering the system at the same angle (characterized by V1) will pass through the same point (characterized by Y2) in the output plane RP2.

Thus, the system brings bundles of parallel rays to focus at points in RP2, which means RP2 is the second focal plane of the system.



(e) It is worth remembering that, if either A or D in a ray-transfer matrix vanishes, the equation (AD - BC) = 1 then requires that BC =-1. Likewise, if either B or C vanishes, A must be the reciprocal of D.

It should also noted that the fact that B vanishes when the object-image relationship holds between the first and second reference planes, and that A or (l/D) is then the transverse magnification, leads to an experimental method for finding the matrix elements of an optical system without dismantling it or measuring its individual components.

EXTENSION OF RAY-TRANSFER METHOD TO REFLECTING SYSTEMS

Discussion of paraxial systems has so far confined itself to through-type optical systems that contain only refracting surfaces. So now we extend our treatment to include reflecting surfaces.

There is a single basic rule that will enable us to achieve this while still using the ray vectors and the T and R-matrices already described *Whenever a light* ray *is travelling in the* - z-direction , *the refractive index of the medium through which it is passing must be taken as negative.*

When interpreting ray data, we must remember that it is not V but V/n which specifies the geometric angle *V*between the ray direction and the optical axis. For example, when a ray is reflected from a plane surface normal to the axis, its ray direction V will be reversed in sign, but so will the value of n for the returning ray, so that it's optical direction-cosine V will remain unchanged. This means that the law of reflection, like the paraxial form of Snell's law of refraction, is already contained in the statement that across any plane mirror or any plane refracting surface $\begin{pmatrix} Y2 \\ V2 \end{pmatrix} = \begin{pmatrix} Y1 \\ V1 \end{pmatrix}$

T-matrices

When light rays are travelling across a gap from the plane z = Z1 to the plane z = Z2*,* the gap value (Z2 – Z1) will be positive for a ray travelling in the + z-direction but negative for a ray travelling in the- Z-direction. On the other hand, in the latter case, the index of the medium must be taken as negative, so that the reduced value of the gap will remain positive, being the ratio of two negative quantities. Provided ,therefore, that a matrix chain is being calculated in the usual way from output back to input, the T-values representing reduced gaps in the various T matrices will all appear positive.

R-matrices

We shall now use the term R-matrix to describe the effect of a reflecting as well as a refracting surface. To calculate the power P of a reflecting curved surface, we modify the formula P = (n2 – n1)/r, replacing the refractive index of the second medium n2 by the negative of the refractive index n for the medium in which the reflector is immersed and in which the ray continues to travel after its reflection. The power P is thus -2n/r and the R matrix is $\begin{pmatrix} 1 & 0 \\ 2n/r & 1 \end{pmatrix}$

It is important to remember here that the radius of curvature of a mirror, like that of a lens surface, is reckoned to be positive if the centre of curvature is located to the right of its surface. This will be the case, for example, if a convex mirror is arranged to face to the left so as to reflect rays back into the – z-direction. If the same mirror is turned round to face the other way, both the curvature radius P and the index n will need to be sign-changed, so the matrix itself remains unchanged. The power P of a diverging (convex) mirror always appears negative, while that of a concave (converging) mirror always appears as a positive quantity. For a succession of plane or curved reflecting surfaces the various T and R matrices need to be telescoped together to determine the desired system matrix.

CONCLUSION

In these ways matrices serve a unique approach in formulating different laws and definitions of geometrical optics and provide a unique method of determining properties of different geometrical optical systems through evaluation of its system matrices. Matrix method can also be used to useful in explaining Optical Resonators and propagation of laser beams.

ACKNOWLEDGEMENT

I would like to express my sincere gratitude to several individuals and the institution for supporting me throughout my execution of project work. First I wish to express my sincere gratitude to my supervisor, Professor Atisdipankar Chakrabarti , for his enthusiasm,patience,insightful comments, helpful information,practical advice and unceasing ideas that have helped me tremendously at all times in my project. Without his support and guidance, this project would not have been possible. I would also like to thank, professor Asok Kumar Pal, head of the department of physics and Swami Kamalasthanandaji Maharaj, principal of the esteemed institution. I could not have imagined a better help in my project work.

BIBLIOGRAPHY

- A.Gerrard,J.M.Burch, *Introduction to Matrix Methods in Optics*, Dover Publications,Inc.,New York,1994
- Ajoy Ghatak, *OPTICS (sixth edition)*, McGraw Hill Education (India) Pvt. Ltd.,Tamil Nadu,2019
- https://www.researchgate.net/publication/279749690_Notes_on_Matrix_of_Geometrical_Optics

Stellar Evolution

The project submitted, in partial fulfilment of the requirement for the assignments in **PHSA CC-XI, PHSA CC-XII, PHSA DSE-I, PHSA DSE-II** *Paper (Semester V) in the Department of Physics*

Submitted by

SUBHAM DAS

Registration No: A01-1112-111-004-2019

Supervisor Teacher: Dr. Rajen Kundu

RAMAKRISHNA MISSION VIVEKANANDA
CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

INTRODUCTION

"The nitrogen in our DNA, the calcium in our teeth, the iron in our blood, the carbon in our apple pies were made in the interiors of collapsing stars. We are made of star stuff"

- Carl Sagan

The cosmos is the background of our existence. It is the chaotic overall view on which we are just a tiny dot trying to learn its machinery. On this learning manifesto stars are an indispensable part of the visible universe. In this project I am trying to briefly review the life of stars, from its birth to its imminent death for both low and high mass stars. Their majestic attire couldn't be crammed in this short write up but I have tried to be as brief as possible. This write up begins with the life of low mass stars (like our sun) in the form of molecular clouds to protostar to the formation of red giant stage, a brief sketch of chaotic internal fusions, followed by its death in the form of planetary nebula. This is followed by the life of high mass stars, their formation of super giants at an accelerated rate followed by their death as supernova explosion (followed by white dwarfs) or as black holes. The last topic briefly skims on the concepts of singularity and the bizarre nature black hole whose escape velocity exceeds the speed of light!

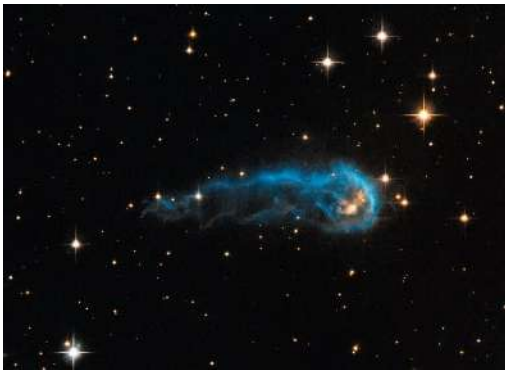
The black holes usually cannot be observed directly but they can be "observed" by the effects of their enormous gravitational fields on nearby matter.

1.HOW DO STARS ARE FORMED?

A star is formed when gravity causes a cloud of interstellar gas to contract to the point at which the central object becomes hot enough to sustain nuclear fusion in its core. Stars are formed in cold, dense clouds of gas whose pressure cannot resist gravitational contraction, these clouds are usually called molecular clouds because they are cold and dense enough to allow atoms to combine and form molecules. Typical molecular cloud temperatures are only 10–30 K.

Stars are usually born in clusters because a typical star- forming cloud is about thousand times massive than a typical star.

1.1 FROM CLOUD TO PROTOSTAR



The formation of stars begins with the collapse and fragmentation of molecular clouds into very dense clumps. The temperature of the material also increases while the area over which it is spread decreases as gravitational contraction continues, forming a more stellar-like object in the process. During this time, and up until hydrogen burning begins and it joins the main sequence, the object is known as a protostar.

This stage of stellar evolution may last for between 10^5 and 10^7 years depending on the size of the star being formed. If the final result is a protostar with more than 0.08 solar masses, it will go on to begin hydrogen burning and will join the main sequence as a normal star. For protostars with masses less than this, temperatures are not sufficient for hydrogen burning to begin and they become brown dwarf stars. Protostars are not yet true stars because their cores are not hot enough for nuclear fusion.

1.2 CONDITIONS FOR STAR BIRTH

Astronomers have found a simple formula that tells us how massive a cloud must be for gravity to start its collapse:

$$M_{\text{minimum}} = 18\ M_{\text{SUN}} \sqrt{\frac{T^3}{n}}$$

where T is the gas temperature in Kelvin and n is the gas density in particles per cubic centimeter. M_{minimum} is the minimum mass for making stars: Gravity can make a cloud collapse and give birth to stars only if the cloud's total mass is greater than M_{minimum}.

A protostar becomes a true star when its core temperature reaches 10 million K, hot enough for hydrogen fusion to operate efficiently. The core temperature continues to rise until fusion in the core generates enough energy to balance the energy lost from the surface in the form of radiation. Gravitational contraction then stops, because the star has achieved energy balance.

1.3 LIMITS ON STELLAR MASSES

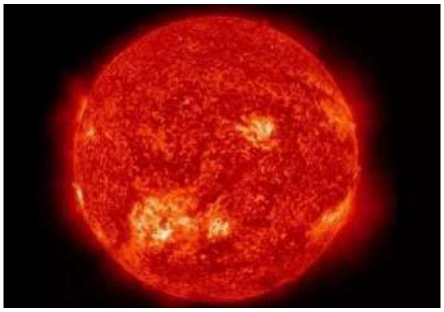
Stars more massive than about $100M_{\text{Sun}}$ blow off their outer layers, while protostars smaller than $0.08M_{\text{Sun}}$ become brown dwarfs that never get hot enough for efficient hydrogen fusion.

2.LIFE AS A LOW-MASS STAR

Low-mass stars are stars born with less than about 2 solar masses ($2M_{\text{Sun}}$) of material. Our sun is an example of a low mass star

2.1 RED GIANT STAGE

When the Sun's core hydrogen is finally depleted, nuclear fusion will cease. With no fusion to replace the energy the star radiates from its surface, the core will no longer be able to resist the inward pull of gravity, and it will begin to shrink.

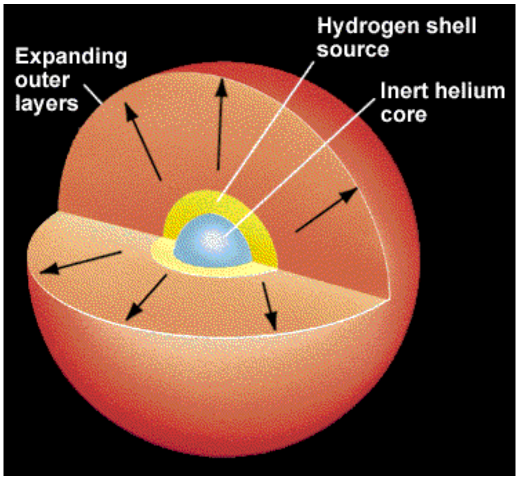


After 10 billion years of shining steadily, the Sun will enter an entirely new phase of life. Somewhat surprisingly, the Sun's outer layers will expand outward during this new life stage, even though its core will be shrinking under the crush of gravity. Over a period of about a billion years— the Sun will grow in size and luminosity to become a red giant. At the peak of its red giant stage, the Sun will be more than 100 times as large in radius and more than 1000 times as luminous as it is today. The star is now a red giant

After the core exhausts its hydrogen, it will be made almost entirely of helium, because helium is the product of hydrogen fusion. However, the gas surrounding the core will still contain hydrogen that has never previously undergone fusion. Because gravity will shrink both the inert (non-fusing) helium core and the surrounding shell of hydrogen, the hydrogen shell will soon become hot enough for hydrogen shell fusion—hydrogen fusion in a shell around the core

The higher fusion rate will generate enough energy to dramatically increase the Sun's luminosity and enough pressure to push the surrounding layers of gas outward.

The situation will grow more extreme as long as the helium core remains inert. Today, the self-correcting feedback process of the solar thermostat regulates the Sun's fusion rate: A rise in the fusion rate causes the core to inflate and cool until the fusion rate drops back downIn contrast, thermal energy generated in the hydrogen fusing shell of a red giant cannot do anything to inflate the inert core that lies underneath. Instead, newly produced helium keeps adding to the mass of the helium core, amplifying its gravitational pull and shrinking it further. The hydrogen-fusing shell shrinks along with the core, growing hotter and denser. The fusion rate in the shell consequently rises, feeding even more helium to the core. The star is caught in a vicious circle with a broken thermostat.



The core and shell will continue to contract and heat up—with the Sun as a whole continuing to grow larger and more luminous—until the temperature in the inert helium core reaches about 100 million K. At that point, it will be hot enough for helium nuclei to begin to fuse together, and the Sun will enter the next stage of its life. Other low-mass stars expand into red giants in the same way, though the time required depends on the star's mass. In some very-low-mass stars, the inert helium core may never become hot enough to fuse helium. In that case, the core collapse will be halted by degeneracy pressure, leaving the star's corpse as a helium white dwarf.

2.2 HELIUM FUSION

Helium fusion requires much higher temperatures, the onset of helium fusion heats the core rapidly without causing it to expand in size, at least at first. Instead, the rising temperature causes the helium fusion rate to spike drastically in what is called a helium flash. The helium flash releases an enormous amount of energy into the core. In a matter of seconds, the temperature rises so much that thermal pressure again becomes dominant. In fact, the thermal pressure becomes strong enough to push back against gravity. The core begins to expand and pushes the hydrogen-fusing shell outward, lowering its temperature and its fusion rate. The result is that, even though helium core fusion and hydrogen shell fusion are taking place simultaneously in the star, energy production falls from the peak it reached during the red giant stage, reducing the star's luminosity and allowing its outer layers to contract somewhat. As the outer layers contract, the star's surface temperature increases, so its color turns back toward yellow from red. After having spent about a billion years expanding into a luminous red giant,

the Sun will decline in size and luminosity as it becomes a helium core-fusion star.

2.3 HOW DOES A LOW-MASS STAR DIE?

It is only a matter of time until a helium core-fusion star converts all its core helium into carbon. The core, now made of the carbon produced by helium fusion, will begin to shrink once more under the crush of gravity. The exhaustion of core helium will cause the Sun to expand once again. This time, the trigger for the expansion will be helium fusion in a shell around the inert carbon core. Meanwhile, hydrogen fusion will continue in a shell atop the helium layer. The Sun will have become a double shell–fusion star. Both shells will contract along with the inert core, driving their temperatures and fusion rates so high that the Sun will expand to an even greater size and luminosity than in its first red giant stage. The Sun's only hope of extending its life at this stage lies with its carbon core, but this is a false hope for a low-mass star like the Sun. Carbon fusion is possible only at temperatures above about 600 million K, and degeneracy pressure will halt the collapse of the Sun's core before it ever gets that hot. With the carbon core unable to undergo fusion and provide a new source of energy, the Sun will finally have reached the end of its life.

2.4 PLANETARY NEBULA

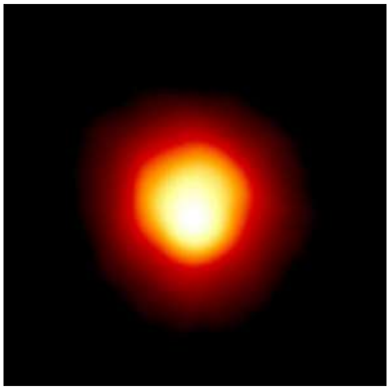
Through winds and other processes, the Sun will eject its outer layers into space, creating a huge shell of gas expanding away from the inert carbon core. The exposed core will still be very hot and will therefore emit intense ultraviolet radiation. This radiation will ionize the gas in the expanding shell, making it glow brightly as a planetary nebula. The nebula will disappear within about 100,000 years, leaving the carbon core behind as a white dwarf. White dwarfs are small in radius and often quite hot. We can now understand why: They are small in radius because they are the exposed cores of dead stars, supported against the crush of gravity by degeneracy pressure. White dwarfs reach this incredible density because they are collapsed so tightly that their electrons are smashed together. The more mass, the greater the pull inward, so a more massive white dwarf has a smaller radius than its less massive counterpart.

They are often hot because some of them were only recently the centers of stars, though over time they will cool and fade from view.

3. LIFE AS A HIGH-MASS STAR

The early stages of a high-mass star's life are similar to the early stages of the Sun's life, except they proceed much more rapidly. But the late stages of life are quite different for high-mass stars.

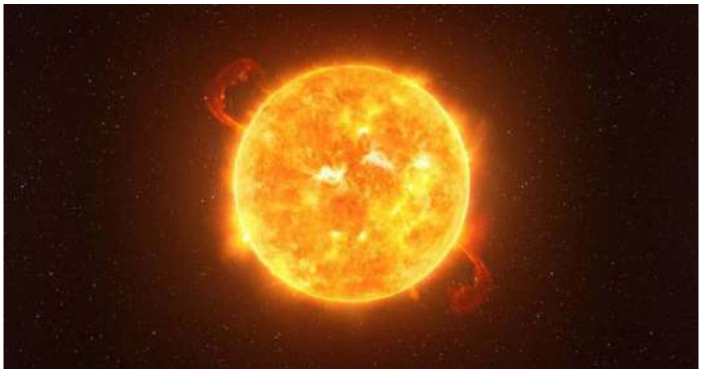
3.1 WHAT ARE THE LIFE STAGES OF A HIGH-MASS STAR?



Like all other stars, a high-mass star forms out of a cloud fragment that gravity forces to contract into a protostar. Just as in a low-mass star, hydrogen fusion begins when the gravitational potential energy released by the contracting protostar makes the core hot enough. However, hydrogen fusion proceeds through a different set of steps inside high-mass stars, which is part of the reason these stars live such brief but brilliant lives. As a specific example, let's investigate the life stages of a star born with 25 times the mass of the Sun ($25M_{Sun}$).

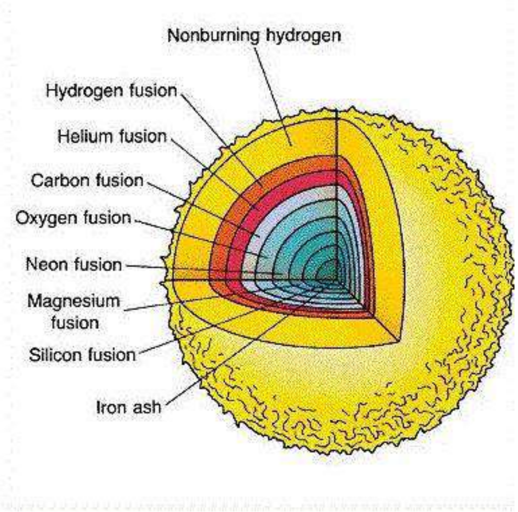
3.2 HYDROGEN FUSION IN A HIGH-MASS STAR

A low-mass star like our Sun fuses hydrogen into helium through the proton–proton chain. In a high-mass star, gravitational compression makes the hydrogen core hotter than in lower-mass stars. The greater core temperature makes it possible for protons to slam into carbon, oxygen, or nitrogen nuclei as well as into other protons. As a result, hydrogen fusion in high-mass stars occurs through a chain of reactions called the CNO cycle (the letters CNO stand for carbon, nitrogen, and oxygen)



3.3 BECOMING A SUPERGIANT

Our $25M_{Sun}$ star will begin to run low on hydrogen fuel after only a few million years. As its core hydrogen runs out, the star responds much like a low-mass star, but much faster. It develops a hydrogen-fusing shell, and its outer layers begin to expand outward, ultimately turning it into a supergiant. At the same time, the core contracts, and this gravitational contraction releases energy that raises the core temperature until it becomes hot enough to fuse helium into carbon. However, there is no helium flash in high-mass stars. Their core temperatures are so high that thermal pressure remains strong, preventing degeneracy pressure from becoming a factor. Helium core fusion therefore ignites gradually, just as hydrogen core fusion did at the beginning of the star's life. Our high-mass star fuses helium into carbon so rapidly that it is left with an inert carbon core after just a few hundred thousand years. Once again, the absence of fusion leaves the core without an energy source to fight off the crush of gravity. The inert carbon core shrinks, the crush of gravity intensifies, and the core pressure, temperature, and density all rise. Meanwhile, a helium fusing shell forms between the inert core and the hydrogen-fusing shell, and the star's outer layers swell further. Eventually, the core will become hot enough to fuse carbon into heavier elements, and the star will undergo another cycle of core fusion followed by shell fusion.



The nuclear reactions in a high-mass star's final stages of life become quite complex, and many different reactions may take place simultaneously. The simplest sequence of fusion stages occurs through successive helium-capture reactions—reactions in which a helium nucleus fuses into some other nucleus.

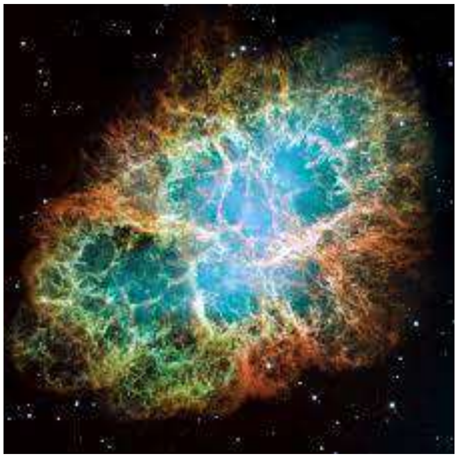
Helium capture can fuse carbon into oxygen, oxygen into neon, neon into magnesium, and so on. At high enough temperatures, a star's core can fuse heavy nuclei to one another. For example, fusing carbon to oxygen creates silicon, fusing two oxygen nuclei creates sulfur, and fusing two silicon nuclei generates iron

The core continues shrinking and heating while iron accumulates within it. Iron is unique among the elements in a very important way: It is the one element from which it is not possible to generate any kind of nuclear energy.

3.4 HOW DOES A HIGH-MASS STAR DIE?

3.4.1 THE SUPERNOVA EXPLOSION

Once gravity pushes the electrons past the quantum mechanical limit, they can no longer exist freely. In an instant, the electrons disappear by combining with protons to form neutrons, releasing neutrinos in the process. The degeneracy pressure provided by the electrons instantly vanishes, and gravity has free rein.

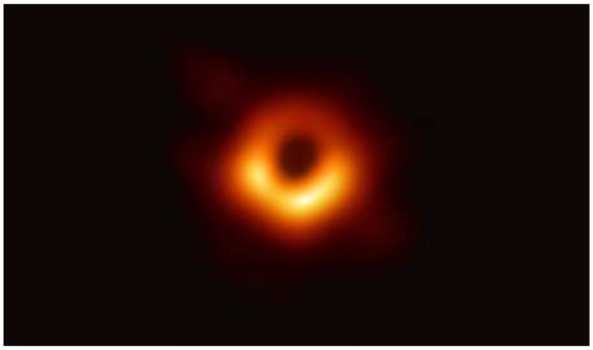


The collapse halts only because the neutrons have a degeneracy pressure of their own. The entire core then resembles a giant atomic nucleus. The gravitational collapse of the core releases an enormous amount of energy. It drives the star’s outer layers into space in a titanic explosion called a supernova.

The ball of neutrons left behind is called a neutron star. In some cases, the remaining mass may be so large that gravity also overcomes the degeneracy pressure associated with neutrons, and the core continues to collapse until it becomes a black hole

3.4.2 BLACK HOLE

The “black” in the name black hole comes from the fact that nothing—not even light—can escape from a black hole. The escape velocity of any object depends on the strength of its gravity, which depends on its mass and size. Decreasing

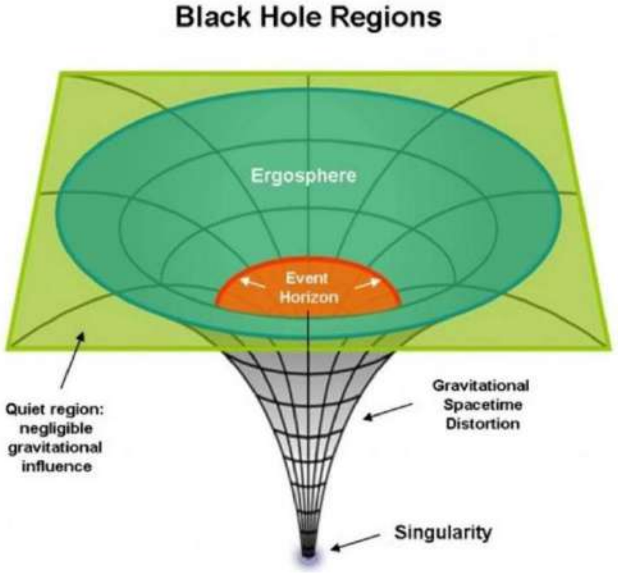


the size of an object of a particular mass makes its gravity stronger and increases its escape velocity. A black hole is so compact that it has an escape velocity greater than the speed of light. Because nothing can travel faster than the speed of light, neither light nor anything else can escape from within a black hole.

The “hole” part of the name black hole comes from the fact that in general theory of relativity, gravity arises from the curvature of spacetime which near a black hole is so great that it forms a bottomless pit as shown in the underlying figure.

The boundary between the inside of the black hole and the universe outside is called the event horizon. The event horizon essentially marks the point of no return for objects entering a black hole

We usually think of the “size” of a black hole as the size of its event horizon. For someone outside the black hole, the event horizon is shaped like a sphere, and the size of the sphere defines what we call the Schwarzschild radius of the black hole. The Schwarzschild radius depends only on the black hole’s mass. In essence, a collapsing stellar core becomes a black hole at the moment it shrinks to a size smaller than its Schwarzschild radius. Because nothing can stop the crush of gravity in a black hole, all the matter that forms a black hole should ultimately be crushed to an infinitely tiny and dense point in the black hole’s center. We call this point a singularity.



CONCLUSION

The story of a star’s life begins in the murky depths of interstellar space. It passes through various phases of red giant, undergoing various nuclear fusion processes ultimately forming a planetary nebula and supernova or forming a black hole.

In this report, we have tried to give an overview of the formation, classification and life-cycle of a star. It further helped us to realize the birth of elements which makes the earth, including us.

Acknowledgement

Primarily I would like to express my profound gratitude to Ramakrishna Mission Vivekananda Centenary College, Rahara for including the project as part of our academic curriculum. This has provided me an opportunity to learn something beyond the text book. I wish to express my deep gratitude and sincere thanks to revered Principal Maharaj Swami Kamalasthananda, Ramakrishna Mission Vivekananda Centenary College, Rahara, for his encouragement and the facilities provided for this project work.

I extend my sincere thanks to Dr. Rajen Kundu, Professor, Ramakrishna Mission Vivekananda Centenary College, Rahara who suggested me the topic and encouraged me to take up this project as an introduction to the subject. I take this opportunity to express my deep sense of gratitude for his valuable support and constructive comments.

I can't forget to offer my sincere thanks to my family and friends whose immense motivation and support had helped me to complete the project in limited time frame.

SUBHAM DAS

PHYSICS HONOURS (5TH SEMESTER)

COLLEGE ROLL NO. : 108

BIBLIOGRAPHY

For the completion of this project and giving it a final shape, I have consulted the following books and websites.

Sites:

https://astronomy.swin.edu.au/cosmos/p/Protostar

https://www.space.com/23756-white-dwarf-stars.html

https://www.nasa.gov

Books :

1. Lecture notes on Stellar Structure and evolution by Jorgen Christensen-Dalsgaard
2. The essential Cosmic Perspective by Jeffrey Bennett, Megan Donahue, Nicholas Schneider, Mark Volt

QUANTUM TUNNELING AND ITS APPLICATIONS

The project submitted, in partial fulfilment of the requirement for the assignments in ***PHSA CC-XI, PHSA CC-XII, PHSA DSE-I, PHSA DSE-II*** *Paper (Semester 5th) in the Department of Physics*

Submitted by

Soumya Halder

Registration No: A01-1112-111-005-2019

Supervisor Teacher: Prof. Bhaskar Haldar

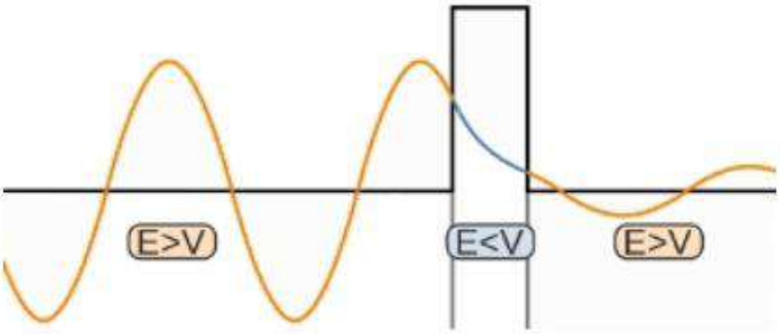
RAMAKRISHNA MISSION VIVEKANANDA CENTENARY COLLEGE

P.O. RAHARA, KOLKATA-700118, WEST BENGAL, INDIA

INTRODUCTION:-

The quantum tunneling effect is a quantum phenomenon which occurs when particles move through a barrier that, according to the theories of classical physics, should be impossible to move through. The barrier may be a physically impassable medium, such as an insulator or a vacuum, or a region of high potential energy.

In classical mechanics, when a particle has insufficient energy, it would not be able to overcome a potential barrier. In the quantum world, however, particles can often behave like waves. On encountering a barrier, a quantum wave will not end abruptly; rather, its amplitude will decrease exponentially. This drop in amplitude corresponds to a drop in the probability of finding a particle further into the barrier. If the barrier is thin enough, then the amplitude may be non-zero on the other side. This would imply that there is a finite probability that some of the particles will tunnel through the barrier.



In regions where the potential energy is higher than the wave's energy, the amplitude of the wave decays exponentially. If the region is narrow enough, the wave can have a non-zero amplitude on the other side. Image Credits: Wikipedia

The tunneling current is defined as the ratio of the current density emerging from the barrier divided by the current density incident on the barrier. If this transmission coefficient across the barrier is a non-zero value, then there is a finite likelihood of a particle tunneling through the barrier.

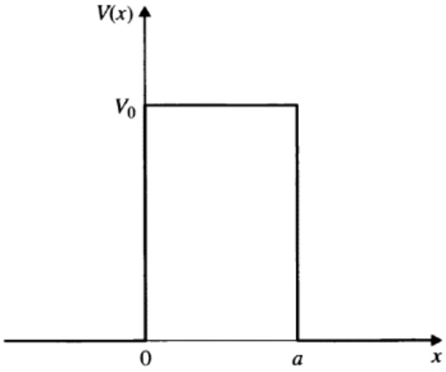
Discovery of Quantum Tunneling:-

The possibility of tunneling was first noticed by F. Hund in 1927, while calculating the ground state energy in a "double-well" potential—a system where two separate states of similar energies are separated by a potential barrier. Many molecules, such as ammonia, are examples of this type of system. "Inversion" transitions between two geometric states are forbidden by classical mechanics but are made possible by quantum tunneling.

In the same year, L. Nordheim noticed another incidence of the tunneling phenomenon while studying the reflection of electrons from a variety of surfaces. In the next few years, Oppenheimer successfully used tunneling to calculate the ionization rate of hydrogen. Meanwhile, studies by Garnow, as well as Gurney and Condon, explained the range of alpha decay rates of radioactive nuclei.

The rectangular potential barrier

If we consider a potential barrier of type shown in figure below



A rectangular potential barrier of height **V_0** and thick ness

Image credits: QUNATUM MECHANICS (2nd edition) B.H. Bransden & C.J Joachain

Classically, for a particle of energy E incident on the barrier, if $E < V_0$ the particle gets reflected by the barrier and if $E > V_0$, the particle gets transmitted through the barrier. We will see that according to quantum mechanics, even for $E < V_0$ there is a finite probability of the particle tunnelling through the barrier.

$E < V_0$ We first consider $E < V_0$. For such a case the solutions of the Schrodinger equation in the three regions are given by:

$$\psi = A\,e^{ikx} + B\,e^{-ikx} \quad x < 0$$

$$= C\,e^{kx} + D\,e^{-kx} \quad 0<x<a \quad (1)$$

$$= F\,e^{ikx} + G\,e^{-ikx} \quad x>a$$

where $k = \left(\frac{2mE}{\hbar^2}\right)^{\frac{1}{2}}$ and $\kappa = \left(\frac{2m(V_0 - E)}{\hbar^2}\right)^{\frac{1}{2}}$ (2)

From Boundary condition of ψ and $\frac{d\psi}{dx}$ at x=0 and at x=a gives us

At x=0

$$A+B=C+D$$

$$\frac{ik}{\kappa}(A-B)=C-D$$

Thus

$$C=\frac{1}{2}\left(1+\frac{ik}{\kappa}\right)A+\frac{1}{2}\left(1-\frac{ik}{\kappa}\right)B \qquad (3)$$

And

$$D=\frac{1}{2}\left(1-\frac{ik}{\kappa}\right)A+\left(1+\frac{ik}{\kappa}\right)B \qquad (4)$$

Boundary condition at x=a gives us

$$C\,e^{\kappa a}+\ D\,e^{-\kappa a}=F\,e^{ika} \qquad (5)$$

$$C\,e^{\kappa a}-D\,e^{-\kappa a}\ \ =\frac{ik}{\kappa}F\,e^{ika} \qquad (6)$$

Thus

$$C=\frac{1}{2}\left(1+\frac{ik}{\kappa}\right)F\,e^{-\kappa a}\ e^{ika}$$

And

$$D=\frac{1}{2}\left(1-\frac{ik}{\kappa}\right)\ F\,e^{\kappa a}\ \ e^{ika}$$

And

$$\frac{C}{D}=\frac{\left(1+\frac{ik}{\kappa}\right)}{\left(1-\frac{ik}{\kappa}\right)}\,e^{-2\kappa a}=\frac{\left(1+\frac{ik}{\kappa}\right)A+\left(1-\frac{ik}{\kappa}\right)B}{\left(1-\frac{ik}{\kappa}\right)A+\left(1+\frac{ik}{\kappa}\right)B}$$

where in the last step we have used Eqs. (3) and (4). Simple manipulations give us

$$\frac{B}{A}=\frac{\left(k^2+\kappa^2\right)\,sinh\;\kappa a}{\left(k^2-\kappa^2\right)\,sinh\;\kappa a+2ik\kappa\;cosh\;\kappa a}$$

We also have

$$\frac{F}{A}=\frac{2ik\kappa\, e^{-ika}}{(k^2-\kappa^2)\, sinh\, \kappa a + 2ik\kappa\, cosh\, \kappa a}$$

Reflection and transmission coefficients

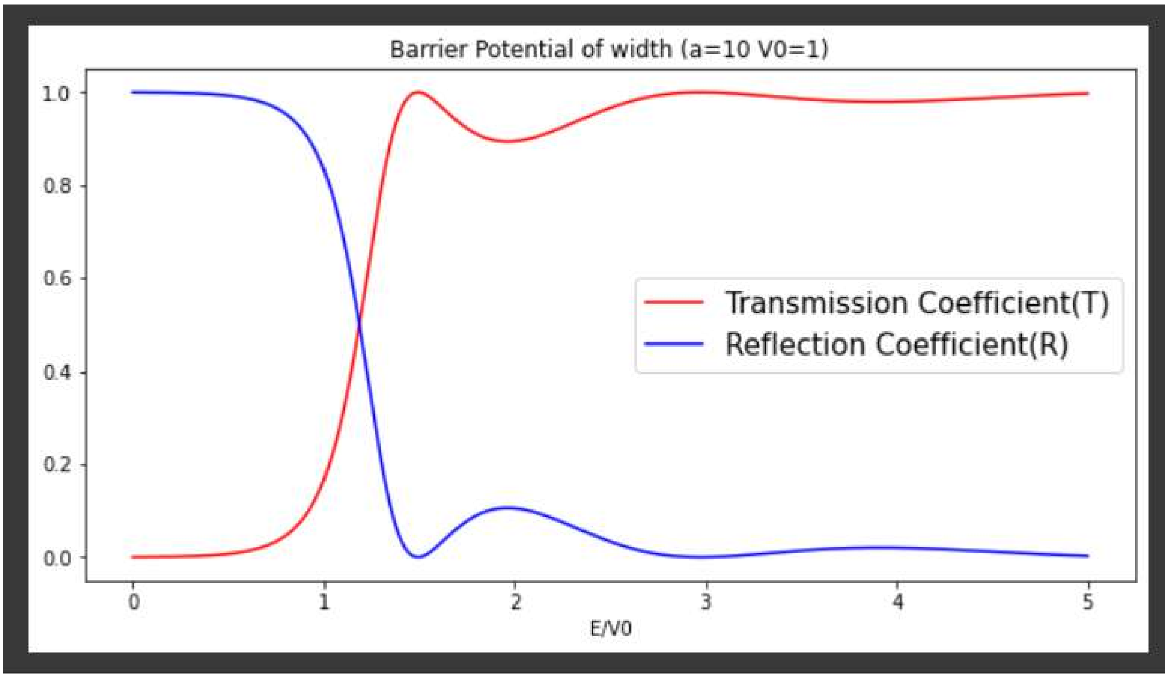
The above equations will lead to the following expressions for the reflection and transmission coefficients:

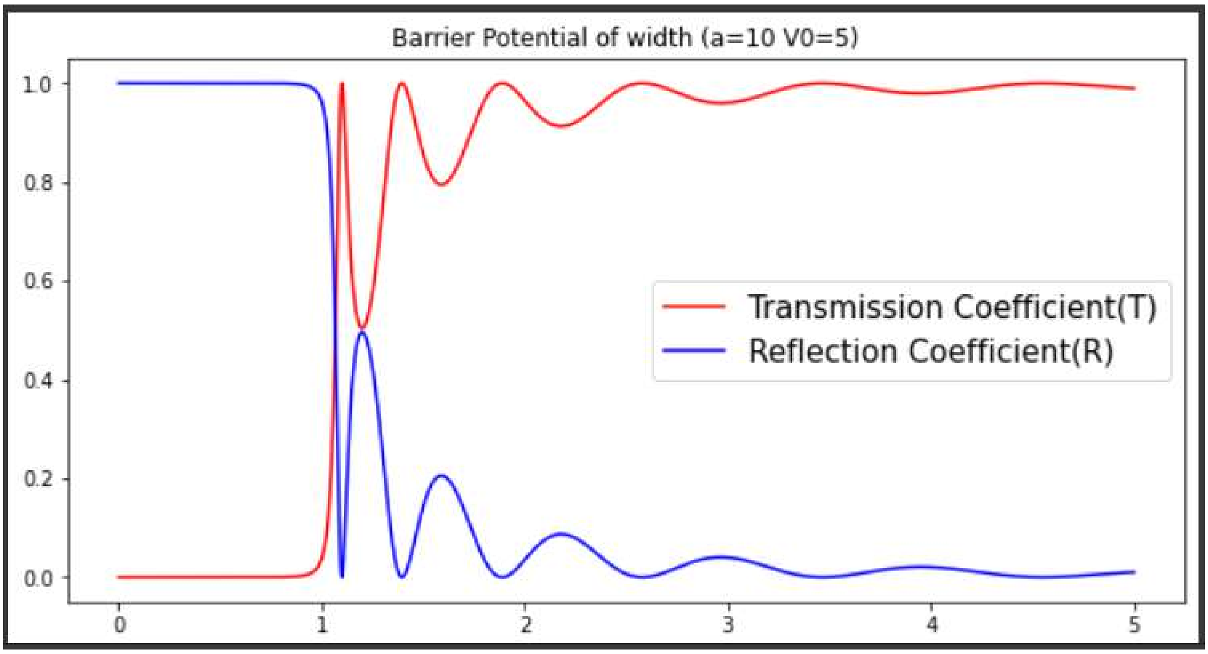
$$R=\frac{J_r}{J_i}=\frac{|B|^2}{|A|^2}=\left[1+\frac{4€(1-€)}{sinh^2\,(\alpha\sqrt{1-€})}\right]^{-1} \quad (7)$$

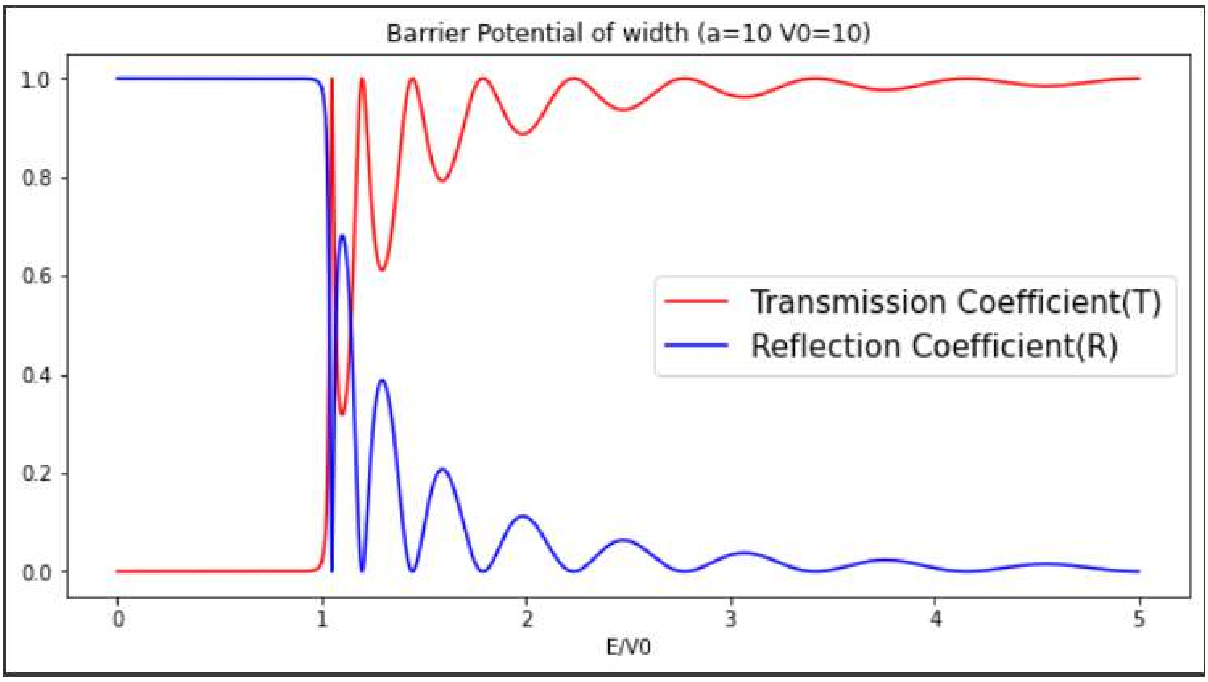
$$T=\frac{J_t}{J_i}=\frac{|C|^2}{|A|^2}=\left[1+\frac{sinh^2(\alpha\sqrt{1-€})}{4€(1-€)}\right]^{-1} \quad (8)$$

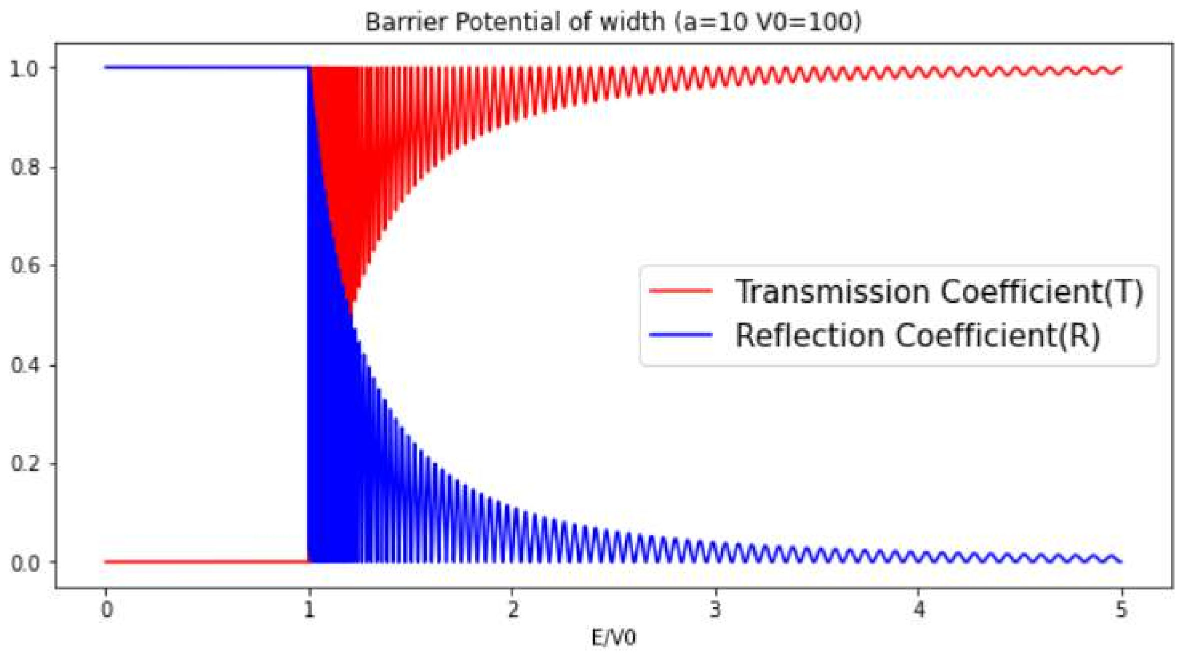
Where $€=\frac{E}{V_0}$ $\alpha=\left(\frac{2mV_0a^2}{\hbar^2}\right)^{\frac{1}{2}}$

If we have plotted variations of the reflection and transmission coefficient with some values of a and V_0




Barrier Potential of width (a=10 V0=5)
1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5
Transmission Coefficient(T)
Reflection Coefficient(R)


Barrier Potential of width (a=10 V0=10)
1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5
E/V0
Transmission Coefficient(T)
Reflection Coefficient(R)



APPLICATION OF QUANTUM TUNNELING:-

Quantum tunneling plays an essential role in physical phenomena, such as nuclear fusion and It has applications in the tunnel diode ,quantum computing and in the scanning tunneling microscope .

Quantum tunneling is also projected to create physical limits to size of the transistors used in microelectronics , due to electrons being able to tunnel past transistors that are too small.

- ### Scanning tunneling microscope:-

A scanning tunneling microscope (STM) is a type of microscope used for imaging surfaces at the atomic level. Its development in 1981 earned its inventors, Gerd Binnig and Heinrich, then at IBM Zurich the Nobel Prize in Physics in 1986. STM senses the surface by using an extremely sharp conducting tip that can distinguish features smaller than 0.1 nm with a 0.01 nm (10pm) depth resolution. This means that individual atoms can routinely be imaged and manipulated. Most microscopes are built for use in ultra high vacuum, at temperatures approaching zero kelvin, but variants exist for studies in air, water and other environments, and for temperatures over 1000 °C.

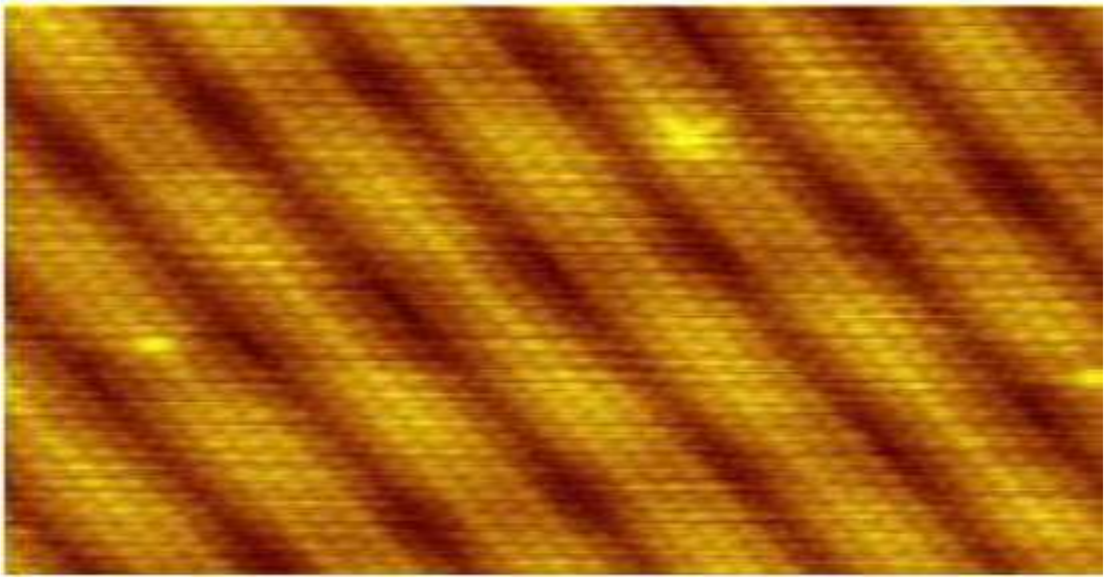


Image of reconstruction on a clean surface of gold

Image credits: Wikipedia

STM is based on the concept of quantum tunneling . When the tip is brought very near to the surface to be examined, a biasvoltage applied between the two allows electronsto tunnel through the vacume separating them. The resulting tunneling current is a function of the tip position, applied voltage, and the local density states (LDOS) of the sample. Information is acquired by monitoring the current as the tip scans across the surface, and is usually displayed in image form.

A refinement of the technique known as scanning tunneling spectroscopy consists of keeping the tip in a constant position above the surface, varying the bias voltage and recording the resultant change in current. Using this technique the local density of the electronic states can be reconstructed. This is sometimes performed in high magnetic fields and in presence of impurities to infer the properties and interactions of electrons in the studied material.

Scanning tunneling microscopy can be a challenging technique, as it requires extremely clean and stable surfaces, sharp tips, excellent vibration section and sophisticated electronics. Nonetheless, many hobbyists build their own microscopes

A large STM setup at the London Centre for Nanotechnology

Image credits: Wikipedia

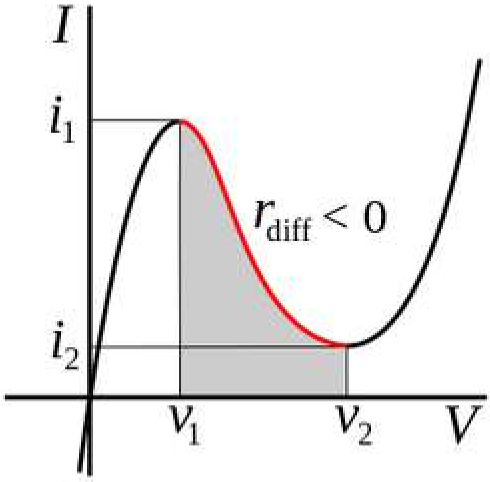
- TUNNEL DIODE:-

A tunnel diode or Esaki diode is a type of semiconductor diode that has effectively "negative resistance" due to the quantum tunneling effect called tunneling. It was invented in August 1957 by Leo Esaki, Yuriko Kurose, and Takashi Suzuki when they were working at Tokyo

Tsushin Kogyo, now known as sony. In 1973, Esaki received the Nobel Prize in Physics , jointly with Barin Josephon, for discovering the electron tunneling effect used in these diodes. Robert Noyce independently devised the idea of a tunnel diode while working for William Shockley , but was discouraged from pursuing it. Tunnel diodes were first manufactured by Sony in 1957, followed by General Electric other companies from about 1960, and are still made in low volume today

Tunnel diodes have a heavily doped positive to negative (p-n) junction that is about 10 nm (100 Å) wide. The heavy doping results in a broken band gap, where conducting band state on the N-side are more or less aligned with valance band state on the P-side. They are usually made from germanium , but can also be made from gallium arsenide and silicone materials.

In a conventional semiconductor diode, conduction takes place while the P-N junction is forward biased and blocks current flow when the junction is reverse biased. This occurs up to a point known as the "reverse breakdown voltage" at which point conduction begins (often accompanied by destruction of the device). In the tunnel diode, the dopant concentrations in the P and N layers are increased to a level such that the reverse breakdown voltage becomes zero and the diode conducts in the reverse direction. However, when forward-biased, an effect occurs called quantum mechanical tunneling which gives rise to a region in its voltage vs. current behavior where an *increase* in forward voltage is accompanied by a *decrease* in forward current. This "negative resistance" region can be exploited in a solid state version of the dynatron oscillator which normally uses a tetrode thermionic valve (vacuum tube).



I vs. *V* curve similar to a tunnel diode characteristic curve. It has "negative" differential resistance in the shaded voltage region, between V_1 and V_2.

Image credits: ANALOGUE ELECTRONICS JACOB MILLMAN & CHRISTOS HALKIAS

CONCLUSION :-

So at last we can draw a conclusion that quantum tunneling is a remarkable theory which advance the Multiple -energy level nature of particles wave function, so it can move in ways that would not be possible in classical physics , and there are many more technological application besides STM(scanning tunneling microscope) , and tunnel diodes like quantum computers , small microchips , Micro-controllers and other Nanotechnological aspect which completely revolutionized our world.

ACKNOWLEDGEMENT:-

In the accomplishment of this project successfully, many people have given me their support and guidance . Now I am utilizing this time to thank all the people who have been concerned with this project .

Primarily I would thank my project supervisor Prof. Bhaskar Haldar for his guidance and support which help me to complete this project in time , Besides him I would like to thank my friends for their help which also help me to complete this project work.

REFERENCE

1. QUNATUM MECHANICS (2nd edition) B.H. Bransden & C.J Joachain
2. BASICS OF QUANTUM MECHANICS AND ITS APPLICATION BY AJOY GHATAK
3. ANALOGUE ELECTRONICS JACOB MILLMAN & CHRISTOS HALKIAS
4. WIKIPEDIA

THANK YOU
FOR YOUR ATTENTION

Title of the Project

*The project submitted, in partial fulfilment of the requirement for the assignments In **PHSA CC-XI,PHSA CC-XII,PHSA DSE-I,PHSA DSE-II**. Paper (**Semester 5th**) in the Department of Physics*

Submitted by

Swarup Kumar Giri

Registration No: A01-1112-111-006-2019

Supervisor Teacher:

Prof. Kalyanbrata Chakroborty

RAMAKRISHNA MISSION VIVEKANANDA CENTENARY COLLEGE

P.O. RAHARA, KOLKATA-700118

WEST BENGAL, INDIA

ASSOCIATED LEGENDRE POLYNOMIALS AND ITS APPLICATIONS

- **EVIDENCE:** Legendre functions are important in physics because they arise when the Laplace or Helmholtz equations (or their generalizations) for central force problems are separated in spherical coordinates. They therefore appear in the descriptions of wave functions for atoms, in a variety of electrostatics problems, and in many other contexts. In addition, the Legendre polynomials provide a convenient set of functions that is orthogonal (with unit weight) on the interval (−1,+1) that is the range of the sine and cosine functions. And from a pedagogical viewpoint, they provide a set of functions that are easy to work with and form an excellent illustration of the general properties of orthogonal polynomials. . We collect here those results, expanding them with additional material that is of great utility and importance. As indicated above, Legendre functions are encountered when an equation written in spherical polar coordinates (r, θ , φ), such as

$$-\nabla^2\psi + V(r)\psi = \lambda\psi,$$

is solved by the method of separation of variables. Note that we are assuming that this equation is to be solved for a spherically symmetric region and that V(r) is a function of the distance from the origin of the coordinate system (and therefore not a function of the three-component position vector r). , we write ψ = R(r)Θ(θ)ϕ(φ) and decompose our original partial differential equation (PDE) into the three one-dimensional ordinary differential equations (ODEs):

∇ *ORIGIN OF ASSOCIATED LEGENDRE EQUATION :*

We started here with Helmhotz equation , Curvilinear coordinate systems introduce additional nuances into the process for separating variables. Again we consider the Helmholtz equation, now in circular spherical coordinates. With our unknown function ψ dependent **on r ,θ, φ,** that equation becomes, **for ∇^2 : ∇^2ψ(r ,θ, φ) + k^2 ψ(r , θ,φ) = 0**

As a final exercise in the separation of variables in PDEs, let us try to separate the Helmholtz equation, again with k^2 constant, in spherical polar coordinates.

$$\frac{1}{r^2\sin\theta}\left[\sin\theta\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right)+\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right)+\frac{1}{\sin\theta}\frac{\partial^2\psi}{\partial\varphi^2}\right]=-k^2\psi.$$

now,

$$\psi(r,\theta,\varphi)=R(r)\Theta(\theta)\Phi(\varphi).$$

by substituting back into above eqution we get

$$\frac{1}{Rr^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+\frac{1}{\Theta r^2\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right)+\frac{1}{\Phi r^2\sin^2\theta}\frac{d^2\Phi}{d\varphi^2}=-k^2.$$

note that all derivatives are now ordinary derivatives rather than partials. By multiplying by (rsinθ)^2 to obtain relates a function of φ alone to a function of r and θ alone. Since r, θ, and φ are independent variables, we equate each side of to a constant. In almost all physical problems, φ will appear as an azimuth angle. This suggests a periodic solution rather than an exponential. With this in mind, let us use m^2 the separation constant, which then must be an integer

$$\frac{1}{\Phi}\frac{d^2\Phi(\varphi)}{d\varphi^2}=-m^2$$

squared. The

$$\frac{1}{Rr^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+\frac{1}{\Theta r^2\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right)-\frac{m^2}{r^2\sin^2\theta}=-k^2.$$

Multiplying by r^2 and rearranging terms, we obtain

$$\frac{1}{R}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+r^2k^2=-\frac{1}{\Theta\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right)+\frac{m^2}{\sin^2\theta}.$$

Again, the variables are separated. We equate each side to a constant, λ, and finally get

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right)-\frac{m^2}{\sin^2\theta}\Theta+\lambda\Theta=0,$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+k^2R-\frac{\lambda R}{r^2}=0.$$

once more we have replaced a partial differential equation of three variables by three ODEs. The ODE for ϕ is the same as that encountered in cylindrical coordinates, with solutions exp(±imφ) or sinmφ, cosmφ. The Θ ODE can be made less forbidding by changing the independent variable from θ to t = cos θ , after which , with Θ(θ) now written as P(cos θ) = P(t), becomes

$$(1-t^2)P''(t)-2tP'(t)-\frac{m^2}{1-t^2}P(t)+\lambda P(t)=0.$$

This is the **associated Legendre equation** (called **the Legendre equation** if m = 0), . We normally require solutions for P(t) that do not have singularities in the region within the range of the spherical polar coordinate θ (namely that it be nonsingular for the entire range 0 ≤ θ ≤ π, equivalent to −1 ≤ t ≤ +1). The solutions satisfying these conditions, called **associated Legendre functions**, are traditionally denoted P_l^m , with l a nonnegative integer. I discussed the Legendre equation as a -D eigenvalue problem, finding that the requirement of nonsingularity at t = ±1 is a sufficient boundary condition to make its solutions well defined.

When Laplace’s equation is separated in spherical polar coordinates , one of the separated ODEs is the associated Legendre equation

With x=cosθ, this becomes

$$(1-x^2)\frac{d^2}{dx^2}P_n^m(x)-2x\frac{d}{dx}P_n^m(x)+\left[n(n+1)-\frac{m^2}{1-x^2}\right]P_n^m(x)=0.$$ **equation (1)**

- **IF THE AZIMUTHAL SEPARATION CONSTANT M = 0 WE HAVE LEGENDRE'S EQUATION, THE REGULAR SOLUTIONS (WITH M NOT NECESSARILY ZERO), RELABELED P" N (X), ARE**

$$P_n^m(x)=(1-x^2)^{m/2}\frac{d^m}{dx^m}P_n(x).$$

equation (2)

These are the associated Legendre functions, Since the highest power of x in Pn(x) is x^n, we must have m ≤ n (or the m-fold differentiation will drive our function to zero). In quantum mechanics the requirement that m ≤ n has the physical interpretation that the expectation value of the square of the z-component of the angular momentum is less than or equal to the expectation value of the square of the angular momentum vector L

$$\langle L_z^2\rangle \le \langle L^2\rangle \equiv \int \psi_{nm}^*\mathbf{L}^2\psi_{nm}d^3r,$$

equation (3)

where m is the eigenvalue of Lz, and n(n+ 1) is the eigenvalue of L^2. From the form of Eq3. , we might expect m to be non negative. However, if Pn(x) is expressed by Rodrigues's formula, this limitation on m is relaxed and we may have −n ≤ m ≤ n, with negative as well as positive values of m being permitted. Using Leibniz's differentiation formula once again, the reader may show that $^{Pm}{}_{n}$(x) and $^{P-m}{}_{n}$ (x) are related by

$$P_n^{-m}(x)=(-1)^m\frac{(n-m)!}{(n+m)!}P_n^m(x).$$

equation (4)

From our definition of the associated Legendre functions, $P^{m}{}_{n}$ (x), $P^{0}{}_{n}$(x) = Pn(x). As with the Legendre polynomials, a generating function for the associated Legendre functions is obtained via Eq. (2) from that of ordinary Legendre polynomials

$$\frac{(2m)!(1-x^2)^{m/2}}{2^m m!(1-2tx+t^2)^{m+1/2}}=\sum_{s=0}^{\infty}P_{s+m}^m(x)t^s.$$

from this formula If we drop the factor (1 − x2)m/2 =sin^m θ and define the the polynomials

$$\mathcal{P}_{s+m}^m(x)=P_{s+m}^m(x)(1-x^2)^{-m/2}$$ **equation (5)**

, then we obtain a practical form of the generating function

$$g_m(x,t) \equiv \frac{(2m)!}{2^m m!(1-2tx+t^2)^{m+1/2}} = \sum_{s=0}^{\infty} \mathcal{P}_{s+m}^m(x)t^s.$$

We can derive a recursion relation for associated Legendre polynomials that is analogous to Eqs.

$$\frac{\partial g(t,x)}{\partial t} = \frac{x-t}{(1-2xt+t^2)^{3/2}} = \sum_{n=0}^{\infty} nP_n(x)t^{n-1}.$$

$$(1-2xt+t^2)\sum_{n=0}^{\infty} nP_n(x)t^{n-1} + (t-x)\sum_{n=0}^{\infty} P_n(x)t^n = 0.$$

equation (6)

This equ. Are used for only legendre polynomials, and so on.From above two equ,differentiate generating function for associate legendre polynomials

$$(1-2tx+t^2)\frac{\partial g_m}{\partial t} = (2m+1)(x-t)g_m(x,t).$$

Substituting the defining expansions for associated Legendre polynomials we get

$$(1-2tx+t^2)\sum_s s\mathcal{P}_{s+m}^m(x)t^{s-1} = (2m+1)\sum_s \left[x\mathcal{P}_{s+m}^m t^s - \mathcal{P}_{s+m}^m t^{s+1}\right].$$

Comparing coefficients of powers of t in these power series, we obtain the recurrence relation

$$(s+1)\mathcal{P}_{s+m+1}^m - (2m+1+2s)x\mathcal{P}_{s+m}^m + (s+2m)\mathcal{P}_{s+m-1}^m = 0.$$

equation (7)

For m = 0 and s = n this relation is,

$$(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x), \quad n = 1, 2, 3, \ldots$$

equation (8)

Before we can use this relation we need to initialize it, that is, relate the associated Legendre polynomials with m = 1 to ordinary Legendre polynomials. We observe that

$$(1-2xt+t^2)g_1(x,t) = (1-2xt+t^2)^{-1/2} = \sum_s P_s(x)t^s$$

Equation (9). so that upon inserting Eq. (7) we

$$\mathcal{P}_{s+1}^1 - 2x\mathcal{P}_s^1 + \mathcal{P}_{s-1}^1 = P_s(x)$$

Equation (10). So that upon inserting Eq. (7) we get the recursionMore generally, we also have the identity

$$(1-2xt+t^2)g_{m+1}(x,t) = (2m+1)g_m(x,t),$$

from which we extract the recursion $\mathcal{P}^{m+1}_{s+m+1} - 2x\mathcal{P}^{m+1}_{s+m} + \mathcal{P}^{m+1}_{s+m-1} = (2m+1)\mathcal{P}^{m}_{s+m}(x),$

equation (11)

which relates the associated Legendre polynomials with superindex m+ 1 to those with m. For m = 0 we recover the initial recursion equ(10).

- ## ORTHOGONAOLITY;

$$\int_{-1}^{1} P_k^m P_\ell^m \, dx = \frac{2(\ell+m)!}{(2\ell+1)(\ell-m)!} \, \delta_{k,\ell}$$

Where $\delta k,\ell$ is the Kronecker delta.

Also, they satisfy the orthogonality condition for fixed ℓ:

$$\int_{-1}^{1} \frac{P_\ell^m P_\ell^n}{1-x^2} dx = \begin{cases} 0 & \text{if } m \neq n \\ \frac{(\ell+m)!}{m(\ell-m)!} & \text{if } m = n \neq 0 \\ \infty & \text{if } m = n = 0 \end{cases}$$

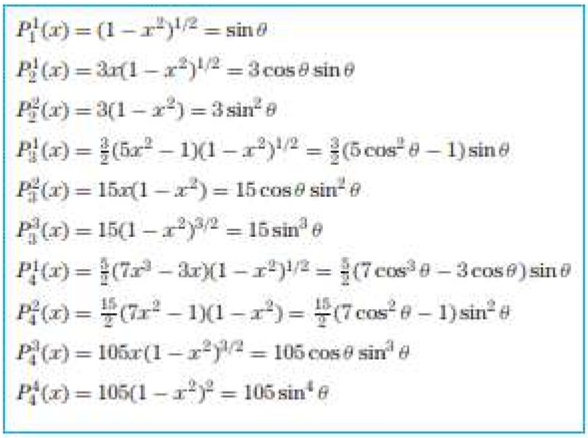
- ## EXAMPLE1;-LOWEST ASSOCIATED LEGENDRE POLYNOMIALS

Now we are ready to derive the entries of Table 1, For m = 1 and s = 0 Eq. (10) yields $P^1_1 = 1$ because $P^1_0 = 0 = P^1_{-1}$ do not occur in the definition, Eq. (7), of the associated Legendre polynomials. Multiplying by (1–x^2)^1/2 = sin θ we get the first line of Table 1. For s = 1 we find from Eq. (10), $\mathcal{P}^1_2(x) = P_1 + 2x\mathcal{P}^1_1 = x + 2x = 3x,$ from which the second line of Table 11.2, 3 cos θ sin θ, follows upon multiplying by sin θ. For s = 2 we get,

$$\mathcal{P}^1_3(x) = P_2 + 2x\mathcal{P}^1_2 - \mathcal{P}^1_1 = \frac{1}{2}(3x^2-1) + 6x^2 - 1 = \frac{15}{2}x^2 - \frac{3}{2},$$

.Equation (12)

- TABLE-1

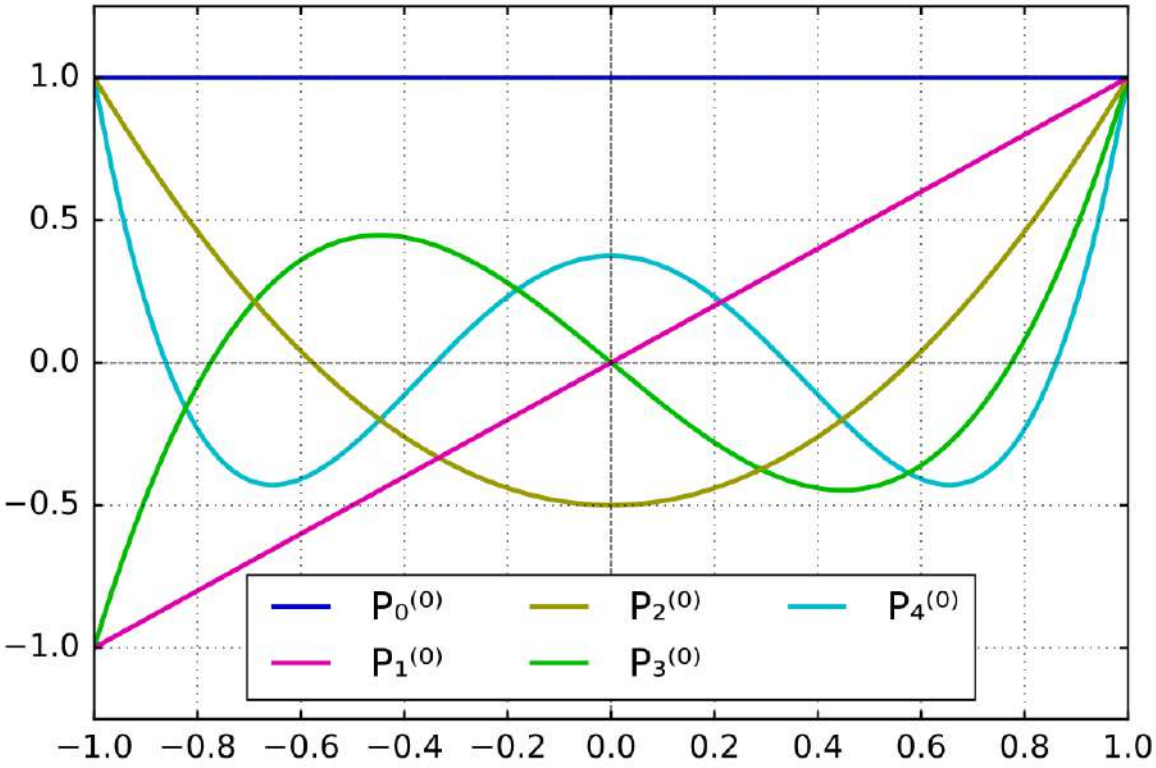


$$P_1^1(x) = (1-x^2)^{1/2} = \sin\theta$$
$$P_2^1(x) = 3x(1-x^2)^{1/2} = 3\cos\theta\sin\theta$$
$$P_2^2(x) = 3(1-x^2) = 3\sin^2\theta$$
$$P_3^1(x) = \frac{3}{2}(5x^2-1)(1-x^2)^{1/2} = \frac{3}{2}(5\cos^2\theta - 1)\sin\theta$$
$$P_3^2(x) = 15x(1-x^2) = 15\cos\theta\sin^2\theta$$
$$P_3^3(x) = 15(1-x^2)^{3/2} = 15\sin^3\theta$$
$$P_4^1(x) = \frac{5}{2}(7x^3-3x)(1-x^2)^{1/2} = \frac{5}{2}(7\cos^3\theta - 3\cos\theta)\sin\theta$$
$$P_4^2(x) = \frac{15}{2}(7x^2-1)(1-x^2) = \frac{15}{2}(7\cos^2\theta - 1)\sin^2\theta$$
$$P_4^3(x) = 105x(1-x^2)^{3/2} = 105\cos\theta\sin^3\theta$$
$$P_4^4(x) = 105(1-x^2)^2 = 105\sin^4\theta$$

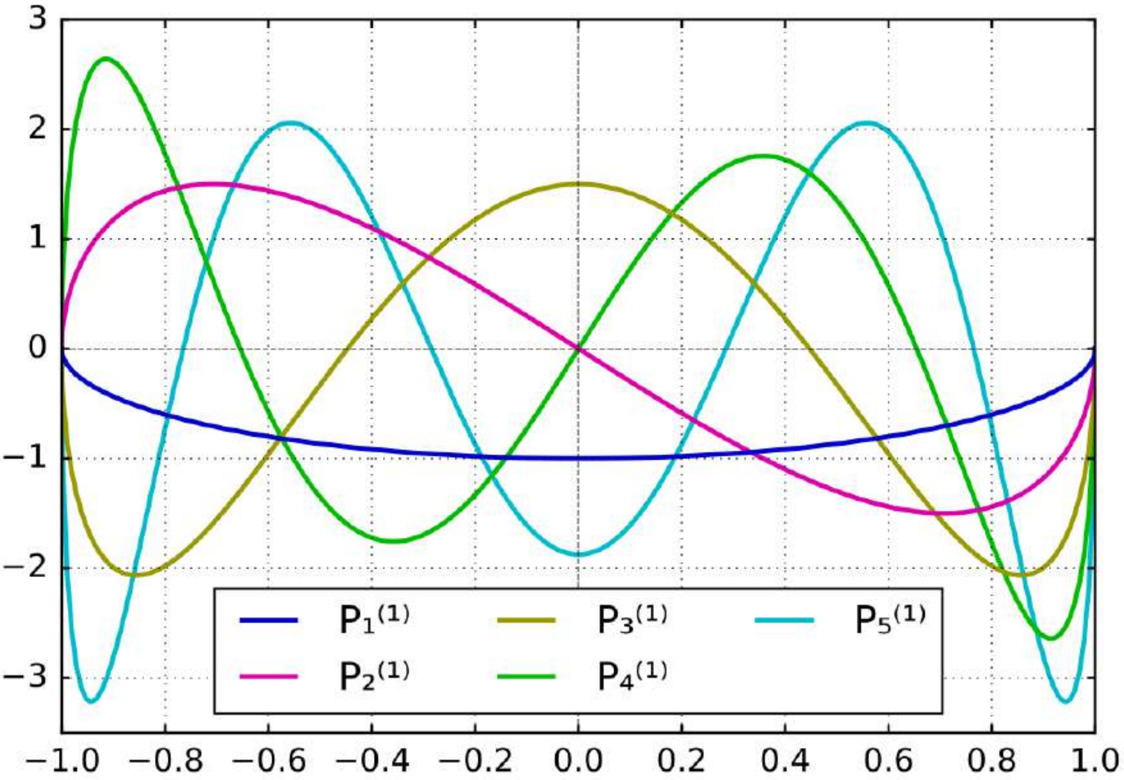
in agreement with line 4 of Table 1. To get line 3 we use Eq. (11). For m = 1, s = 0, this gives$P^2_2(x) = 3P^1_1 (x) = 3$, and multiplying by $1–x$^2 =(sin θ)^2 reproduces line 3 of Table 1. For lines 5, 8, and 9, Eq. (7.) may be used, which we leave as an exercise.

The first few associated Legendre function

Associated Legendre functions for *m* = 0:

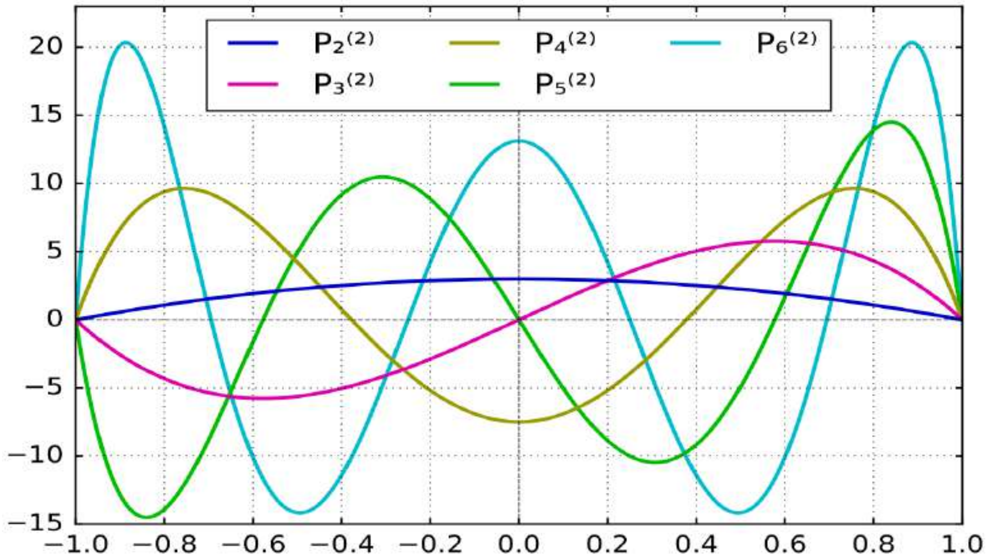


Associated Legendre functions for *m* = 1



Associated Legendre functions for *m* = 2

$$\mathcal{P}^{m}_{s+m}(1)=\frac{(2m)!}{2^{m}m!}\binom{-2m-1}{s}$$



➢ EXAMPLE-2: SPECIAL VALUES

For x=1 we use

$$(1-2t+t^2)^{-m-1/2} = (1-t)^{-2m-1} = \sum_{s=0}^{\infty} \binom{-2m-1}{s} t^s$$ **Equation (13)**

For m = 1, s = 0 we have $P^1_1(1)$ = $\binom{-3}{0}$=1;for s=1,$P^1_2(1)$= −$\binom{-3}{1}$ = 3; and for s = 2, $P^1_3(1)$ = $\binom{-3}{2}$ = (−3)*(−4)/ 2 = 6 = 3 /2 (5 − 1). These all agree with Table 1..For x = 0 we can also use the binomial expansion, which we leave as an exercise.

➢ EXAMPLE-3,PARITY:

From the identity $g_m(-x, -t) = g_m(x, t)$ we obtain the parity relation

$$\mathcal{P}^m_{s+m}(-x) = (-1)^s \mathcal{P}^m_{s+m}(x).$$

Equation (14) We have the **orthogonality integral**

$$\int_{-1}^{1} P^m_p(x) P^m_q(x)\, dx = \frac{2}{2q+1} \cdot \frac{(q+m)!}{(q-m)!} \delta_{pq}$$

or, in spherical polar coordinates,

$$\int_0^{\pi} P^m_p(\cos\theta) P^m_q(\cos\theta) \sin\theta \; d\theta = \frac{2}{2q+1} \cdot \frac{(q+m)!}{(q-m)!} \delta_{pq}.$$ **Eqation (15) and**

(16)he orthogonality of the Legendre polynomials is a special case of this result, obtained by settingmequal to zero; that is, form = 0, Eq. (15.) reduces to

$$\int_{-1}^{1} [P_n(x)]^2\, dx = \frac{2}{2n+1}.$$

$$\int_{-1}^{1} P_m(x) P_n(x)\, dx = \frac{2\delta_{mn}}{2n+1}.$$

In both Eqs. (15) and (16) our Sturm–Liouville theory(not explain here) of could provide the Kronecker delta. A special calculation is required for the normalization constant.

❖SPHERICAL HARMONICS:

Our earlier discussion of separated-variable methods for solving the Laplace, Helmholtz, or Schrödinger equations in spherical polar coordinates showed that the possible angular solutions Θ(θ)ϕ(ϕ) are always the same in spherically symmetric problems; in particular we found that the solutions for ϕ depended on the single integer index m, and can be written in the form

$$\Phi_m(\varphi) = \frac{1}{\sqrt{2\pi}}\, e^{im\varphi}, \quad m = \ldots, -2, -1, 0, 1, 2, \ldots,$$

the functions $\phi_m(\phi)$= e^imϕ are orthogonal when integrated over the azimuthal angle ϕ, whereas the functions P^m_n (cos θ) are orthogonal upon integrating over the polar angle θ. We take the product of the two and define

$$Y_n^m(\theta, \varphi) \equiv (-1)^m \sqrt{\frac{2n+1}{4\pi} \frac{(n-m)!}{(n+m)!}} P_n^m(\cos\theta) e^{im\varphi}$$

♣ Table -2

spherical harmonics (condon-shortly phase)

$$Y_0^0(\theta, \varphi) = \frac{1}{\sqrt{4\pi}}$$

$$Y_1^1(\theta, \varphi) = -\sqrt{\frac{3}{8\pi}} \sin\theta e^{i\varphi}$$

$$Y_1^0(\theta, \varphi) = \sqrt{\frac{3}{4\pi}} \cos\theta$$

$$Y_1^{-1}(\theta, \varphi) = +\sqrt{\frac{3}{8\pi}} \sin\theta e^{-i\varphi}$$

$$Y_2^2(\theta, \varphi) = \sqrt{\frac{5}{96\pi}} 3\sin^2\theta e^{2i\varphi}$$

$$Y_2^1(\theta, \varphi) = -\sqrt{\frac{5}{24\pi}} 3\sin\theta\cos\theta e^{i\varphi}$$

$$Y_2^0(\theta, \varphi) = \sqrt{\frac{5}{4\pi}} \left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^{-1}(\theta, \varphi) = +\sqrt{\frac{5}{24\pi}} 3\sin\theta\cos\theta e^{-i\varphi}$$

$$Y_2^{-2}(\theta, \varphi) = \sqrt{\frac{5}{96\pi}} 3\sin^2\theta e^{-2i\varphi}$$

to obtain functions of two angles (and two indices) that are orthonormal over the spherical surface. These $Y^m_n(\theta, \varphi)$ are spherical harmonics. The complete orthogonality integral becomest

$$\int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi} Y_{n_1}^{m_1 *}(\theta, \varphi) Y_{n_2}^{m_2}(\theta, \varphi) \sin\theta \, d\theta \, d\varphi = \delta_{n_1 n_2} \delta_{m_1 m_2}$$

Equation (17). And explains the presence of the complicated normalization constant in Eq. (17).

The extra (–1^)m included in the defining equation of $Y^m_n(\theta, \phi)$ with $-n \le m \le n$ deserves some comment. It is clearly legitimate since Eq. (1) is linear and homogeneous. It is not necessary, but in moving on to certain quantum mechanical calculations, particularly in the quantum theory of angular momentum, it is most convenient. The factor (–1)^m is a phase factor often called the Condon–Shortley phase after the authors of a classic text on atomic spectroscopy. The effect of this (–1)^m for $P^{-m}_n(\cos\theta)$ is to introduce an alternation of sign among the positive m spherical harmonics. This is shown in Table 2.

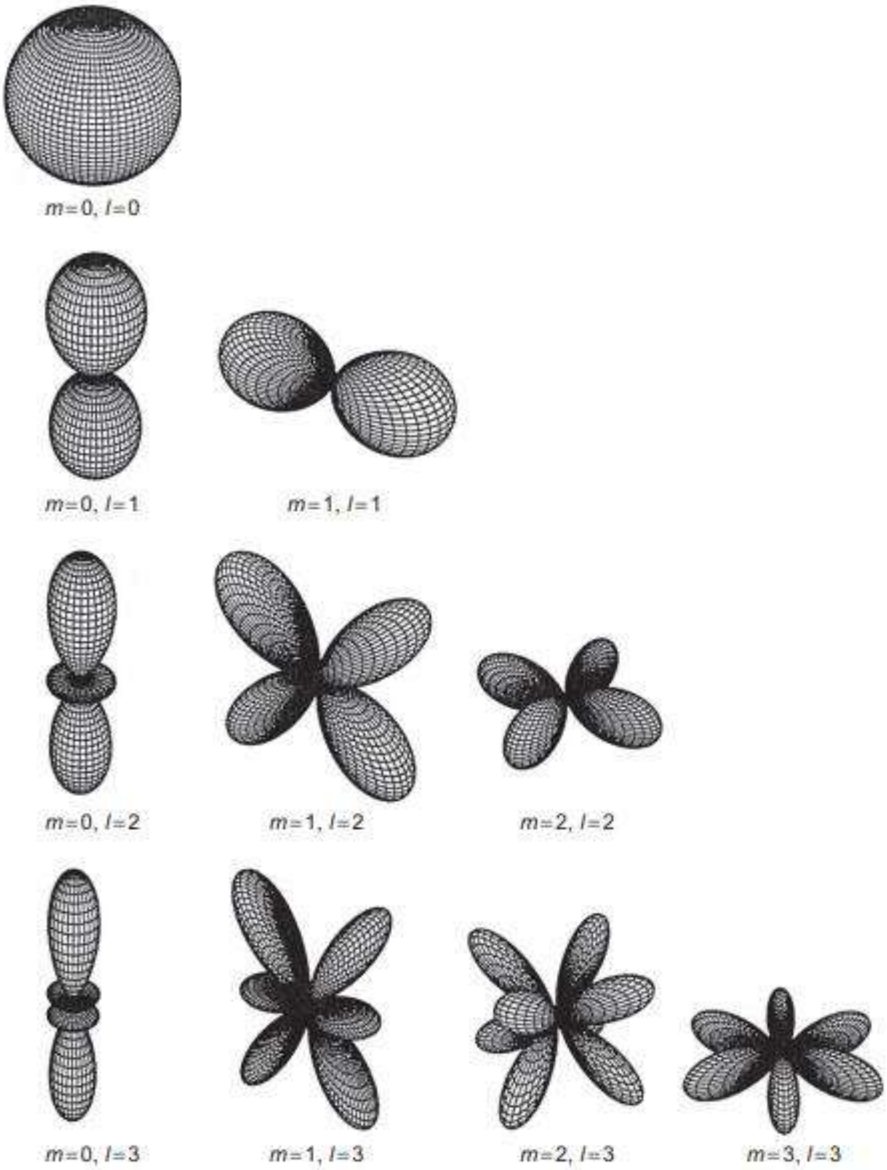
The functions $Y^m_n(\theta, \phi)$ acquired the name spherical harmonics because they are defined over the surface of a sphere with θ the polar angle and ϕthe azimuth. The "harmonic" was included because solutions of Laplace's equation were called harmonic functions and $Y^m_n(\cos\phi)$ is the angular part of such a solution.

CERTESIAN REPRESENTATION:

For some purposes it is useful to express the spherical harmonics using Cartesian coordinates, which can be done by writing exp(±iφ) as cosφ ± i sinφ and using the formulas for x, y,z in spherical polar coordinates (retaining, however, an overall dependence on r, necessary because the angular quantities must be independent of scale). For example, cos θ = z/r, sin θ exp(±iφ) = sin θ cosφ ± i sin θ sinϕ=$\frac{x}{r}$±i$\frac{y}{r}$.

these quantities are all homogeneous (of degree zero) in the coordinates. Continuing to higher values of l, we obtain fractions in which the numerators are homogeneous products of x, y, z of overall degree l, divided by a common factor r^l includes the Cartesian expression for each of its entries.

➢ SOME SHAPES OF SPHERICAL HARMONICS:



Shapes of $|\Re e Y_l^m(\theta, \varphi)|^2$ for $0 \leq l \leq 3, m = 0 \ldots l$.

❖ LAPLACE EXPANSION:

Part of the importance of spherical harmonics lies in the completeness property, a consequence of the Sturm-Liouville form of Laplace's equation. Here this property means that any function f (θ , ɸ) (with sufficient continuity properties) evaluated over the surface of a sphere can be expanded in a uniformly Convergent double series of spherical harmonics.

EXAMPLE 4:LOWEST SPHERICAL HARMONICS:

For n = 0 we have m = 0 and Y^{0}_{0}= 1/√ 4π from Eq. (17). For n = 1 we have m = ±1, 0 and Y^{0}_{1} =$\sqrt{3/4\pi}$ cos θ, whereas for m = ±1 we see from Table 1 that cos θ is replaced by sin θ and we have the additional factor (∓1) e±iφ /√2 , which checks with Table . In the framework of quantum mechanics Eq. (3) becomes an orbital angular momentum momentum eigenfunction, with L being the angular momentum quantum number and M the z-axis projection o

EXAMPLE 5:SPHERICAL SYMMETRY OF PROBABILITY DENSITY OF ATOMIC STATES:

What is the angular dependence of the probability density of the degenerate atomic states with principal quantum number n = 2?

Here we have to sum the absolute square of the wave functions for n = 2 and orbital angular momentum l = 0, m = 0; l = 1, m = −1, 0, +1; that is, s and three p states. We ignore the radial dependence. The s state has orbital angular momentum l = 0 and m = 0 and is independent of angles. For the p states with l = 1 and m = ±1,0

equation and the solution Y^{M}_{L} (θ, φ) (n replaced by L and m by M) is an angular

$$\psi_{200} \sim Y_0^0 = \frac{1}{\sqrt{4\pi}}, \quad \psi_{21m} \sim Y_1^m,$$

We have to evaluate the sum

$$\sum_{m=-1}^{1} |Y_1^m|^2 = \frac{3}{4\pi}\left[2\left(\frac{1}{\sqrt{2}}\sin\theta\right)^2 + \cos^2\theta\right] = 1$$

alone or the sum of the three p states. upon substituting the spherical harmonics from Table 2. This result is spherically symmetric, as is the density for the s state These results can be generalized to higher orbital angular momentum.

EXAMPLE 6:*SPHERICAL HARMONIC EXPANSION:*

Consider the problem of determining the electrostatic potential within a charge-free spherical region of radius r0, with the potential on the spherical bounding surface specified as an arbitrary function V(r0, θ , φ) of the angular coordinates θ and φ. The potential V(r, θ , φ) is the solution of the Laplace equation satisfying the boundary condition at r = r0 and regular for all r ≤r0 **.**This means it must be of the form of with the coefficients blm set to zero to ensure a solution that is nonsingular at r = 0. We proceed by obtaining the spherical harmonic expansion of V(r0, θ ,ϕ).

$$V(r,\theta,\varphi)=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}c_{lm}\left(\frac{r}{r_0}\right)^{l}Y_l^m(\theta,\varphi).$$

SUMMARY:

Legendre polynomials are naturally defined by their generating function in a multipole expansion of the Coulomb Zpotential. They appear in physical systems with azimuthal symmetry. They also arise in the separation of partial differential equations with spherical or cylindrical symmetry or as orthogonal eigenfunctions of the Sturm–Liouville theory of their second-order differential equation. Associated Legendre polynomials appear as ingredients of the spherical harmonics in situations that lack in azimuthal symme

BIBLIOGRAPHY:

- (1) Abramowitz, M., and I. A. Stegun, eds., Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables (AMS-55). Washington, DC: National Bureau of Standards (1972), reprinted, Dover (1974).
- (2) Hobson, E. W., The Theory of Spherical and Ellipsoidal Harmonics. New York: Chelsea (1955). This is a very complete reference, which is the classic text on Legendre polynomials and all related functions.
- Jackson, J. D., Classical Electrodynamics, 3rd ed. New York: Wiley (1999).
- Margenau, H., and G. M. Murphy, The Mathematics of Physics and Chemistry, 2nd ed. Princeton, NJ: Van Nostrand (1956).
- Morse, P. M., and H. Feshbach, Methods of Theoretical Physics, 2 vols. New York: McGraw-Hill (1953). This work is detailed but at a rather advanced level.
- Smythe, W. R., Static and Dynamic Electricity, 3rd ed. New York: McGraw-Hill (1968), reprinted, Taylor & Francis (1989), paperback. Advanced, detailed, and difficult. Includes use of elliptic integrals to obtain closed formulas.
- Whittaker, E. T., and G. N. Watson, A Course of Modern Analysis, 4th ed. Cambridge, UK: Cambridge University Press (1962), paperback.

HYBRID ELECTRIC VEHICLES

&

A comparison with conventional Vehicles

The project submitted, in partial fulfilment of the requirement for the assignments in CC XI, CC XII, DSE I, DSE II Paper (Semester V) in the Department of Physics

Submitted by:

Deep Sarkar

Registration No: A01-1112-111-007-2019

Supervisor Teacher: Prof. Anjan Kumar Chandra

RAMAKRISHNA MISSION VIVEKANANDA CENTENARY COLLEGE

P.O. RAHARA, KOLKATA-700118

WEST BENGAL, INDIA

Aim of the Project

Driving is one of our daily activities with the most significant impact on the environment therefore choosing a green option is an essential part of a sustainable lifestyle. Hybrid cars are the most popular auto green options. The purpose of hybrid cars is to offer a cutting-edge and practical solution to contemporary drivers and gas-electric hybrid cars are the most popular options. Some models produce as much as 90% less toxic gases than conventional, non-hybrid vehicles.

The purpose of hybrid cars is to combine a gas engine and an electric motor that assists the engine when accelerating. The batteries that power the electric motor are recharged automatically while driving. Therefore the main purpose of hybrid cars is to cut down on fossil fuels while maintaining excellent performance and saving money in the process as the government offers tax incentives to those who purchase a hybrid car.

The government of India has also adopted the Faster Adoption and Manufacturing of Hybrid and EV (FAME) scheme in 2015 with an outlay of INR 8.95 billion (USD 130 million), which provided subsidies for electric 2- and 3-wheelers, hybrid and e-cars and buses.

The scaled up FAME II, effective from April 2019, has an outlay of INR 100 billion (USD 1.4 billion) to be used for upfront incentives on the purchase of EVs (INR 85.96 billion) and for supporting the deployment of charging infrastructure (INR 10 billion).

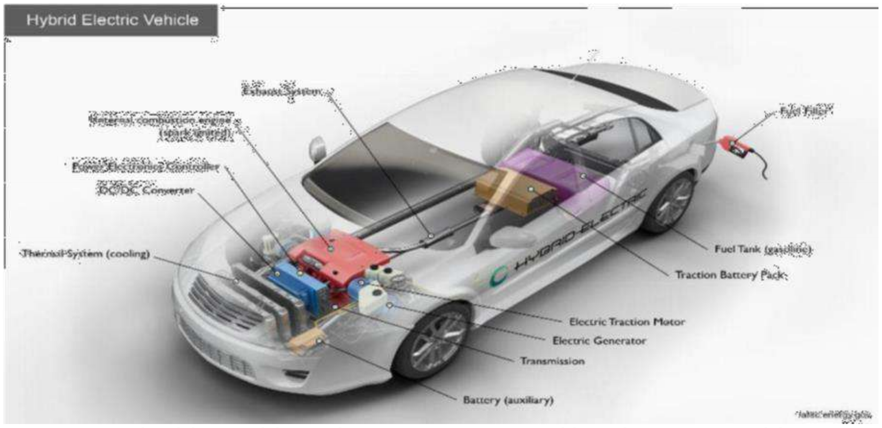
This project is a study of Hybrid Vehicles and how much effective they are in reducing pollution. The aim of this project is to build awareness about Hybrid Cars and show all the important statistical studies related to it.

History of HEV’s

The **history of hybrid cars** stretches back over 100 years. Hybrid cars are defined as any car that runs on two sources of power. The most common hybrid powertrain combines a gasoline engine with an electric motor. These cars are known as hybrid electric vehicles (HEVs). While it may seem that hybrids are a recent phenomenon, the technology has been around since the creation of the automobile. In fact, auto manufacturers have been developing and building hybrids since the beginning of the auto industry.

- 1830’s
  - Battery electric vehicle invented by Thomas Davenport, Robert Anderson, others using non-rechargeable batteries
- 1890’s
  - EV’s outsold gas cars 10 to 1, Oldsmobile and Studebaker started as EV companies
- 1904’s
  - Krieger company builds first Hybrid Vehicle
- 1910’s
  - Mass-produced Ford cars undercut hand-built EVs
  - EV’s persist as status symbols and utility vehicles until Great Depression

Components of a Hybrid Electric Vehicle



Battery (auxiliary): In an electric drive vehicle, the low-voltage auxiliary battery provides electricity to start the car before the traction battery is engaged; it also powers vehicle accessories.

DC/DC converter: This device converts higher-voltage DC power from the traction battery pack to the lower-voltage DC power needed to run vehicle accessories and recharge the auxiliary battery.

Electric generator: Generates electricity from the rotating wheels while braking, transferring that energy back to the traction battery pack. Some vehicles use motor generators that perform both the drive and regeneration functions.

Electric traction motor: Using power from the traction battery pack, this motor drives the vehicle's wheels. Some vehicles use motor generators that perform both the drive and regeneration functions.

Exhaust system: The exhaust system channels the exhaust gases from the engine out through the tailpipe. A three-way catalyst is designed to reduce engine-out emissions within the exhaust system.

Fuel filler: A nozzle from a fuel dispenser attaches to the receptacle on the vehicle to fill the tank.

Fuel tank (gasoline): This tank stores gasoline on board the vehicle until it's needed by the engine.

Internal combustion engine (spark-ignited): In this configuration, fuel is injected into either the intake manifold or the combustion chamber, where it is combined with air, and the air/fuel mixture is ignited by the spark from a spark plug.

Power electronics controller: This unit manages the flow of electrical energy delivered by the traction battery, controlling the speed of the electric traction motor and the torque it produces.

Thermal system (cooling): This system maintains a proper operating temperature range of the engine, electric motor, power electronics, and other components.

Traction battery pack: Stores electricity for use by the electric traction motor.

Transmission: The transmission transfers mechanical power from the engine and/or electric traction motor to drive the wheels.

Vehicle Type

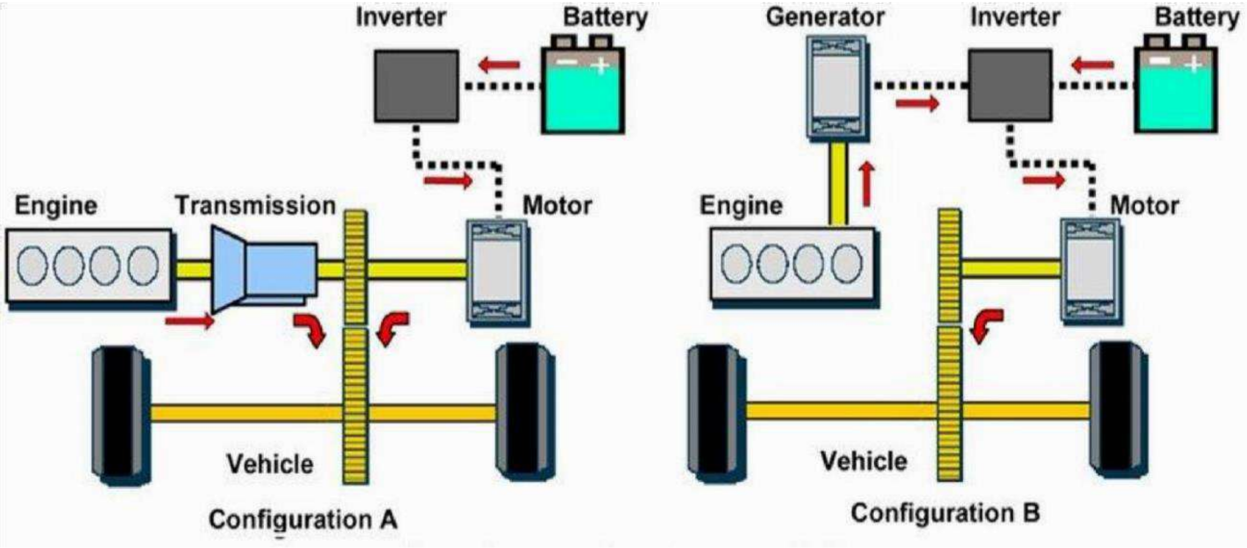
Two power sources found in Hybrid Vehicles may be combined in different ways, either in parallel or series.

PARALLEL HYBRID

- Gasoline motor
- Batteries which powers an electric motor
- Both can power the transmission at the same time
- Electric motor supplements the gasoline engine

SERIES HYBRID

- Gasoline motor turns a generator
- Generator may either charge the batteries or power an electric motor that drives the transmission
- At low speed is powered only by the electric motor



Configuration A: Parallel, Configuration B: Series

Working Principle

To know the working of a hybrid car, we must understand the basics of Mild Hybrid cars and Full Hybrid cars.

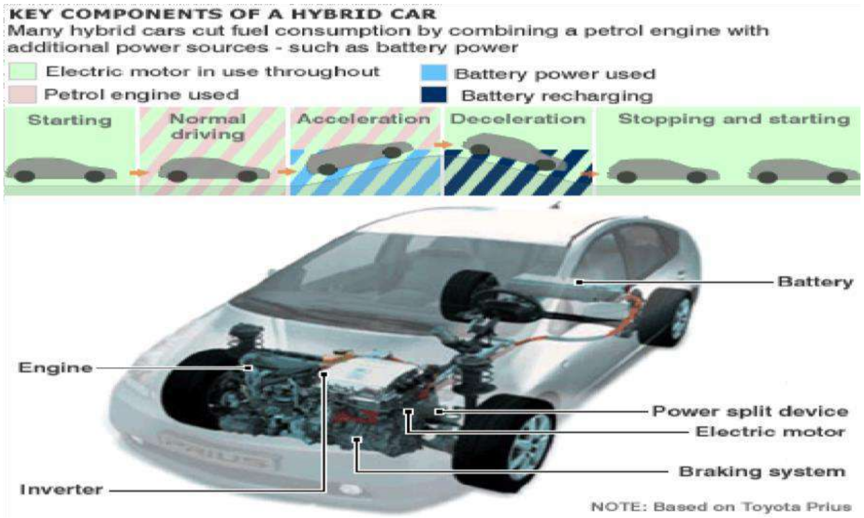
In mild hybrid cars, the electrical motor is used only when additional power is needed. The conventional engine is used to provide most of the power. The electrical motor alone cannot operate the vehicle. Whenever power is needed the electric motor acts as a side-kick to the conventional engine. Some vehicles that carry this concept is the Honda Civic and Insight.

In a full hybrid car, the electrical energy is used while the car needs less power. The gasoline energy is used when the car needs less power. Thus at lower speeds the battery drives the vehicle and at higher speed the gasoline drives the vehicle. This technology has been used in cars like Toyota Prius and Ford Escape.

Both of them though have a little different mode of operation provide the same amount of efficiency.

Since both electric motor and an engine are used simultaneously, the size of the engine will be considerably smaller than the usual ones. But they will be a lot more advanced than the usual ones. The motor, on the other hand is also used to give power for the air conditioner, power windows, water pump and also power steering.

Let’s take a look at the diagram given below. It shows the actual working of the hybrid car Toyota Prius. During the starting position, none of the system is working. After the car starts to move, it is in the normal driving mode. Thus the car will automatically change to the use of electric motor. Later when the car is accelerated and gains speed, it switches from the use f motor to the use of engine. Thus the gasoline engine supplies the required power. This switching is carried out automatically, with the help of an on-board computer. Since the battery has lost some of its charge, it needs to be immediately recharged. This is also done automatically. When the car starts to go in a uniform speed or when it is descending a road, the generator starts charging the battery.



Comparison between a hybrid car and a gasoline car

If you compare the power drive of both the cars, you will see that both of them are equally efficient. While a gasoline car has a bigger engine, the hybrid car has a smaller engine. Conventional cars have enough power to attain the required speed, and that too at the required time. In a hybrid car, as the engine is smaller, it is more efficient. It has lighter parts and reduced number of cylinder. The fuel required for smaller engines is lesser than the other ones. Since bigger engines have all the pistons in a bigger size, they need more energy when they make an up and down movement in the cylinder. Even if the car is not moving, the engine may be on. At this time also the big engine cylinders use fuel.

If both the cars are moving equally the car with gas engine will use its whole single power to drive the car. The hybrid car will also need the same output power to drive the car. But, as it is smaller, it makes lesser power than the bigger one. This is when the electric motor comes into play. They provide the rest of the power from the battery to compensate the balance.

Hybrid Vehicles in India

Cost concerns have deterred manufacturers from introducing hybrid electric vehicle (HEVs) and battery electric vehicle (BEV) technologies in India until recently, but this seems poised to change following the introduction of incentives to boost the penetration of these vehicles. In FY 2015-16, hybrid and electric passenger vehicles constituted approximately 1.3% of all passenger vehicle sales in India, up from essentially zero in FY 2012-13.

The flagship program to boost hybrid and electric technologies in India is the Faster Adoption and Manufacturing of (Hybrid &) Electric Vehicles (FAME) scheme from the Central Government, launched in April 2015

In 2015, the Government of India announced corporate average weight-based fuel-consumption standards for passenger cars, with phase-wise targets declared for FY 2017-18 to FY 20121-22, and FY 2022-23 onward. As depicted in **Figure 1**, the fuel consumption for the eligible models under the FAME scheme all come well under the 2017-18 standard limit of about 5.5 gasoline equivalent liters/100 km (or 130 g/km when expressed in terms of CO_2 emissions). All of the above models also fall under the 5-star fuel efficiency label as per the star labeling methodology proposed by the government's Bureau of Energy Efficiency (BEE).

Efficiency gains of the registered hybrid and electric models under the FAME scheme compared with their base models are presented in **Table 1**.

Sales of hybrid and electric passenger cars in India in FY 2015-16 resulted in fuelconsumption reductions of approximately 2.97 million gasoline equivalent liters. This is based on the assumption that a unit sale of a hybrid or electric model displaced a unit sale of a corresponding base model from the market.

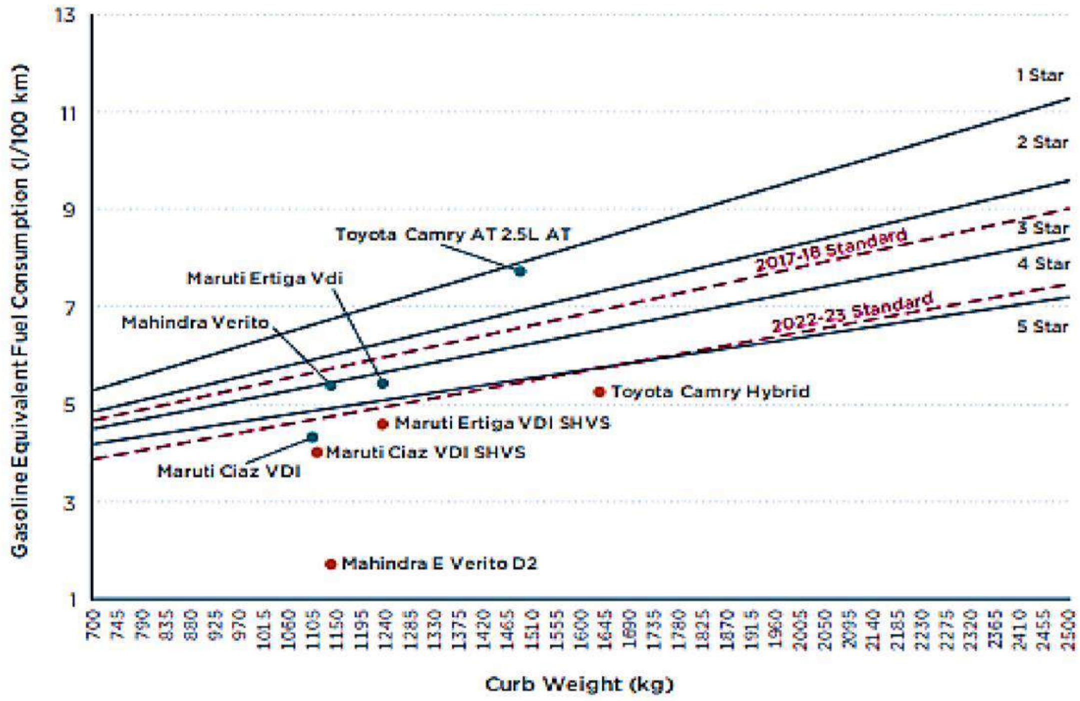
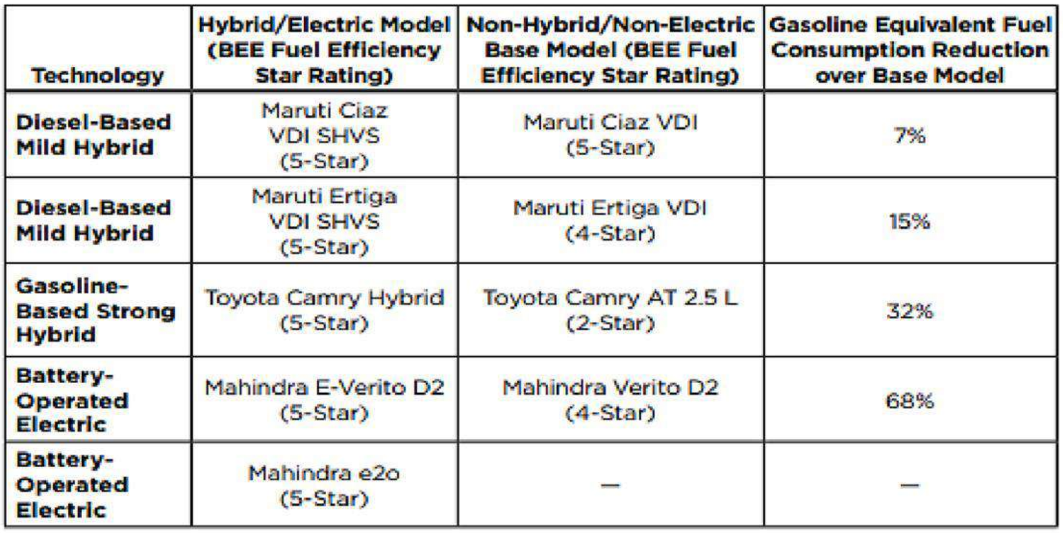


Figure 1 Fuel Consumption of Hybrid and Electric Models and Their Base Models

Table 1 Fuel Consumption Savings of Models Under FAME Scheme Compared with Base Models.



Technology	Hybrid/Electric Model (BEE Fuel Efficiency Star Rating)	Non-Hybrid/Non-Electric Base Model (BEE Fuel Efficiency Star Rating)	Gasoline Equivalent Fuel Consumption Reduction over Base Model
Diesel-Based Mild Hybrid	Maruti Ciaz VDI SHVS (5-Star)	Maruti Ciaz VDI (5-Star)	7%
Diesel-Based Mild Hybrid	Maruti Ertiga VDI SHVS (5-Star)	Maruti Ertiga VDI (4-Star)	15%
Gasoline-Based Strong Hybrid	Toyota Camry Hybrid (5-Star)	Toyota Camry AT 2.5 L (2-Star)	32%
Battery-Operated Electric	Mahindra E-Verito D2 (5-Star)	Mahindra Verito D2 (4-Star)	68%
Battery-Operated Electric	Mahindra e2o (5-Star)	—	—

Life Cycle Emission Analysis

As can be seen in Figure 3, emissions for electric models are due to electricity consumption from the Indian grid (including transmission and distribution efficiencies), while for the gasoline- and diesel-based models, the majority of the emissions are on account of fuel consumption during vehicle operation.

As outlined in Figure 4, the current Indian grid is powered by predominantly thermal sources with coal-fired power plants forming the majority of the total installed capacity from 36 GW in 2015 to 175 GW by 2022, as per India's

Intended Nationally Determined Contribution (INDC) under the United Nations Framework Convention on Climate Change This is expected to result in an abatement of an estimated 326 million tonnes of CO_2 per year. Further, it is reported that the efficiencies of power generation as well as transmission and distribution in India are significantly lower than global averages, and improvement of efficiencies in the power sector have also been recognized as a priority in the INDC. Thus, the life-cycle emissions intensity of electric vehicles in India are poised for substantial reductions in the near term in alignment with India's post 2020 climate action plans. Thus, it is important to build up markets for electric mobility in a parallel timeframe as the renewable energy footprint and energy efficiency of India's power sector undergo transformational improvements.

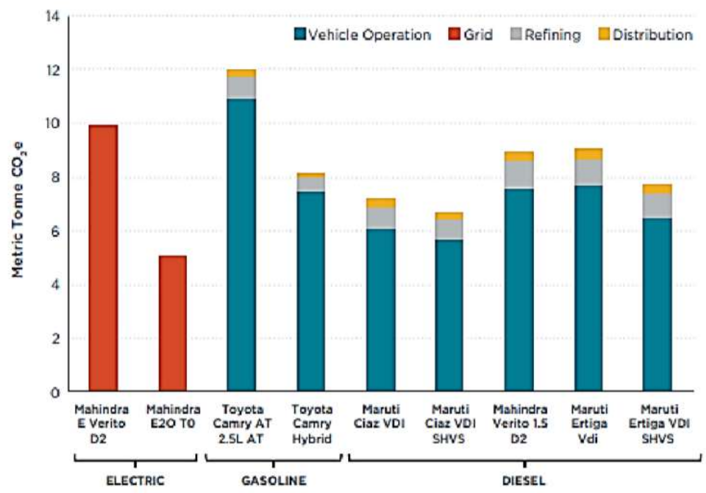


Figure 3. Total Life Cycle Emissions Over 5-year Use Period of Electric and Hybrid Cars and Their Base Models

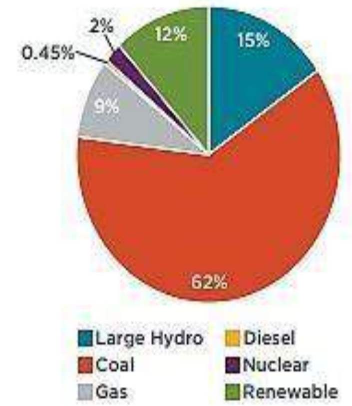


Figure 4. Composition of Indian Grid by Source as of FY 2014-15[30]

Advantages and Disadvantages

Advantages

- Very less pollution.
- Better mileage.
- More reliable and comfortable.
- Very clean cars due to less emissions.
- Batteries need not be charged by an external source.
- Warranties available for batteries as well as motors.
- Less dependence on fuels.

Disadvantage

- The initial cost will be very high – higher than other cars.
- Since a lot of batteries will be needed, the car will be very heavy.
- As there are electrical components, there is risk of shock during an accident.
- The vehicle can be repaired only by professionals.
- Spare parts will be very costly and rare.

Discussion

Internal combustion engines operate best when under a constant load. Engines operating at their most efficient configuration will have increased longevity, be less complex, cost less to manufacturer and emit lower emissions. Hybrid drivetrains will combine the ICE and electric motor to take advantage of the best characteristics of both. The hybrids of the future will circumvent the trade-off between power and efficiency that current internal combustion engine powered cars are subject to. Hybrids have been sold in the U.S. for over 20 years and there is opportunity for many future refinements. Hybrid vehicles will bridge the gap between fuel-powered only to all-electric power. The hybrid of the future will be less expensive to own and operate, plus provide consumers with a fun, environmentally friendly driving experience.

Sources

- https://afdc.energy.gov/fuels/electricity_benefits.html
- https://www.edfenergy.com/for-home/energywise/electric-cars-and-environment
- https://beeindia.gov.in/content/e-mobility
- https://www.mikesmithhonda.com/gas-or-hybrid-car

Acknowledgement

The success and final outcome of this project required a lot of guidance and assistance from many people and I am extremely privileged to have got this all along the completion of my project. All that I have done is only due to such supervision and assistance and I would not forget to thank them.

I respect and thank our Principal, Swami Kamalasthananda and Vice Principal, Swami Vedanuragananda for providing me an opportunity to do the project work. I am extremely thankful to the Head of the Department of Physics, Prof. Asok Kumar Pal for providing such a nice support and guidance although he had a very busy schedule.

I owe my deep gratitude to our project guide Prof. Anjan Kumar Chandra, who took keen interest on my project work and guided me all along, till the completion of the project work by providing all the necessary information.

I heartily thank all my friends and classmates for their encouragement and more over for their timely support and guidance till the completion of my project work.

I am thankful to and fortunate enough to get constant encouragement, support and guidance from all Teaching staffs of RKMVCC Physics Department which helped me in successfully completing my project work.

Deep Sarkar, Physics Dept.

STELLAR STRUCTURE AND EVOLUTION

A dissertion submitted to the Department of Physics Faculty

RAMAKRISHNA MISSION VIVEKANANDA CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118 WEST BENGAL, INDIA

by
Debasish Ghosh
B.Sc. (hons) , semester v
Registration no :A01-1112-111-008-2019

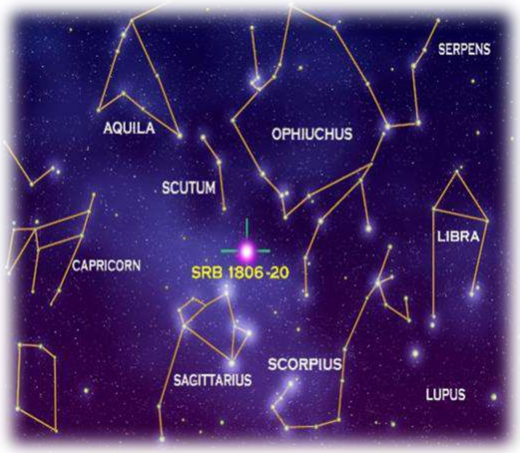
Under the supervision
of
Dr. Palash Nath

January 2022

Table of Contents

WHAT IS STAR?

A star is an astronomical object consisting of luminous spheroid of plasma held together in its own gravity. The nearest star is the Sun .Many of other stars are visible to the naked eye at night, but due to their immense distance from earth they appear as fixed points of light in the sky .The most prominent stars are grouped into *constellations* and *asterism* and many other of the biggest stars have proper names .According to the latest data the observable universe contain an estimate 10^22 to 10^24 stars. For most of its active life , a stars shines due to thermonuclear fusion of hydrogen into helium in its core ,releasing energy that traverses the star interior and then radiates into outer space .A star life begins with the gravitational collapse of a gaseous nebula of material composed primarily of hydrogen , along with helium and trace amounts of heavier elements when the stellar core is sufficiently dense ,hydrogen becomes steadily converted into helium through nuclear fusion, releasing energy in this process .Every Stars have an expiry date and die out like all entities in the universe. When they die, they go out with a large explosion which is called a “supernova”. Stars usually collapse due to the weight of gravity when they run out of fuel, which produces a strong explosion. Additionally, some stars simply morph into a black hole. These are the two ways for stars to go out.



Constellations

*A **constellation** is an area on the celestial sphere in which a group of visible stars forms a perceived outline or pattern, typically representing an animal, mythological person or creature, or an inanimate object*

(Credits: NASA)

Asterism

*In observational astronomy, an **asterism** is a pattern or group of stars that can be seen in the night sky. Asterisms range from simple shapes of just a few* stars to more complex collections of many stars covering large portions of the sky.

(Credits: COSMOS)

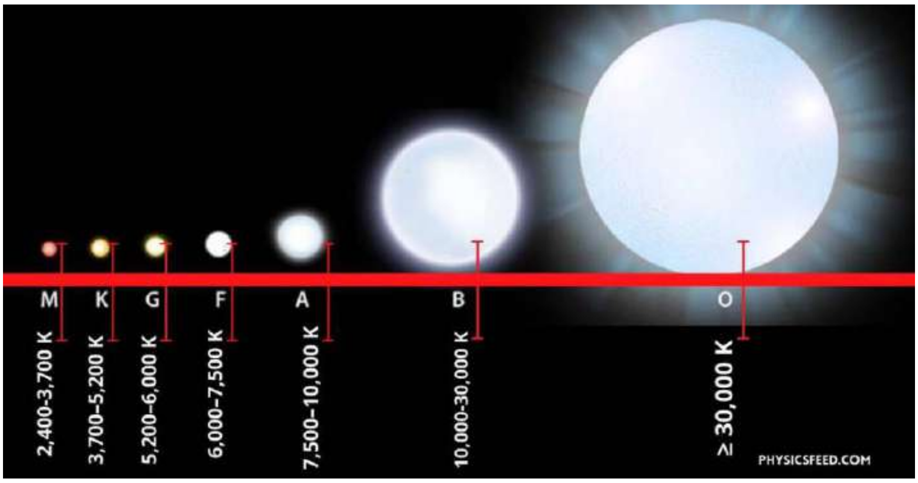
Types of stars:

Stars are classified by their spectra (the elements that they absorb) and their temperature. There are seven main types of stars , in order of decreasing temperature is O,B,A,F,G,K and M .Stellar classification is a scheme assigning stars their type on the basis temperature as calculated from their spectra .The classification of temperature is done on the basis of Wien's law regarding black body radiation .The universally accepted system of stellar classification is a combination of 2 schemes, the ***Harvard system*** which is based on the stars surface temperature and the ***Morgan Keenan*** (MK) system which is based on the stars luminosity .

Harvard spectral classification :

The absorption features present in stellar spectra allow us to divide stars into several spectral types depending on the temperature of the star. The scheme in use today is the *Harvard spectral classification* scheme which was developed at Harvard college observatory in the late 1800s. So, the classification is -

Types of stars	O	B	A	F	G	K	M
Colour of stars	Violet	Blue	Blue	Blue-white	White-yellow	Orange-red	Red
Temperature	≥28000k	10000-28000k	7500-10000k	6000-7500k	5000-6000k	3500-5000	≤3500k



Each class is subdivided into 10 subclasses, numbered from 0 to 9 (our Sun is a G2 star with characteristics part-way between G and F).

(Source: https://physicsfeed.com/post/harvard-spectral-classification/)

Morgan Keenan (MK) system :

Morgan Keenan classification of star is the classification of star depending on their luminosity. The Harvard classification scheme does not completely describe the star as it cannot distinguish between stars with the same temperature but different luminosities. In other words, it cannot distinguish between main sequence (dwarf) stars, giant stars and supergiant stars.

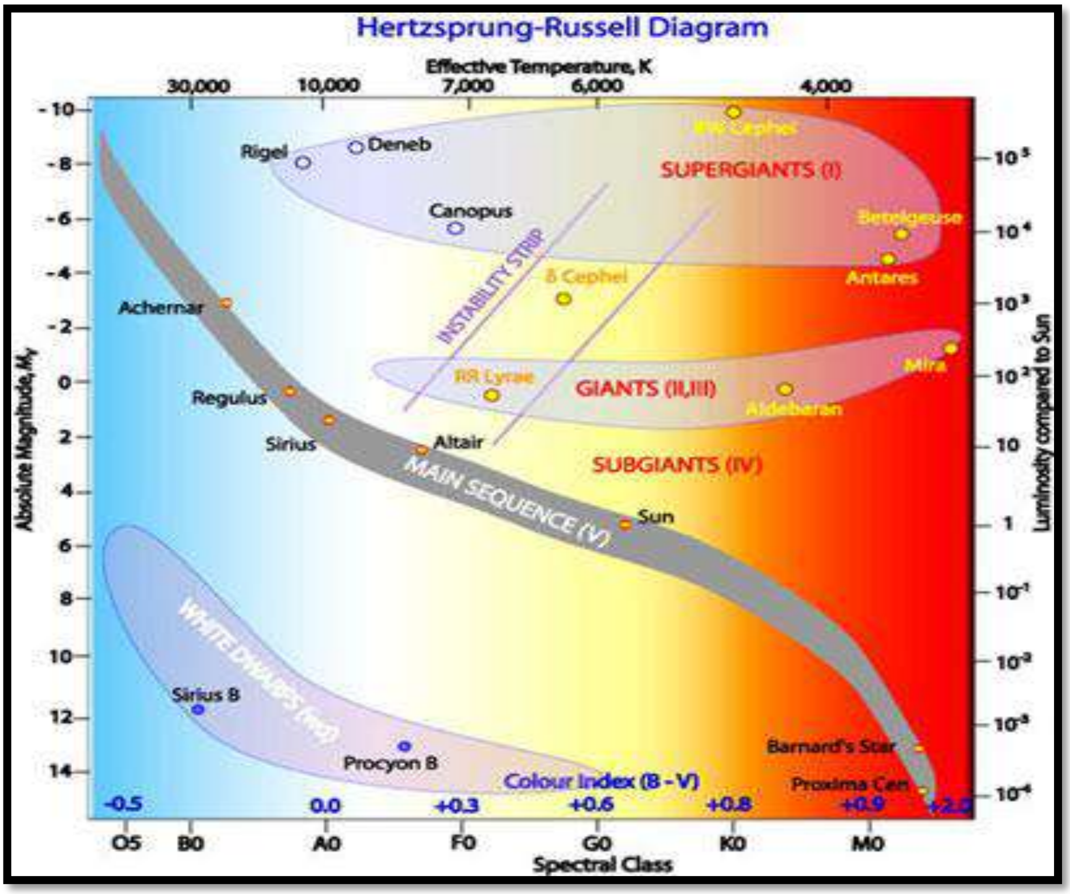
For this reason, the Morgan-Keenan luminosity class (MK or MKK) was established. Originally containing roman numerals between I (supergiant star) and V (main sequence), these days, class I stars have been subdivided into Ia-O, Ia and Ib, and classes VI (sub-dwarf) and D (white dwarf) have been added.In luminosity scale, stars are classified by-

Class	Star
Ia-O	extremely luminous supergiants
Ia	luminous supergiants
Ib	less luminous supergiants
II	bright giants
III	normal giants
IV	Subgiants
V	main sequence dwarf stars
VI	Sub dwarfs
D	White dwarfs

(Source: cosmos)

Hertzsprung-Russell Diagram:

The Hertzsprung-Russell diagram (HR diagram) is one of the most important tools in the study of stellar evolution. Developed independently in the early 1900s by *Ejnar Hertzsprung* and *Henry Norris Russell,* it plots the temperature of stars against their luminosity (the theoretical HR diagram), or the colour of stars (or spectral type) against their absolute magnitude (the observational HR diagram, also known as a colour-magnitude diagram).Depending on its initial mass, every star goes through specific evolutionary stages dictated by its internal structure and how it produces energy. Each of these stages corresponds to a change in the temperature and luminosity of the star, which can be seen to move to different regions on the HR diagram as it evolves. This reveals the true power of the HR diagram – astronomers can know a star's internal structure and evolutionary stage simply by determining its position in the diagram.



(Source:cosmos)

There are 3 main regions (or evolutionary stages) of the HR diagram:

- The main sequence stretching from the upper left (hot, luminous stars) to the bottom right (cool, faint stars) dominates the HR diagram. It is here that stars spend about 90% of their lives burning hydrogen into helium in their cores. Main sequence stars have a Morgan-Keenan luminosity class labelled V
- Red giant and supergiant stars (luminosity classes I through III) occupy the region above the main sequence. They have low surface temperatures and high luminosities which, according to the Stefan-Boltzmann law, means they also have large radii. Stars enter this evolutionary stage once they have exhausted the hydrogen fuel in their cores and have started to burn helium and other heavier elements.
- White dwarf stars (luminosity class D) are the final evolutionary stage of low to intermediate mass stars, and are found in the bottom left of the HR diagram. These stars are very hot but have low luminosities due to their small size.

FORMATION OF STAR:

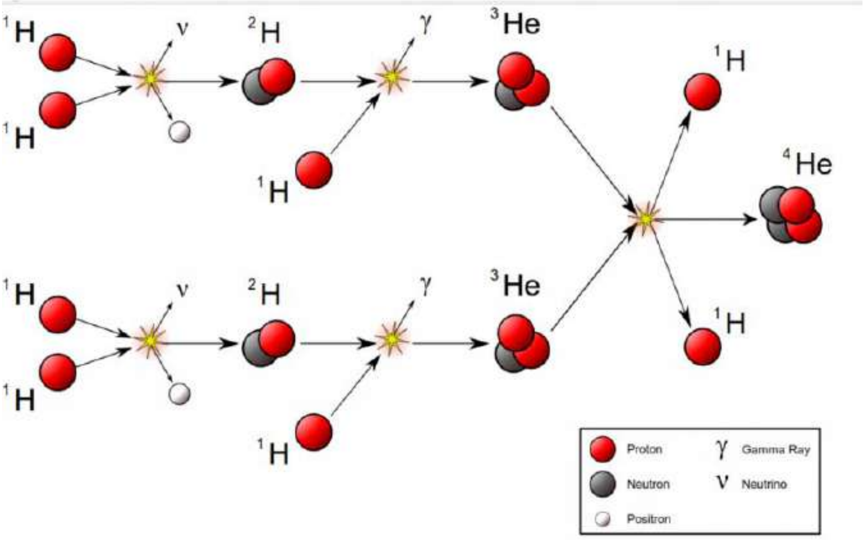
The life cycle of any star, from birth to death, and all the stages in between, will span millions or even billions of years. The path that will be followed by a particular star depends mainly on its mass, or how much gas collected and collapsed to form the star because that material will serve as the stars fuel. As we may remember from physics and chemistry, when nuclei collide with enough energy so as to overcome the electromagnetic repulsion between them, the strong nuclear force takes over , and they fuse , with a small fraction of their mass converting into huge amounts of pure energy , as dictated by equals MC^2. Therefore, only by colliding nuclei together and fusing them in its ultra hot core can a star release enough outward energy to counter the effects of gravity relentlessly crushing inward this means that the amount of matter that forms the star determines the amount of fuel, and through a variety of other factors the lifetime and eventual fate of the star.

FORMATION OF LOW MASS STAR:

- The life cycle of a low mass star will begin from a large cloud of gas and dust at least a few light years across.
- The first, stars were formed from clouds made of almost only hydrogen and helium.
- Gravity causes the gas cloud to contract until fusion begins.

Fusion process:

These fusion reactions begin with two protons fusing followed by subsequent beta decay, to get a proton and a neutron, and we call this a deuteron, which is a nucleus of heavy hydrogen then deuterons are involved in reactions that make helium, which has two protons and to neutrons such a star will continue in this manner.



(Fusion reaction in star: 2 proton – helium)

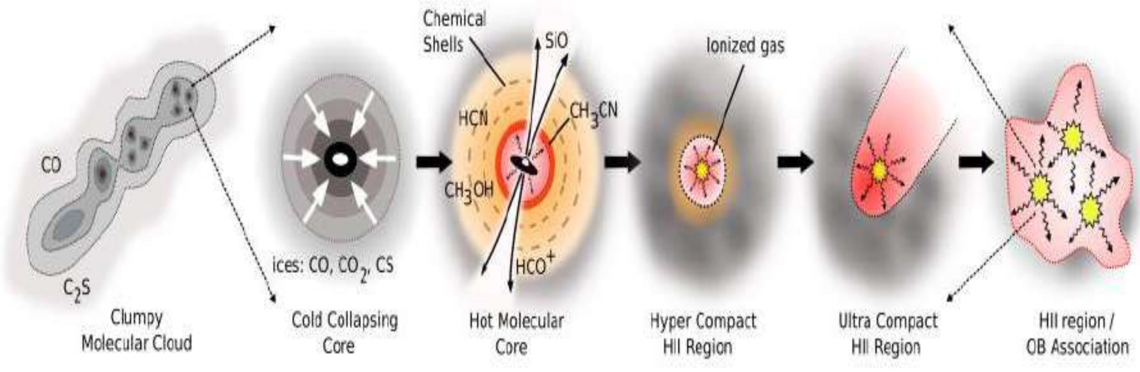
(Source: Science at Your Door step)

- Fusion in the core continues as long as there is hydrogen to fuel it. When hydrogen runs out the core will begin to shrink and the outer layers are pushed away from the core to produce a red giant.

The star can maintain this new status for a little while longer, around a billion years.

FORMATION OF HIGH MASS STAR:

Now for a high mass star, once much more massive than our sun, things are quite different. Their demise will not be so quiet. Big stars go out with a bang. Things start out normally, with a gas cloud collecting under the influence of gravity. It is simply that this cloud will be much larger than those that form low mass stars, so it will contain much more mass means more gravity which means the force pushing inward is much stronger ,and the star gets much hotter . A hotter temperature means faster fusion, which generates greater outward pressure to counteract the greater inward pull of gravity. This will result in a main sequence star that is hot big, bright, and blue. This is where things start to go differently from low mass stars. whereas low mass stars take billions of years to use up all their fuel, high mass stars are much hotter and burn their fuel much faster .that means they use up all the hydrogen in their course in around just a fleeting 100 million years, or even 10 million if big enough.



(Step by step formation of high mass star)

(Source: University of KENT)

Source of energy:

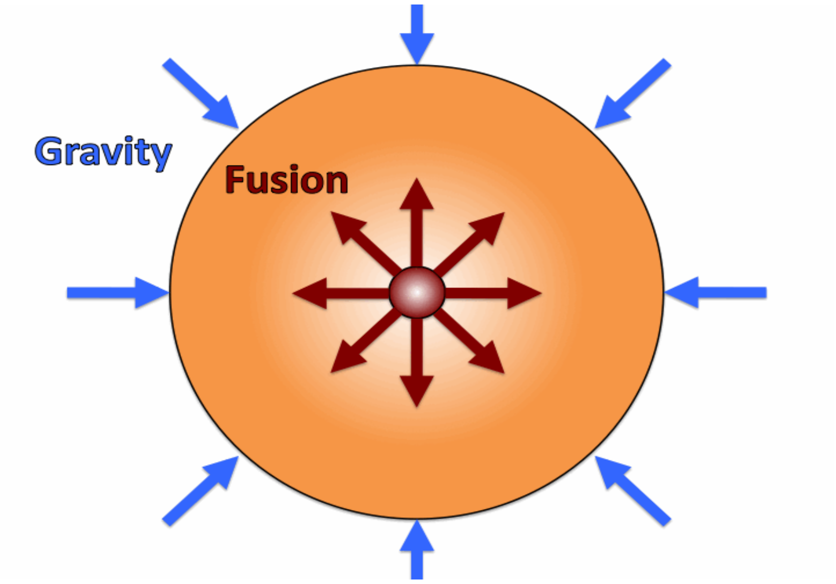
The energy source for all stars is nuclear fusion .Stars are made mostly of hydrogen and helium, which are packed so densely in a star that in the star’s centre the pressure is great enough to initiate nuclear fusion reactions. In a nuclear fusion reaction, the nuclei of two atoms combine to create a new atom. Most commonly, in the core of a star, two hydrogen atoms fuse to become a helium atom. Although nuclear fusion reactions require a lot of energy to get started, once they are going they produce enormous amounts of energy .In a star, the energy from fusion reactions in the core pushes outward to balance the inward pull of gravity. This energy moves outward through the layers of the star until it finally reaches the star's outer surface. The outer layer of the star glows brightly, sending the energy out into space as **electromagnetic radiation**, including visible light, heat, ultraviolet light, and radio waves.

Dynamic equilibrium:

A star spends most of its lifetime at a relatively constant size, temperature, luminosity, etc. while it fuses some fraction of its hydrogen into helium. During this time there is a balance between the forces inward and the forces outward.

Forces inward — due to gravity, without the forces acting, outwards the star would collapse.

Forces outward — due to nuclear fusion, without the forces acting, inward the star would collapse.



(Diagram showing the balance of gravitational forces (contraction) with fusion (expansion) that is maintained to keep Sun stable and fusion burning.) (Ref: Stanford University, winter 2011)

INTRODUCTION TO THE DEATH OF STAR:

Do stars die with a bang or a whimper? In the earlier part, we followed the life story of stars, from the process of birth to the brink of death. Now we are ready to explore the ways that stars end their lives. Sooner or later, each star exhausts its store of nuclear energy. Without a source of internal pressure to balance the weight of the overlying layers, every star eventually gives way to the inexorable pull of gravity and collapses under its own weight.

The fate of a star depends on its mass. Low mass stars like the sun in their dying stages shed their outer layers transferring most of their mass into the interstellar medium. Massive stars go out with a bang as supernovas ejecting heavy elements into the interstellar medium.

- **Low mass stars end up as white dwarf stars and eventually black dwarf stars.**
- **High mass stars end up as neutron stars and in some cases black holes.**

Chandrasekhar Limit:

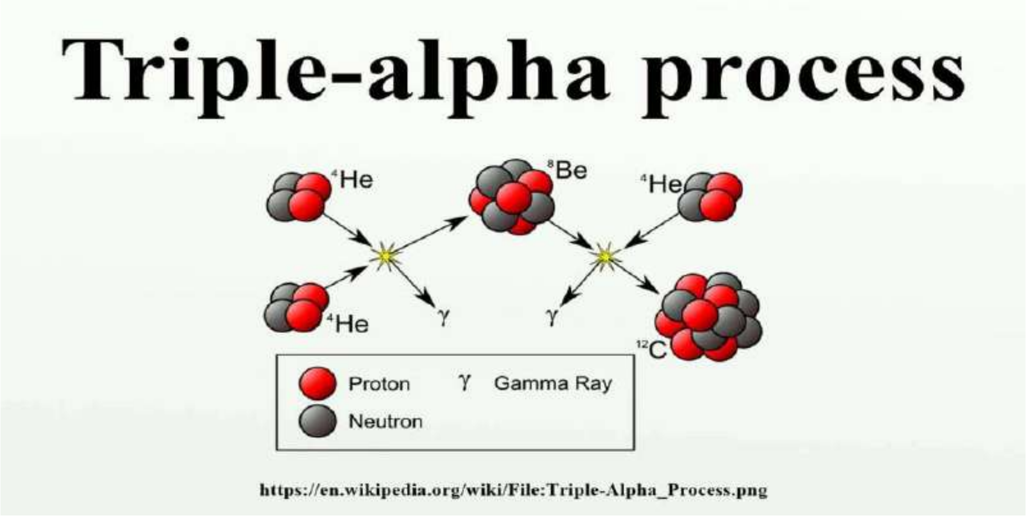
White dwarf stars are the end products of the stellar evolution of low to medium mass stars like our Sun. They are extremely dense objects (1 teaspoon of white dwarf material would weigh several tonnes!) and are supported against further gravitational collapse by electron degeneracy pressure.

The Chandrasekhar Limit of 1.44 solar masses, is the theoretical maximum mass a white dwarf star can have and still remain a white dwarf. Above this mass, electron degeneracy pressure is not enough to prevent gravity from collapsing the star further into a neutron star or black hole.

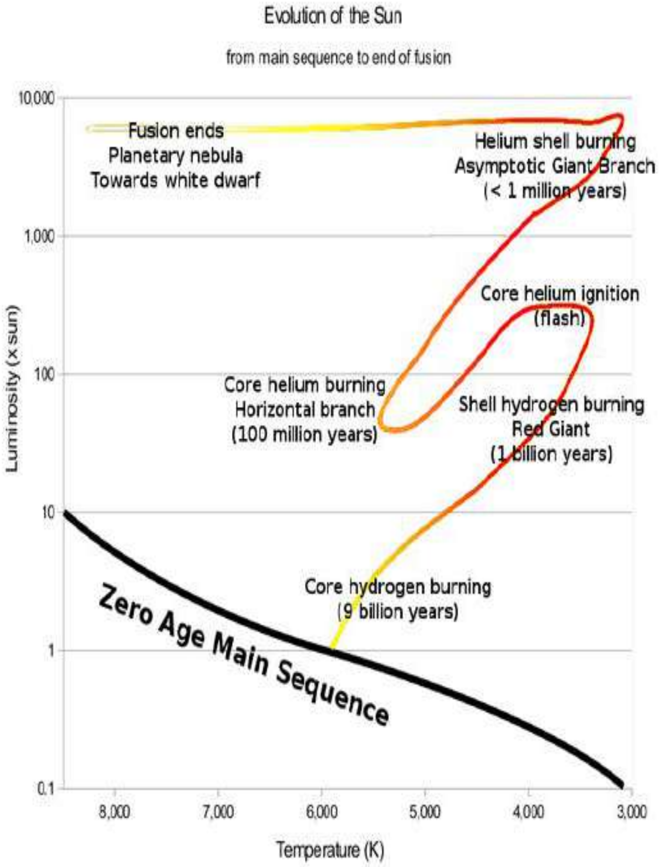
The limit is named after the Nobel laureate Subrahmanyan Chandrasekhar, who first proposed the idea in 1931, using Albert Einstein's special theory of relativity and the principles of quantum physics.

Death of low mass star:

At the end of the formation stage of low mass star, I said that, Fusion in the core of red giant star continues as long as there is hydrogen to fuel it. But after almost all the hydrogen is gone, the core gets even smaller and even hotter. At this stage of phase is called helium flash, things are so hot that the star is able to fuse these heavier helium nuclei into larger nuclei like carbon, and then oxygen, through something called the triple alpha process.



https://en.wikipedia.org/wiki/File:Triple-Alpha_Process.png



This means that the star has a whole new source of fuel in all the helium it has been making for billions of years. The star begins pulsating as it runs through its final energy reserve, entering what we call the horizontal branch, and in this time it becomes smaller, hotter, and blue, are until at last much of the helium has been fused into larger nuclei. Once the core is predominantly carbon and oxygen, with just a shell of helium around it and a shell of hydrogen around that the star has very little material left to burn so the core will collapse and the star enters the asymptotic giant branch. This means it will grow rapidly and become a giant star again, until the last bursts of energy injected the outer layer, pushing it away from the core and back into the interstellar medium, leaving only a tiny, very hot, bare core behind, about the size of earth. This will gradually cool, as it has no more fuel to burn, not being hot enough to fuse carbon or oxygen nuclei, and it will contract further until we are left with a white dwarf star. The ejected shell is called a planetary nebula.

(White dwarf)

(https://www.dreamstime.com)

(Planetary nebula)

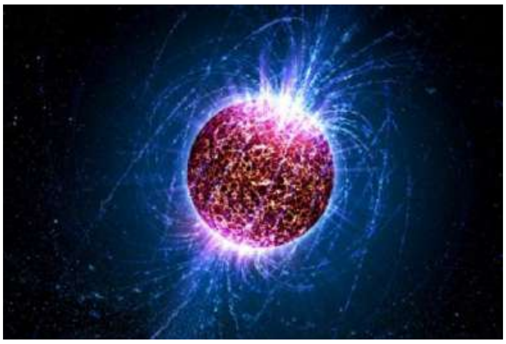
(https://www.nasa.gov)

Death of a massive star:

- At the formation time, the core of the massive star is made of hydrogen, and the hydrogen made helium. But when the core of a massive star becomes mainly made of iron, it has no more nuclear fuel to continue with fusion and can no longer remain hot. So the process to make hydrogen to iron is -

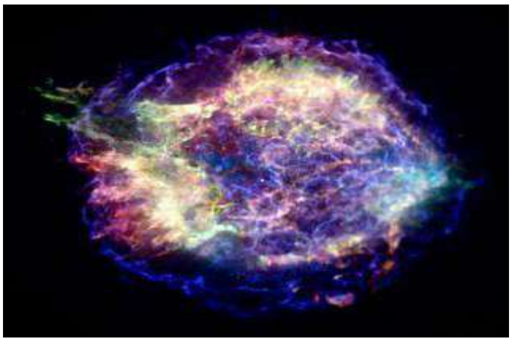
🟥	Hydrogen to Helium Fusion	(main sequence)	
		super giant	
🟨	Helium to Carbon Fusion	↓	million years
🟩	Carbon to Neon Fusion	↓	thousand yrs
▨	Neon to Oxygen Fusion	↓	a few years
🟦	Oxygen to Silicon Fusion	↓	a year
🟪	Silicon to Iron Fusion	↓	days
🟨	iron (fe)		

- Iron is nuclear ash .It has no energy to give and cannot be fused .The fusion suddenly stops and the balance ends .Without the outer pressure from fusion ,the core is crushed by the enormous weight of the star above it .particle like electrons and protons ,really don’t want to be near each other .But the pressure of the collapsing star is so great that electrons and protons fuse into neutrons ,Which then get squeezed together as tightly as in atomic nuclei ,an iron ball ,the size of earth is squeezed into a pure ball of pure nuclear matter ,size of a city .But not just ,the whole star implodes ,gravity pulling the outer layer in at 25% the speed of light .The implosion bounces off the iron core ,producing shock wave that explodes outwards catapults the rest of the star into space .This is known as supernova explosion .What remains of the star is now a neutron star .Its mass is around a million times the mass of the earth but compressed to an object about 25 km wide .



(neutron star)

(https://astronomy.com)



(supernova explosion)

(https://www.space.com)

Black hole:

A black hole is a place in space where gravity pulls so much that even light cannot get out. The gravity is so strong because matter has been squeezed into a tiny space. This can happen when a star is dying .Because no light can get out, people can't see black holes. They are invisible. Space telescopes with special tools can help find black holes. The special tools can see how stars that are very close to black holes act differently than other stars.

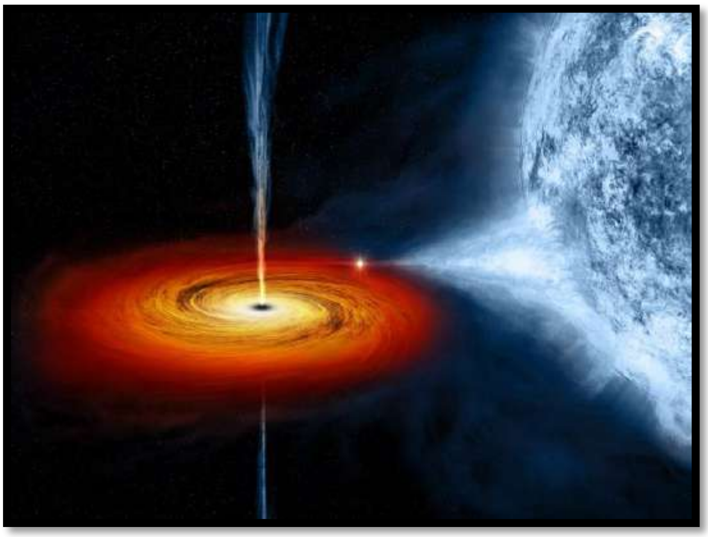
Black holes can be big or small. Scientists think the smallest black holes are as small as just one atom. These black holes are very tiny but have the mass of a large mountain. Mass is the amount of matter, or "stuff," in an object.

Another kind of black hole is called "stellar." Its mass can be up to 20 times more than the mass of the sun.

Scientists think the smallest black holes formed when the universe began.

Stellar black holes are made when the center of a very big star falls in upon itself, or collapses. When this happens, it causes a supernova. A supernova is an exploding star that blasts part of the star into space.

Scientists think supermassive black holes were made at the same time as the galaxy they are in.



(black hole)

(https://www.nasa.gov)

Conclusion:

Aftar all this study, it is quite evident that stars which are quite a major part of our survival, have lives quite similar to ours. Stellar evolution is a description of the way that stars change with time. On human timescales, most stars do not appear to change at all, but if we were to look for billions of years, we would see how stars are born, how they age, and finally how they die.

Acknowledgement:

The success and final outcome of this project required a lot of guidance and assistance from many people and I am extremely privileged to have got this all along the completion of my project. All that I have done is only due to such supervision and assistance and I would not forget to thank them.

I respect and thank our Principal, Swami Kamalasthananda and Vice Principal, Swami Vedanuragananda for providing me an opportunity to do the project work. I am extremely thankful to the Head of the Department of Physics, Prof. Asok Kumar Pal for providing such a nice support and guidance although he had a very busy schedule.

I owe my deep gratitude to our project guide Prof. Palash Nath, who took keen interest on my project work and guided me all along, till the completion of the project work by providing all the necessary information for developing a good system. I heartily thank all my friends and classmates for their encouragement and more over for their timely support and guidance till the completion of my project work. I am thankful to and fortunate enough to get constant encouragement, support and guidance from all Teaching staffs of RKMVCC Physics Department which helped me in successfully completing my project work.

Debasish Ghosh, Physics dept.

Reference :

1. cosmos
2. https://www.mpia.de
3. https://nasa.gov
4. https://www.nasa.gov/black-hole

Supermassive Black Holes as the Power House of Quasars

The project submitted, in partial fulfilment of the requirement for the assignments in **PHSA CC-XI, PHSA CC-XII, PHSA DSE-I, PHSA DSE-II** *Paper (Semester V) in the Department of Physics*

Submitted by

Niladri Chakraborty

Registration No: A01-1112-111-009-2019

Supervisor Teacher: Dr. Sankhasubhra Nag

RAMAKRISHNA MISSION VIVEKANANDA CENTENARY COLLEGE

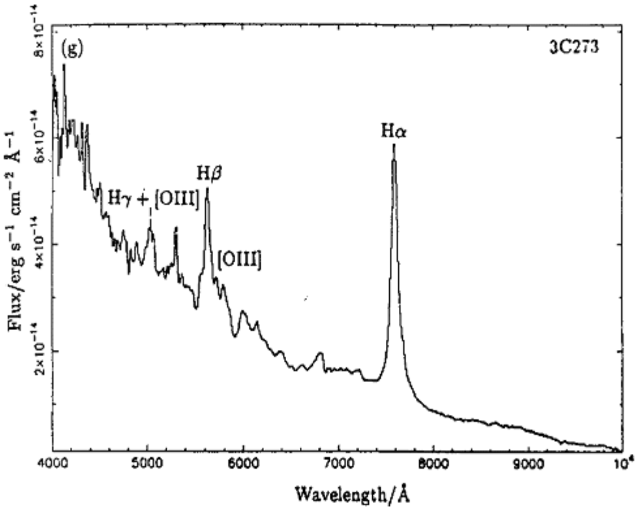
P.O. RAHARA, KOLKATA-700118, WEST BENGAL, INDIA

Table of Contents

- **Abstract:** A quasar is an “ultra-luminous” active galactic nucleus having rapidly variable and mysterious emission lines. Quasars seem to be linked to galaxy formation for which study of them can be utilized to get a better picture of the primordial universe. Relying upon elementary physics, an attempt is made in this project work to show that it is reasonable to hypothesize that quasars are powered by supermassive black holes surrounded by gaseous accretion disk.

1. Introduction

Astronomers in the early twentieth century thought that the Milky Way Galaxy is the entire Universe. Gradually, it became clear that galaxies are the building blocks of the universe, heralding the study of extragalactic astronomy. A typical galaxy is made up of stars and interstellar matters. But, in the late 1950s, some galaxies were found having additional compact nuclei at their centre giving out enormous amount of radiation in several bands of electromagnetic spectra composed of strong, broad emission lines, which are entirely different from the spectra of “normal galaxies” that contains prominent absorption lines of known elements like calcium, magnesium, sodium etc. Such galaxies are called active galaxies and their nuclei are called active galactic nuclei (**AGN**). One of the most extreme examples of AGN are radio sources called quasars which have completely different spectra compared to that of stars [fig.1 & fig.2].



- **Fig.1: Spectra of quasar 3C 273, 2.5-m telescope Isaac Newton (Palma).**

Source: Astrosurf

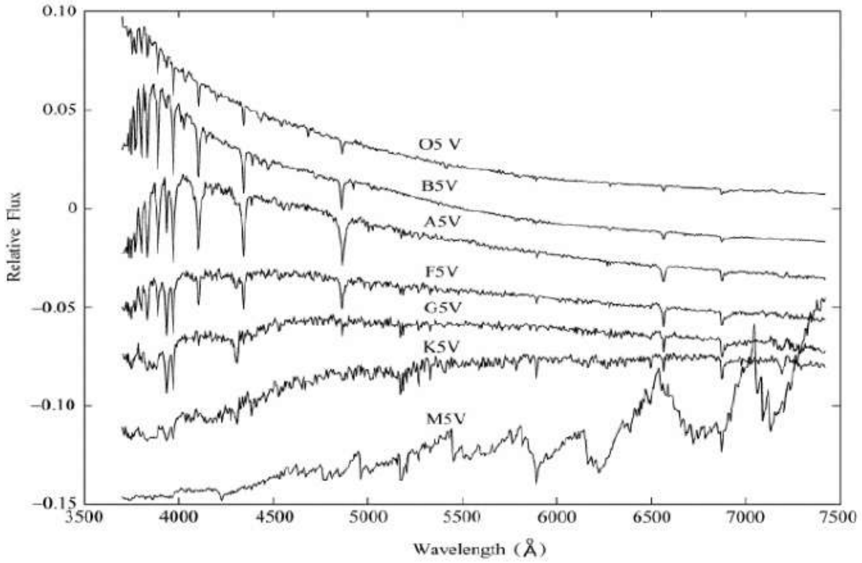


Fig.2: Spectra of stars

Source: ResearchGate, uploaded by Yongheng Zhao, CAS-National Astronomical Observatories of China

2. How far the Quasars are

Change in frequency of any kind of waves due to the relative motion of the source and the observer is known as Doppler shift. When a source moves towards (or away from) the observer, frequency of electromagnetic radiation increases (or decreases). This phenomenon is called blue shift (or red shift). Considering the expanding picture of the Universe, it is obvious that quasars must have been red or blue shifted. Now, relativistic treatment of the Doppler effect enables us to write,

a) Transverse effect (when the observer is displaced in a direction perpendicular the direction of motion):

$$\nu_{obs}^{(trans)} = \nu_{em}^{(trans)} \sqrt{1 - \left(\frac{v}{c}\right)^2} \quad \_\_\_\_\_\_\_\_\_\_\_\_ eqn(1)$$

b) Longitudinal effect (when the source is moving towards or away from the observer along a straight line):

$$\nu_{obs}^{(long)} = \nu_{em}^{(long)} \sqrt{\frac{c - v}{c + v}} \quad \_\_\_\_\_\_\_\_\_\_\_\_ eqn(2)$$

where c is the velocity of light in free space, v is the receding velocity of the source with respect to the observer, ν_{obs} and ν_{em} are the frequencies of the radiation in the frame of observer and emitter respectively. Considering $v \ll c$, eqn (1) and eqn (2) can be written as:

$$\nu_{obs}^{(trans)} = \nu_{em}^{(trans)}$$

(i.e., there is no shift due to transverse effect)

and

$$\nu_{obs}^{(long)} = \nu_{em}^{(long)}(1 - \frac{v}{c})$$

$$or, \qquad \frac{\lambda_{obs}}{\lambda_{em}} = \frac{\nu_{em}^{(long)}}{\nu_{obs}^{(long)}} \approx 1 + \frac{v}{c} \equiv 1 + z \quad \underline{\hspace{3cm}} eqn(3)$$

Thus, measuring the redshifts of spectral lines in the spectrum of a quasar, its receding velocity can be measured. From observations, it has been found that most of the galaxies show redshift in their spectra indicating that they are receding from us. In 1929, astronomer Edwin Hubble proposed that,

$$v = H_0\, l \quad \underline{\hspace{3cm}} eqn(4)$$

where $H_0 = 72 \pm 8\ km\ s^{-1}\ Mpc^{-1}$, v and l are the velocity and distance of the galaxy respectively. This is known as Hubble's law and H_0 is known as Hubble constant. Utilizing this law, we can get an estimation of the distance of a quasar.

3. Facts that Baffle the Intuition

One of the most interesting property of quasars is the rapid variability of their continuum and line spectra. Emission from quasars vary by substantial amounts over time scales of the order of as short as years, months or days. If Δt is the time scale of variation, the size of the emitting region has to be smaller than $c\Delta t$. Since no stationary source of radiation can vary in brightness faster than the time it takes for light to cross it, rapid variability implies a compact source size. Thus, quasars should not be located so far and hence not intrinsically luminous enough.

But observational data shows that spectral lines in the spectra of quasars are redshifted by unbelievably large amount. For example, quasar 3C 273 is found to have redshifted hydrogen spectral lines by an amount, $z = 0.158$ [fig.3]. So, from $eqn(3)$ and $eqn(4)$ we can estimate the distance (l) of 3C 273 from us,

$$592.5\, Mpc \leq l \leq 740.625\, Mpc$$

which indicates that quasars are lying far away, beyond the distances of most "normal galaxies".

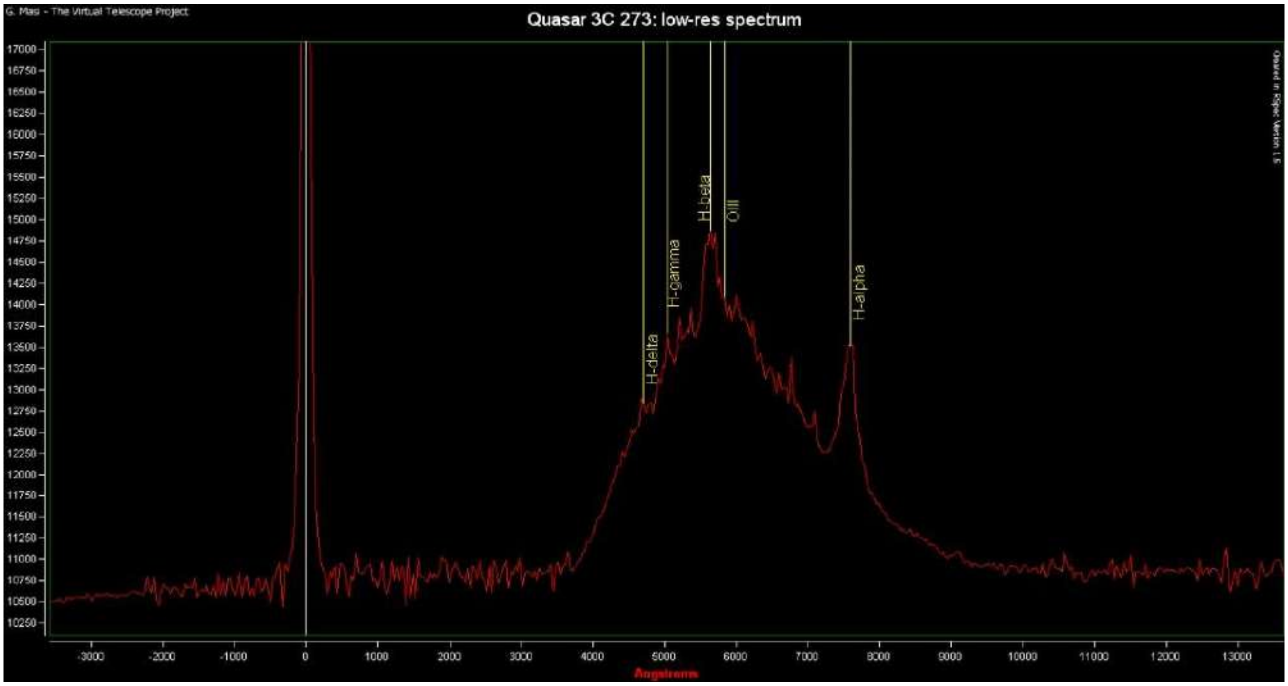


Fig.3: Spectral lines of hydrogen redshifted by an amount of z = 0.158

Source: The Virtual Telescope Project

On the other hand, luminosity of a typical quasar is,

$$L_q \sim 10^{14} L_{\odot}$$

$$or, \qquad L_q \sim \ 10^{40}\ Watts$$

i.e., it is about trillion times as bright as sun and more than hundred times brighter than the Milky Way. That means quasars are intrinsically “ultra-luminous” which is also against the conclusion drawn from the rapid variability of emission.

Again, hydrogen fusion, the primary process that generates stellar energy, occurs mainly from thermonuclear proton-proton cycle:

1. ${}_1^1H + {}_1^1H \rightarrow {}_1^2H + \beta^+ + \nu + 0.42\ MeV$
2. ${}_1^1H + {}_1^2H \rightarrow {}_2^3He + \gamma + 5.5\ MeV$
3. ${}_2^3He + {}_2^3He \rightarrow {}_2^4He + 2{}_1^1H\ + 12.8\ MeV$

So, the net result is

$$4{}_1^1\mathrm{H} \rightarrow {}_2^4\mathrm{He} + 2\beta^+ + 2\nu + 2\gamma + 24.64\ MeV$$

Since the neutrinos escape, we are left with 24.3 MeV energy after the fusion of 4 hydrogen nuclei. Hence, hydrogen required by the quasars to produce enormous amount energy per second is $\sim 10^{22} kg\ s^{-1}$. Such a large amount of hydrogen can be burnt every second in a very small emission region, is not plausible at all. So, quasars, in comparison with the normal stellar cores, must have some unique process for generation of energy.

4. In Search of a “Central Engine”

Now, it is clear that the actual astrophysical problem is to explain how quasars can generate enormous energy from very compact region using the observational key pieces – rapid variability of emission and “ultra-luminosity”.

Let’s assume, the emitting region is spherical (having radius R) and variation of radiation happens at its centre and propagates towards the limbs. Time interval between which radiation from point A and B [fig.4] reaches an observer at point O is Δt. So, the time scale of variability is $\sim \Delta t$. From this toy model, it can be written that:

$$l_2 \cos\theta = l_1 + R$$

$$or, \qquad \Delta l \equiv l_2 - l_1 \approx R$$

$$[\text{assuming } \theta \text{ is sufficiently small so that} \cos\theta \approx 1]$$

Hence,

$$R \approx c\,\Delta t \,\underline{\hspace{3em}}\, eqn(5)$$

Since the luminosity of any spherically symmetric object in equilibrium cannot exceed the Eddington’s luminosity, we can write

$$L_q \leq L_{Edd} \Rightarrow L_q \leq 1.3 \times\ 10^{31}\left(M'/M_\odot\right) Watts$$

$$or, \qquad M' >\ 7.7 \times 10^{8}\, M_\odot \quad [\because L_q \sim\ 10^{40}\, Watts]$$

where M' is the mass of the emitting region.

Therefore, the “central engine” of a quasar must have huge amount of mass in a very compact region. So, we may hypothesize the “central engine” is nothing but a supermassive black hole (**SMBH**). To check whether the presumption is rational, we can compare eqn (5) with Schwarzschild radius,

$$R_s = 2GM/ c^2 \_\_\_\_\_\_\_\_\_\_ eqn(6)$$

and can write that,

$$M = c^3\, \Delta t/ 2G \Rightarrow\ M \sim 10^9 M_\odot \quad [\,taking\ \Delta t \sim 1\ year]$$

which is consistent with the order of M'.

How this assumption disposes of the problem, is as follows:

Diffused material in orbital motion around SMBH forms a rapidly spinning accretion disk. As matter enters the accretion disk, it follows an inward spiral trajectory and kinetic energy provided by the infalling matter and frictional effects within the disk raise the temperature. Thus, deep gravitational potential well possessed by SMBH makes the quasar "ultra-luminous" which is also consistent with the fact that eqn (6) ensures gravitational potential energy, $U = GMm/r$ to be converted into $E \sim mc^2$ when $r \sim R_s$ (symbols have their usual meanings).

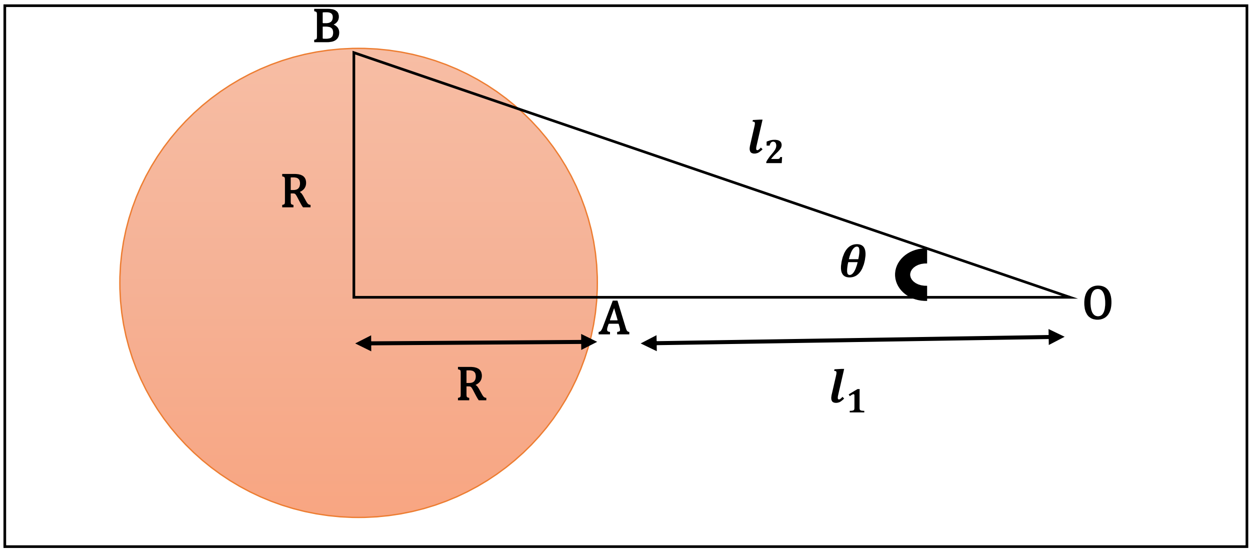


Fig.4

5. Current Problems and Future Works

Quasars may be linked with the galaxy formation. It may be possible that at the early age of the universe all the galaxies were active and powered by SMBH-s but for some obvious reasons gradually a bunch of them became “dead” and got converted into “normal galaxies”. Thus, studies on quasars are really useful to get the actual picture of the primordial universe. Though the general theory of relativity and quantum physics can explain how the universe works on the largest and smallest scales respectively, it is still beyond mankind’s understanding how to unify them into a single physical theory. So, studying the SMBH-s living inside the quasars may also enable us to verify the fundamental theories which are present as well as those which will rise in near future.

The flow dynamics of the accreted fluid and the radiation mechanisms are two main aspects of the problem covered here, which occur in nature in various degrees of variety and complexity. The continuation of the current reading project may be directed towards the theoretical explorations of those aspects, in future.

Acknowledgement

I would like to thank my respected supervisor **Dr. Sankhasubhra Nag** for his support and guidance. Primarily, I am grateful to him for giving me an opportunity and continuous encouragement to work on such an interesting topic. Besides, he helped me out to accomplish the project work by discussing and clearing my doubts. I am also thankful to the college authority and the Faculty of Physics department for their overall support.

Bibliography

- Astrophysics for Physicists by Arnab Rai Choudhuri, Cambridge University Press
- Astrosurf
- ResearchGate
- The Virtual Telescope Project

Quantum Theory of Paramagnetic Behaviour of Solids for Static Applied Field

The project submitted, in partial fulfilment of the requirement for the assignments in PHSA CC XI, PHSA CC XII, PHSA DSE I and PHSA DSE II Papers (Semester V) in the Department of Physics.

Submitted by

Soumyo Kheto

Registration No: **A01-1112-111-010-2019**

Supervisor Teacher: Dr. Asok Kumar Pal

RAMAKRISHNA MISSION
VIVEKANANDA CENTENARY COLLEGE

P.O. RAHARA, KOLKATA-700118

WEST BENGAL, INDIA

Preface

In this context I want to derive the quantum mechanical theory of paramagnetism briefly. First of all Hare discuss about what the normal people think about magnetic interaction. And the actual theoretical phenomenon of these types of introduction. Then a mathematical in production of paramagnetism is given in this context full stop and at last I give a brief thought from quantum mechanical point of view that how paramagnetism theory developed.

To make this context I got help from some books and internet. Without all of these I cannot complete this paragraph.

Content

Introduction

What happened when a chunk of iron towards a magnetic material? Maximum people know this answer whatever no magnetism or not. The answer is the magnet attract on the iron pieces. But why? If anyone have the basic idea about how magnetism work they can't answer this phenomena because they the theory of mine says that magnetic force occur only when changes are in motion or in accordance follow through a circuit. Yet all magnetic phenomena are due to electric charges in motion. But here there is no current flowing through the piece of iron or the magnetic material then what happened here? Actually , if you could examine a piece of magnetic material on an atomic scale you would find tiny currents: electrons orbiting around nuclei and spinning about their axes. For macroscopic purposes, these current loops are so small that we may treat them as magnetic dipoles. Ordinarily, they cancel each other out because of the random orientation of the atoms. But when a magnetic field is applied, a net alignment of these magnetic dipoles occurs, and the medium becomes magnetically polarized, or magnetized. Unlike electric polarization, which is almost always in the same direction as E, some materials acquire a magnetization parallel to B (paramagnets) and some opposite to B (diamagnets). A few substances (called ferromagnets, in deference to the most common example, iron) retain their magnetization even after the external field has been removed.

Paramagnetism from mathematical point of view

Magnetism under the action of an applied magnetic field is a universal property of all solids. In most of them, the magnetisation induced by the external magnetic field is feeble. It may be quite high if M is the magnetism field we can write:

M=X.H

X is called magnetic succeptability in an isotropic substance, M is in the direction of H and X is a constant. For an anistropic material X is a tensor rank 2. In this case M and H are not in the same direction as a system in S.I. system.

Magnetic Induction B is defined as:

B=u0(H+M)=u0H(1+X)=u0.ur.H

u0=4.pie.10^-7

u=u0(1+X)

ur=(1+X)

In an isotropic material B,H and M are parallel and u either scaler. All solids can be classified according to their magnetic properties in to three groups: a. diamagnetics b.paramagnetics

c. ferromagnetics.

The classification is based on the values and behavior of X under different conditions.

a. Diamagnetic: In this case X is less than equal to zero has very little value i.e X is less than less than one 19.1 a the nature of variation of M & H is shown graphically. Straight line with negative slope which a free Souls independent of H. Some typical values of X for diamagnetic substances are given below:

Material	X
Bi	-15*10^-5
Ge	-0.8*10^-5
Si	-0.3*10^-5
Cu	-0.9*10^-5

b. Paramagnetic: In case also X is very very less than 1. But it is positive (X) is greater than 0 for these substances. In fig 19.1 b the variation of M with H for a typical paramagnetic substances is shown graphically. The is again a straight line but with a positive slope. Thus X is a constant, independent of H in this case also. However unlike in this case of diamagnetics , X depends on temperature in the case of a magnetic substance and is given by

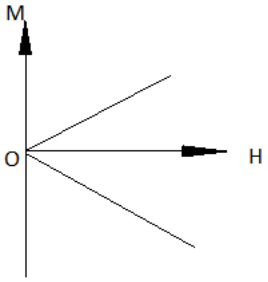
X=C/T

Here T is absolute temperature end C is a constant equation in 19.1-3 IS known as curies law.

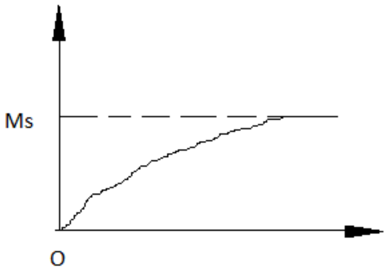
X for paramagnetics is independent of H only for relatively low magnetic fields. At very high fields and at very low temperatures the magnetisation M change to saturation as H is increased which source that X -> 0 as H increases fig 19.1 b. Values of X for a few typical paramagnetic at room temperature (300K) are given below:

Material	X
CaO	580*10^-5
FeCl2	360*10^-5
NiSO4	120*10^-5
Pt	26*10^-5

c. Ferromagnetic: For these substances X is positive and has is really very high value X is very very greater than one. Apart from the first transition group elements iron Z=26, Cobalt Z= 27, and erbium Z =68, as also alloys exhibit ferromagnetism.



19.1.a Variation of M with H



19.1.b Saturation magnetisation in paramagnetic material

QUANTUM THEORY OF PARAMAGNETISM

The magnetic moment of an atom or ion in free space is given by where the total angular rriorrleriturn h.J is the surn of the orbital h.L and spin fiS angular momenta. The constant y is the ratio of the magnetic moment to the angular momentum; y is called the gyromagnetic ratio or magnetogyric ratio. For electronic systems a quantity g called the g factor or the spectroscopic splitting factor is defined by ~PB=-$. (12) For an clcctron spin g = 2.0023, us~~ally taken as 2.00. For a Gee atom the g factor is given by the Land6 equation

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} \ .$$

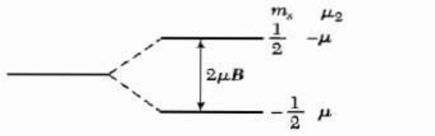


Figure 2 Energy level splitting for one electron in a magnetic field B directed along the positive z axis. For an electron the magnetic moment μ is opposite in sign to the spin S, so that $\mu = -g\mu_B S$. In the low-energy state the magnetic moment is parallel to the magnetic field.

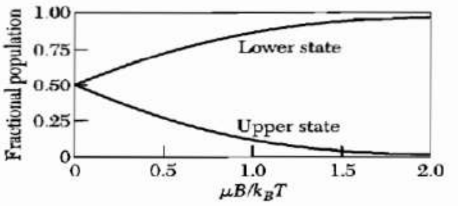


Figure 3 Fractional populations of a two-level system in thermal equilibrium at temperature T in a magnetic field B. The magnetic moment is proportional to the difference between the two curves.

The Bohr magneton p, is defined as efi/2m in CGS and efiflm in SI. It is closely equal to the spin magnetic moment of a free electron. The energy levels of the system in a magnetic field are

$$U = -\boldsymbol{\mu} \cdot \mathbf{B} = m_J g \mu_B B \ ,$$

where ml is the azimuthal quantum number and has the values J, J - 1, . . . , -1. For a single spin with no orbital moment we have VL] = 2; and g = 2, whence U = u$_B$B. This splitting is shown in Fig. 2. If a system has only two levels the equilibrium populations are, with 7 = k$_B$T,

$$\frac{N_1}{N} = \frac{\exp(\mu B/\tau)}{\exp(\mu B/\tau) + \exp(-\mu B/\tau)} \ ;$$
$$\frac{N_2}{N} = \frac{\exp(-\mu B/\tau)}{\exp(\mu B/\tau) + \exp(-\mu B/\tau)} \ ;$$

here N,, Nz are the populations of the lower and upper levels, and N = N, + NP is the total number of atoms. The fractional populations are plotted in Fig. 3. The projcction of the magnetic moment of the upper state along the field direction is -p and of the lower state is p. The resultant magnetization for N atoms per unit volume is, with x = pB/k,T, M = (N, - N2)p = Np . - Fr-e-r - Np tanhx . eX + eKX For x < 1, tanh x = x, and we have

$$M \cong N\mu(\mu B/k_B T) \ .$$

In a magnetic field an atom wit11 angular momentum quantum niimher] has 2J + 1 equally spaced energy levels. The magnetization (Fig. 4) is given by

$$M = NgJ\mu_B B_J(x) \ , \qquad (x = gJ\mu_B B/k_B T) \ ,$$

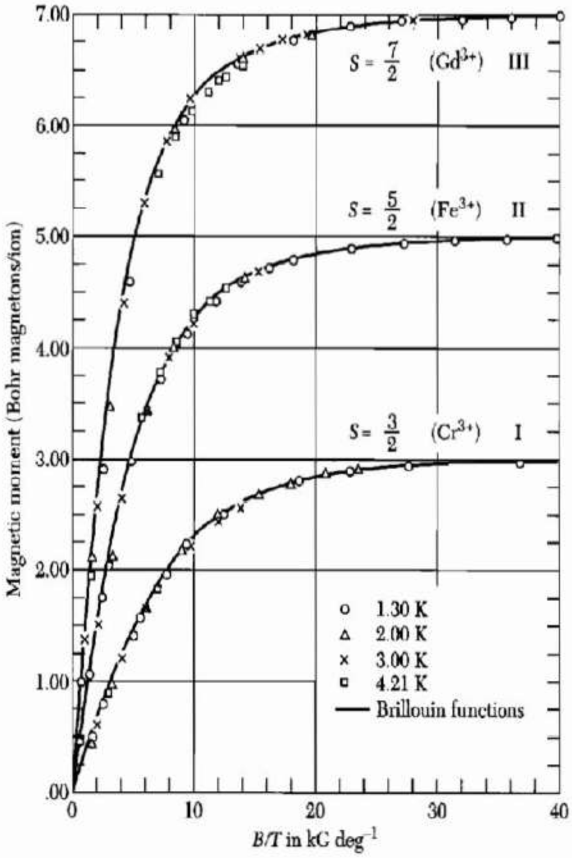


Figure 4 Plot of magnetic moment versus *B/T* for spherical samples of (I) potassium chromium alum, (II) ferric ammonium alum, and (III) gadolinium sulfate octahydrate. Over 99.5% magnetic saturation is achieved at 1.3 K and about 50,000 gauss (5T). After W. E. Henry.

where the Brillouin function B, is defined by

$$B_J(x) = \frac{2J+1}{2J} \operatorname{ctnh}\left(\frac{(2J+1)x}{2J}\right) - \frac{1}{2J} \operatorname{ctnh}\left(\frac{x}{2J}\right) \ .$$

Equation (17) is a special case of (20) for] = i.

For $x = \mu B/k_B T \ll 1$, we have

$$\operatorname{ctnh} x = \frac{1}{x} + \frac{x}{3} - \frac{x^3}{45} + \cdots \ , \qquad (21)$$

and the susceptibility is

$$\frac{M}{B} \cong \frac{NJ(J+1)g^2\mu_B^2}{3k_B T} = \frac{Np^2\mu_B^2}{3k_B T} = \frac{C}{T} \ . \qquad (22)$$

Here p is the effective number of Bohr magnetons, defined as

$$p \equiv g[J(J+1)]^{1/2} \ .$$

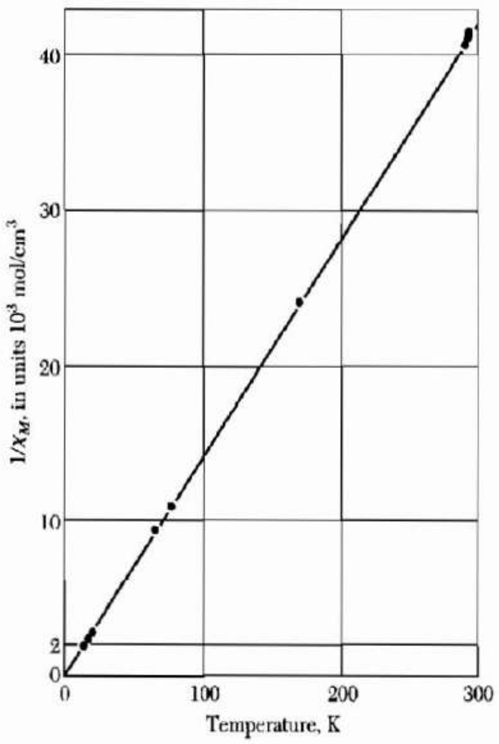


Figure 5 Plot of $1/\chi$ *vs* T for a gadolinium salt, $Gd(C_2H_3SO_4)_3 \cdot 9H_2O$. The straight line is the Curie law. (After L. C. Jackson and H. Kamerlingh Onnes.)

The constant C is know1 as the Curie constant. The form (19) is known as the Curie-Brillouin law, and (22) is known as the Curie law. Results for the paramagnetic ions in a gadolinium salt are shown in Fig. 5.

ACKNOWLEDGEMENT

First of all I would like to graciously thank our rkmv CC road who helped me to find the right track to write a paper and also our other professor they also help me very much. They all give me a huge knowledge of Physics. I also think our honorable principal Maharaj Swami Kamalasthananda to give a full support for this project work. And specially leave to thanks those books without which I cannot complete this write up.

Thank you

Bibliography

1. introduction to electrodynamics by David J prefix 4th edition- Pearson

2. Atomic Physics by Dr S.N. Ghoshal, S. Chand

3.Solid state Physics by AJ Dekhkar-macmillan India Limited

4. Introduction to solid state Physics by Charles kittle 7th edition -John wiley and sons

QUANTUM THEORY OF BLACKBODY RADIATION

The project submitted, in partial fulfilment of the requirement for the assignments in **CC-XI, CC-XII, DSE-I, DSE-II** Paper (Semester **V**) in the Department of Physics.

Submitted by

CHAYAN KUMAR MANDAL

Registration No.: A01-1112-111-011-2019

Supervisor Teacher: Prof. CHANDAN KUMAR DAS

RAMAKRISHNA MISSION VIVEKANANDA
CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

Introduction

At the end of the nineteenth century, physics consisted essentially of classical mechanics, the theory of electromagnetism and thermodynamics. Classical mechanics was used to predict the dynamics of material bodies, and Maxwell's electromagnetism provided the proper framework to study radiation; matter and radiation were described in terms of particles and waves, respectively. As for the interactions between matter and radiation, they were well explained by the Lorentz force or by thermodynamics. The overwhelming success of classical physics— classical mechanics, classical theory of electromagnetism, and thermodynamics—made people believe that the ultimate description of nature had been achieved. It seemed that all known physical phenomena could be explained within the framework of the general theories of matter and radiation. But there is a problem when people try to describe the fact of black body radiation, so, we try to describe how historically quantum mechanics is developed.

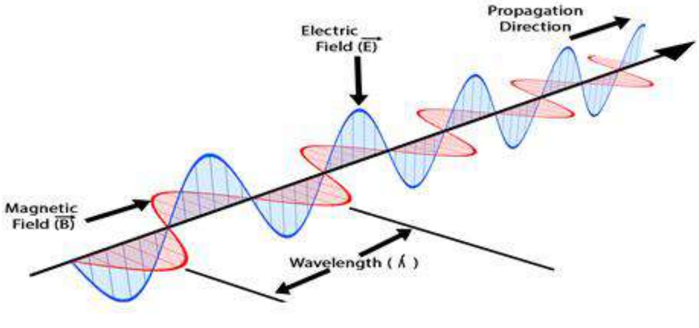
Max Planck (Father of quantum mechanics)

Wave and particle

In our everyday life there is nothing mysterious about the concept of particle and wave Classical physics, which mirrors the "**physical reality**" of our sense impressions, treats particles and wave as separate component of the reality. The mechanics of particles and optics of wave are traditionally independent disciplines, each have its own theory.

According to classical physics, a particle is characterized by an energy E and a momentum $\vec{p}$,whereas a wave is characterized by an amplitude and a wave vector $\vec{k}$; ($|\vec{k}|$ = 2π/λ) that specifies the direction of propagation of the wave. Particles and waves exhibit entirely different behaviours; for instance, the "particle" and "wave" properties are mutually exclusive. We should note that waves can exchange any (continuous) amount of energy with particles

In 1864, the British physicist **James Clerk Maxwell** (1831-1879), know from earlier work of **Faraday** that changing magnetic field induced electric field, and **Maxwell** invent that accelerated electric charges generate coupled electric and magnetic disturbances that can travel indefinitely through space. If the charges oscillate periodically, the disturbances are wave whose electric and magnetic field components are perpendicular to each other and the direction of propagation is perpendicular to both of them.

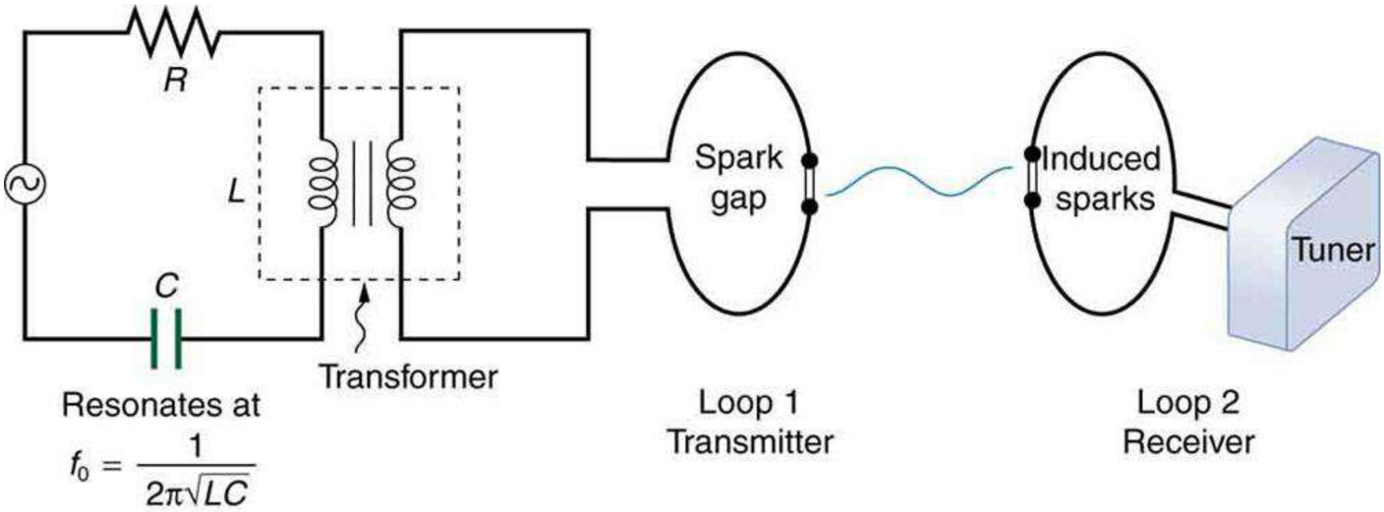


Electromagnetic Wave

Maxwell was able to show that the speed of c of electromagnetic waves in free space is given by $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 2.998 \times 10^8\ m/s$.

Where ϵ_0 is the electric permittivity of free space and μ_0 is the magnetic permeability of free space. This is the same as speed of light. The correspondence was too great to be accidental, and Maxwell concluded that light consist of electromagnetic wave.

The existence of electromagnetic wave and the behaviour exactly as Maxwell had predicted was confirmed experimentally by **Hertz** in 1888. This experiment is based on the fact that an oscillating electric charge radiates electromagnetic waves. The energy of these waves is due to kinetic energy of the oscillating charge.

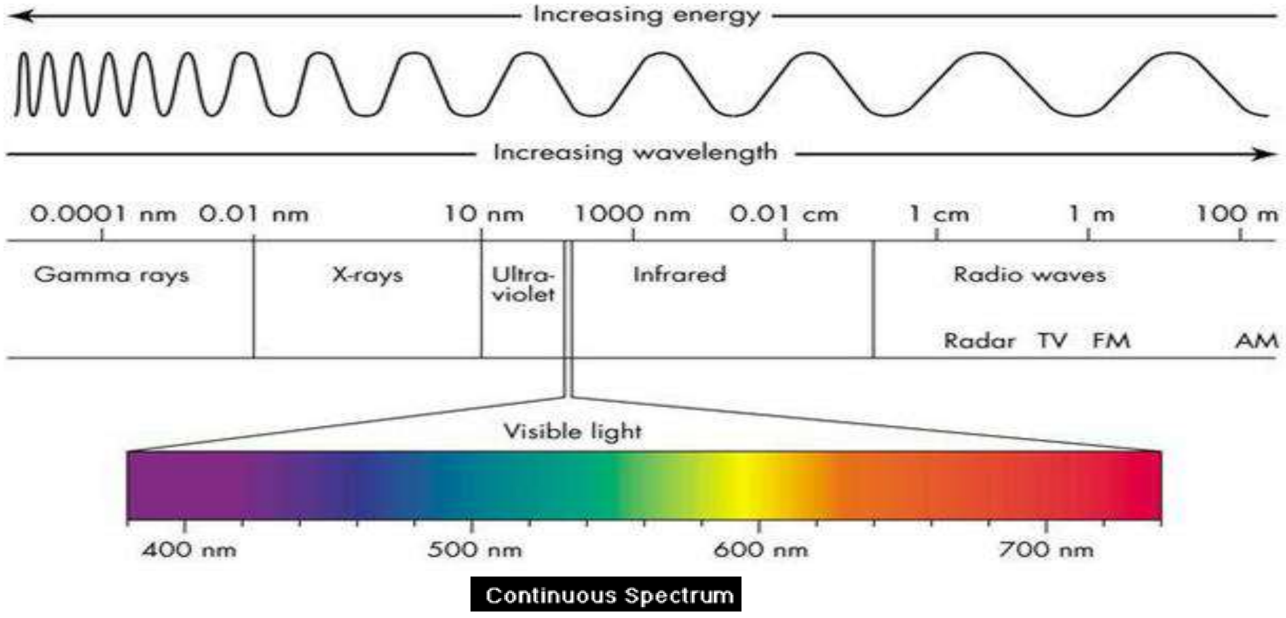


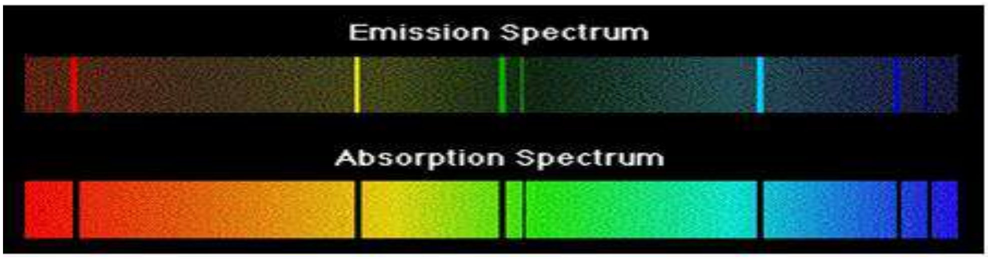
Hertz generated the waves by applying an alternating current to air gap between two metal plate at a distance of 60 cm from each other, The metal plates are connected to two polished metal spheres by means of metal rods. Using an induction coil a high potential difference is applied across the small gap between the spheres. Due to high potential difference across the metal balls, the air in small gap between the spheres get ionized and provides a path for the discharge of the plates. A spark is produced between the metal balls and electromagnetic waves of high frequency radiated. The width of the gap was such that a spark occurred each time the current reached a peak. Hertz was able to produce electromagnetic waves of frequency about 5×10^7Hz. Thus the whole circuit and loop 1 constitutes and LC combination. The high frequency oscillation of charges between the plates is given by $f_0 = \frac{1}{2\pi\sqrt{LC}}$. Wavelength of the electromagnetic produced is given by $\lambda = \frac{c}{f_0} = \frac{3\times10^6}{5\times10^7} = 6m$. An open metallic ring of diameter 0.70 m having small metallic loop (loop 2) acts as a detector. This constitutes another LC combination whose frequency can be varied by varying its diameter. Hertz observe a spark being created in the receiving loop due to the charge in the loop 1 being made to oscillate. Hertz also did experiment to show the wave being reflected from a metal mirror and refracted. He also confirmed the polarisation of light.

BLACKBODY RADIATION

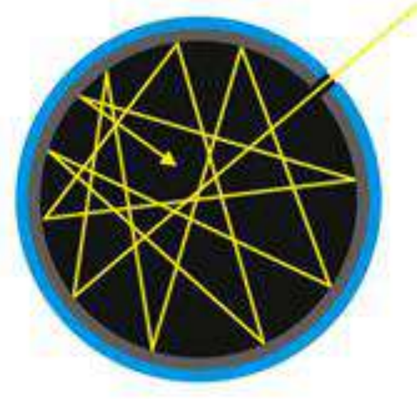
After Hertz's experiment the fundamental nature of light was clear: light consisted of electromagnetic wave that obeyed Maxwell theory. This certainty lasted only 12 years. The uncertainty come when there a serious issue come in radiation emitted by bodies of matter.

When heated, a solid object glows and emits thermal radiation. As the temperature increases, the object becomes red, then yellow, then white. The thermal radiation emitted by glowing solid objects consists of a *continuous* distribution of frequencies ranging from infrared to ultraviolet. The continuous pattern of the distribution spectrum is in sharp contrast to the radiation emitted by heated gases; the radiation emitted by gases has a discrete distribution spectrum: a few sharp (narrow), coloured lines with no light (i.e., darkness) in between. The intensity of this radiation depends on its frequency





and on the temperature. When radiation falls on the surface of body some is reflected and some absorbed. Like dark body absorb most of the radiation falling on them while white coloured bod reflected most of it. A idealized "**blackbody**" is defined as a body which absorbs all the radiant energy falling on it. Thermal radiation absorbed or emitted by a black body is called ***black body radiation.*** A perfect black body in real life is very idealisation. Consider a cavity kept at constant temperature. To an outside observer, a small hole made in wall of such cavity behaves like a blackbody surface because any radiation incident from the outside on the hole will almost completely absorbed after multiple reflections on the interior surface of the cavity. So that the hole has an effective absorption coefficient near to unity. Since the cavity is in thermal equilibrium, the radiation within it and that escaping from small opening can thus be closely treated as the thermal radiation from the black body. **Kirchhoff** proved using classical thermodynamics that the ratio of **emissive power** (power emitted per unit area at a given wavelength) to the **absorption coefficient** is the same for all bodies at the same temperature, and equal to the emissive power of a black body at that temperature ($E(\lambda, T)$), this is called the **Kirchhoff's law**.



Conceptual Black Body

$$\frac{R_\lambda(T)}{a_\lambda(T)} = E(\lambda, T)$$

$R_\lambda(T)$ = emissive power

$a_\lambda(T)$ = absorption coefficient

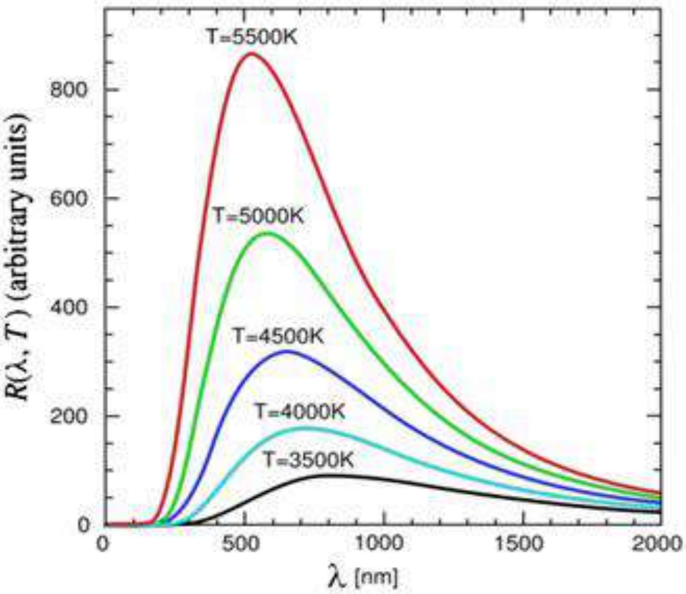
$R(\lambda, T)$ is power emitted per unit area of a black body at the absolute temperature T and corresponding to radiation wavelength between λ and $\lambda + d\lambda$. So $R(T) = \int_0^\infty R(\lambda, T)\, d\lambda$. In 1879 **J. Stefan** found an empirical relation between the quantity R (emissive power) and the absolute temperature of black body.

$$R(T) = \sigma T^4$$

Where σ = $5.67 \times 10^{-8}\, W m^{-2} K^{-4}$ is a proportionality constant is called the

Stefan's constant. In **1884** L. **Boltzmann** deduced this relation from thermodynamics; it is now called ***Stefan-Boltzmann*** law.

And in 1899 the first accurate measurement of $R(\lambda, T)$ were made by **O.Lummer and E. Pringsheim.**



Spectral distribution of black body radiation

The peak of the radiation spectrum occurs at a wavelength (λ_{max}) *that is inversely proportional to the temperature.* This is the underlying reason behind the change in colour of a heated object as its temperature increases, from red to yellow to white. It turned out that the explanation of the blackbody spectrum was not so easy. **Wilhelm Wien** in **1889** find this experimentally. This is **Wien's displacement law.**

$$\lambda_{max}T = b$$

The constant b is displacement constant and the value of $b = 2.898 \times 10^{-3}\, mK$.

But instead of using the $R(\lambda, T)$, we use the radiation inside cavity in terms of the quantity $\rho(\lambda, T)$ which is called *spectral distribution function* or *monochromatic energy density*. $\rho(\lambda, T)d\lambda$ is the energy per unit volume (energy density) of the radiation in the wavelength interval $(\lambda, \lambda + d\lambda)$, at the absolute temperature. And the relation between $R(\lambda, T)$ and $\rho(\lambda, T)$ is

$$\rho(\lambda, T) = \frac{4}{c} R(\lambda, T)$$

After this the main focus of physicist is to find out the functional form of $\rho(\lambda, T)$. In 1895 **Wilhelm Wien** showed that the functional form of $\rho(\lambda, T)$ is

$$\rho(\lambda, T) = \frac{f(\lambda \mathrm{T})}{\lambda^{-5}}$$

Where $f(\lambda\mathrm{T})$ is the function of single variable $\lambda\mathrm{T}$. This cannot be determined from theory (Thermodynamics). This is called **Wien's law**.

ULTRAVIOLET CATASTROPHE

To understand the behaviour of $\rho(\lambda,T)$ you have to know the function $f(\lambda\mathrm{T})$

Lord Rayleigh and J. H. Jeans after some attempts developed an equation by using classical electromagnetic theory, it follows that the thermal radiation within the cavity must exist in the form of standing electromagnetic waves. *The big assumption which Rayleigh and Jean implied was that infinitesimal amounts of energy were continuously added to the system when the frequency was increased.* The number of such wave in the cavity within the wavelength interval λ to $\lambda + d\lambda$ and per unit volume can be shown to be $n(\lambda)d\lambda = (8\pi/\lambda^4)d\lambda$ .So the number of such wave per unit wavelength range and per unit volume is $n(\lambda)$. If each of these wavs carry $\bar{\varepsilon}$ average energy with wavelength λ, the function $\rho(\lambda,T)$ is simply $\rho(\lambda,T) = n(\lambda) \times \bar{\varepsilon} = \frac{8\pi}{\lambda^4}\,\bar{\varepsilon}$. But how does one calculate $\bar{\varepsilon}$? Each of these wavs can take energy value 0 to ∞. As the system is in thermal equilibrium the average energy of these wave can be obtained from classical statistical mechanics with the Boltzmann probability distribution factor $exp(-\varepsilon/KT)$. Where K is Boltzmann constant, ε is each energy value. We have

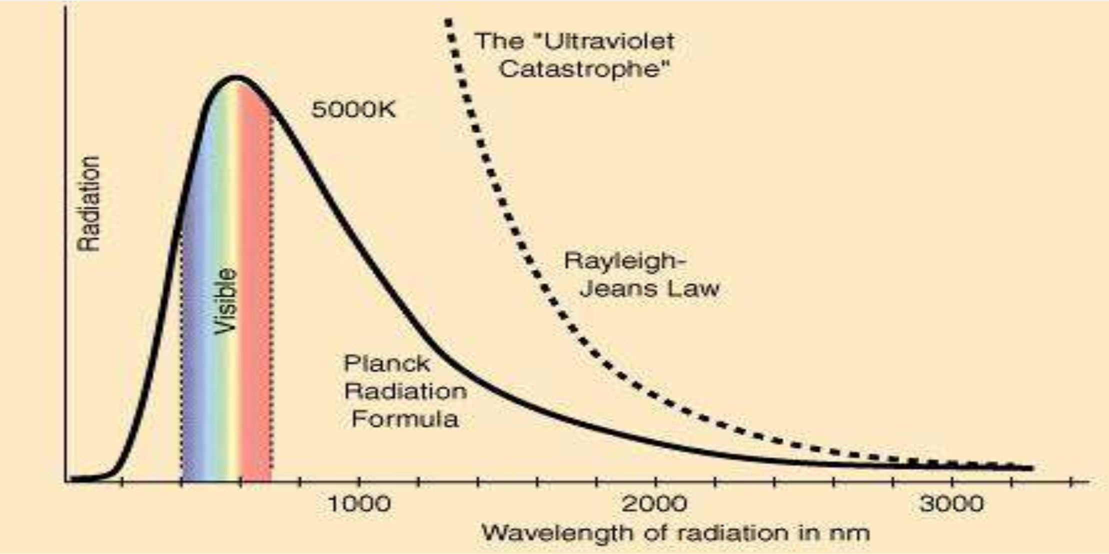
$$\bar{\varepsilon}_{\lambda(T)} = \int_0^\infty P(\varepsilon_\lambda)\,\varepsilon_\lambda\,d\varepsilon_\lambda \qquad P(\varepsilon_\lambda) = Ce^{(-\varepsilon/KT)}$$

$$\bar{\varepsilon}_{\lambda(T)} = \frac{\int_0^\infty \varepsilon\, e^{(-\beta\varepsilon)}d\varepsilon}{\int_0^\infty e^{(-\beta\varepsilon)}d\varepsilon} \qquad C = \int_0^\infty e^{(-\beta\varepsilon)}d\varepsilon$$

$$= \frac{\partial}{\partial\beta}\left[\ln\int_0^\infty e^{(-\beta\varepsilon)}d\varepsilon\right] \qquad \beta = \frac{1}{KT}$$

$$= \frac{1}{\beta} = KT$$

So we get $\rho(\lambda,T) = \left(\frac{8\pi}{\lambda^4}\right)\cdot KT$. And this follows that $f(\lambda\mathrm{T}) = 8\pi\mathrm{K}(\lambda\mathrm{T})$. In the limit of long wavelength this result approaches to the experimental value. But when $\lambda \to 0$ the curve does not exhibit maximum and diverges (i.e., in the *ultraviolet* range). This result is absurd. Historically this was called the *ultraviolet catastrophe*. a real catastrophic failure of classical physics indeed!



Planck's Quantum theory

In **December 1900**, the German physicist **Max Planck** (1858–1947), as there cannot be found any solution of ultraviolet catastrophe using classical physics, represent a new theory of $\rho(\lambda, T)$, based on a revolutionary *"lucky guesswork"*.

Planck's formula was announced at a meeting of the Berlin Physics Society in October 1900. For the next two months he immersed himself in the problem of finding a physical basis for the law, trying out different combination of physical assumption to see which ones matched the mathematical equations. He later said that this was the most intensive period of work of his entire life. Many attempts failed, until at last Planck was left with only one, to him ***unwelcome***, alternative.

In sharp contrast to Rayleigh's assumption that a standing wave can take *any* amount (continuum) of energy, Planck considered that the energy of an standing wave of a given frequency ν cannot take arbitrary values between 0 and ∞, but can only take *discrete* values $n\varepsilon_0$.

$$\varepsilon_n(\lambda) = \frac{nch}{\lambda} = nh\nu = n\varepsilon_0 \qquad n = 0,1,2,3,\cdots$$

Where n is a positive integer or zero, and ε_0 is finite amount, or called of *quantum* energy. It may depend on the frequency ν(and also λ). The letter h

stands for a new constant of nature introduced by Planck and now called "Planck's constant." This new constant plays the same of role in quantum theory that the speed of light plays in relativity theory; it tells us when quantum effects will be important. The number is very small, $h \ = \ 6.626 \times 10^{-34} JS$. Note that Classical mechanics, however, puts no restrictions whatsoever on the frequency. So, assuming that the energy of an oscillator (each standing wave) is quantized. So according to Planck's suggestion

$$\overline{\varepsilon(\lambda)} \ = \ \sum_0^\infty \varepsilon_n(\lambda)\, P(\varepsilon_n(\lambda))$$

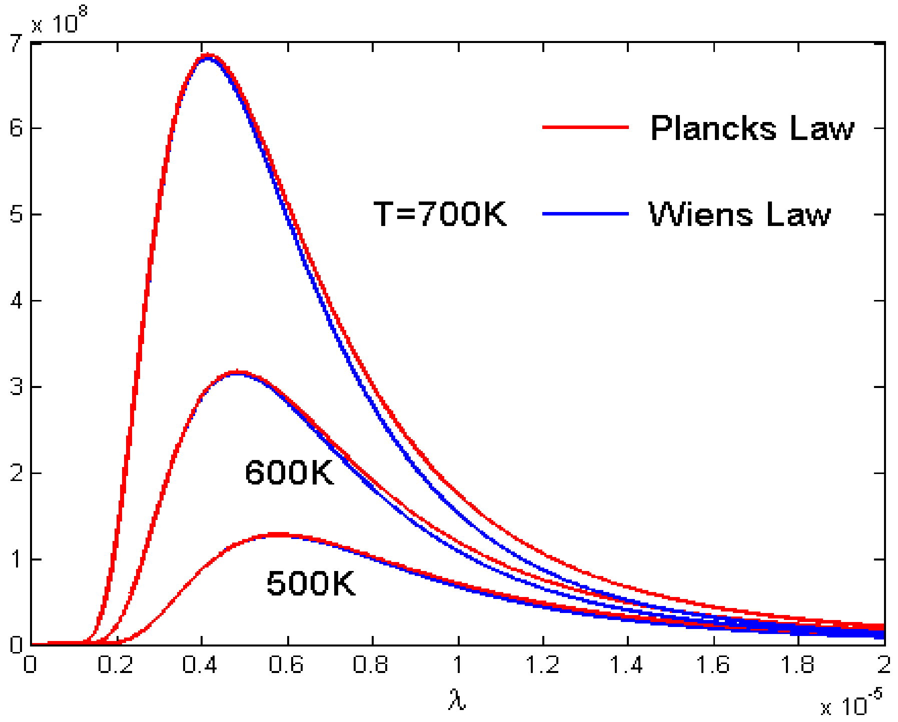
$$\overline{\varepsilon(\lambda)} \ = \ -\frac{d}{d\beta}[ln \sum_0^\infty exp(-\beta\varepsilon_n)] \ \cdots\cdots \ (1)$$

Now we know, $\sum_0^\infty e^{-\beta\varepsilon_n} \ = \ 1 + \ e^{-\beta\varepsilon_0} + \ e^{-2\beta\varepsilon_0} + e^{-3\beta\varepsilon_0} \ + \cdots \ = \ \frac{1}{(1-e^{-\beta\varepsilon_0})}$

From eq(1), $\overline{\varepsilon(\lambda)} \ = \ \frac{d}{d\beta}\left[ln\left(1 - e^{-\beta\varepsilon_0}\right)\right] \ = \ \frac{\varepsilon_0 . e^{-\beta\varepsilon_0}}{(1-e^{-\beta\varepsilon_0})} = \frac{\varepsilon_0}{(e^{\beta\varepsilon_0}-1)} \ = \ \frac{hc/\lambda}{e^{\beta\varepsilon_0}-1}$

$$\text{So, } \ \rho(\lambda, T) = \frac{8\pi}{\lambda^4}\ \frac{hc/\lambda}{e^{\beta\varepsilon_0}-1} \ = \frac{8\pi}{\lambda^5}\ \frac{hc/\lambda}{e^{\frac{\beta hc}{\lambda}}-1}$$

This is Planck's radiation formula. This successfully describe the curve which was get experimentally.



CONCLUSION:

We have seen above that the quantisation postulate introduced in 1900 by Planck was successful in explaining the blackbody radiation problem.

Quantum theory that describes the dynamics of matter at the microscopic scale. Fine! But is it that important to learn? This is no less than an otiose question, for quantum mechanics is the only valid framework for describing the microphysical world. It is vital for understanding the physics of solids, lasers, semiconductor and superconductor devices, plasmas, etc.

Acknowledgement:

I would like to thank our principal, Swami Kamalasthananda Maharaj and our physics departmental professor Dr. Chandan Kumar Das, whose valuable guidance has ones that helped me to complete this. Their instructions and suggestions help me very much. Besides this I would like to thank my parents and friends who helped me with their suggestions.

BIBLIOGRAPHY :

- www.google.com
- www.wikipedia.com
- www.scribd.com
- www.researchgate.net
- www.elprocus.com
- www.allprojectreports.com

Anharmonic oscillation

The project is submitted ,in partial fulfillment of the requirement for the assignment in PHSA CC-XI, CC-XII, PHSA DSE-I and DSE-II of 5th semester in the Department of Physics

Submitted by *Akash Ghosh*

RegistrationNo. :A01-1112-111-012-2019

Supervisor teacher :

Atisdipankar Chakrabarti

RAMAKRISHNA MISSION VIVEKANANDA
CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

ANHARMONIC OSCILLATION

For a simple harmonic oscillation we know that the restoring force is proportional to the displacement of the oscillation. In this case, the time period of oscillation doesn't depend on angle of oscillation θ. In most of practical cases the restoring force doesn't properly proportional with the displacement only. It also depends on θ.

For small oscillation in taylor series we can ignore higher order terms cause the displacement from the equilibrium position is so small. But for large enough displacement we can't ignore the higher order terms of displacement .In this case now the time period now also starts depending on θ. then this oscillation become an anharmonic oscillation.

Now the equation of this oscillation becomes

$$ml^2\ddot{\theta} + mgl\ \sin\theta = 0$$

------(1) θ

Here we can't replace sinθ by θ as its still not a small oscillation now.

Now integrating (1) we get

$$\frac{1}{2}\dot{\theta}^2 + \frac{g}{1}.\cos\theta =$$ constant ……(2)

If θ_0 be the displacement then $\theta=\theta_0$ then

$$\dot{\theta} = 0$$

.and the value of the constant become

So mgl.$\cos\theta_0$ is the total energy.In $\theta=0$ the potential energy is -mgl and at $\theta= \frac{\pi}{2}$ the value of potential energy will be 0.From equation (2) we get

$$\sqrt{\frac{g}{1}}\int_0^t dt = \frac{1}{\sqrt{2}}\int_0^\theta \frac{d\theta}{\sqrt{\cos\theta - \cos\theta_0}}$$ …….(3)

Here we considered at $\theta=0$ the value of t=0.From (3) we get

$$\sqrt{\frac{g}{1}}t = \frac{1}{2}\int_0^\theta \frac{d\theta}{\left[\sin^2\left(\theta_0/2\right) - \sin^2\left(\theta/2\right)\right]^{1/2}}$$

…….(4)

Now a new variable ϕ is introduced to simplify our mathematical calculation

$$\sin\phi \equiv \frac{\sin\left(\theta/2\right)}{\sin\left(\theta_0/2\right)}$$

- ……..(5)

Now from (4) by using (5)we get

$$\sqrt{\frac{g}{l}}.t = \int_0^{\phi}\frac{d\varphi}{\left(1-\sin^2\frac{\theta_0}{2}.\sin^2\phi\right)^{1/2}}$$

$$= \int_0^{\phi}\left(1+\frac{1}{2}\ \sin^2\frac{\theta_0}{2}.\sin^2\varphi+\frac{3}{8}\ \sin^4\frac{\theta_0}{2}.\sin^4\varphi....\right)$$

……(6)

Now taking only the first term we can write

$$\sqrt{\frac{g}{l}}.t = \phi = \sin^{-1}\left[\frac{\sin\left(\frac{\theta}{2}\right)}{\sin\left(\frac{\theta_0}{2}\right)}\right]$$

taking $\sin\frac{\theta_0}{2}=\frac{\theta_0}{2}$ and $\sin\frac{\theta}{2}=\frac{\theta}{2}$,

$$\theta = \theta_0 \sin\sqrt{\frac{g}{l}}.t$$

We getting back simple harmonic oscillation taking the first two terms of (6) we get

$$\sqrt{\frac{g}{l}}.t=\int_0^{\phi}\left(1+\frac{1}{2}\sin^2\left(\theta_0/2\right).\sin^2\varphi\right)d\varphi$$

$$=\left|\phi+\frac{1}{4}\sin^2\left(\theta_0/2\right)\left(\phi-\frac{1}{2}\sin 2\varphi\right)\right|_0^{\phi}$$

.......(7)

Displacement $\theta=\theta_0$ for $\phi=\frac{\pi}{2}$ .So if then higher value of ϕ is $\frac{\pi}{2}$ taken then the time period will be got one forth.If time period is T ,

$$\sqrt{\frac{g}{l}}\cdot\frac{T}{4}=\left[\frac{\pi}{2}\left(1+\frac{1}{4}\sin^2\theta_0/2\right)\right]$$

$$T=2\pi\sqrt{l/g}\left(1+\frac{\theta_0^2}{16}\right)$$

....(8)

In last step we replaced sin $\theta_0/2$ by$\theta_0/2$. Now we can see that the time period is not as same as the previous ,rather it has slightly increased and this increment is proportional to the square of θ_0 .From (7) we get

$$\sqrt{\frac{g}{l}}\,t=\phi\left(1+\frac{\theta_0^2}{16}\right)-\frac{\theta_0^2}{32}\sin 2\varphi$$

...(9)

From equation (5) we approximately get ,

$$\phi = \sin^{-1}\left(\theta/\theta_0\right) \text{ এবং } \sin 2\varphi = \frac{2\theta}{\theta_0}\left(1-\frac{\theta^2}{\theta_0^2}\right)^{1/2}$$

So the equation(9) becomes ,

$$t = \sqrt{l/g}\cdot\left(1+\frac{\theta_0^2}{16}\right)\sin^{-1}\left(\theta/\theta_0\right) - \frac{\theta\theta_0}{16}\left(1-\frac{\theta^2}{\theta_0^2}\right)^{1/2}\cdot\sqrt{\frac{l}{g}}$$

.........(10)

From the above equation the term with $\sin^{-1}$ (θ/θ_0) ***we can see the change of time period. From the second term of R.H.S. we can see the change of θ is not simple harmonic now. We taking θ not equal to sinθ we can write ,***

Sinθ = θ - $\theta^3/3!$

Now form (1) we get ,

$\ddot{\theta}+p^2\theta-p^2\theta^3/6 = 0$ **(**P^2**=g/l)......(11)**

We can ignore the term $p^2\theta^3$ **/6 being small and will get the result θ =**θ_0 **sin pt .Now the value of θ putting in the small term of equation (11) we get**

$$\ddot{\theta} + p^2\theta = p^2 \frac{\theta_0^3}{6} \sin^3 pt$$

$$= \frac{p^2}{8}\theta_0^3 \sin pt - p^2\frac{\theta_0^3}{24} \sin 3pt \quad .(12)$$

The term sin pt will create the resonance and the amplitude of the resonance will be infinite .Our common sense tells us its impossible, for a small change can't give us such a infinite amplitude.So to get a reasonable solution we taking ω for angular frequency p.The significance of ω is now we don't have to worry about the change of time period by the change of the term $p^2\theta^3$/6. So we now getting from equation (12),

$$\ddot{\theta} + p^2\theta = \frac{p^2}{8}\theta_0^3 \sin \omega t - p^2 \frac{\theta_0^3}{24} \sin 3\omega t$$

.....(13)

If its a equation of forced vibration then the external force is the combination of two sine forces the solution looks like ,

θ=$\theta_0$sin ωt+$\theta_1$sin 3ωt

Here $\theta_1$is the angular displacement of 2nd S.H.M. and its constant.

From equation (11) and (12) we get two types of terms sinωt and sin 3ωt which a time dependent.

$$-\omega^2 + p^2 = p^2 \frac{\theta_0^2}{8} \qquad (\sin \omega t$$

Now,

$$\omega = p\left(1-\frac{\theta_0^2}{8}\right)^{1/2} \approx p\left(1-\frac{\theta_0^2}{16}\right)$$

So, the time period

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{p\left(1-\theta_0^2/16\right)} = 2\pi\sqrt{1/g}\left(1+\frac{\theta_0^2}{16}\right)$$

From comparing the term sin 3ωt we get ,

$$(9\omega^2+p^2)\theta_1 = -p^2\frac{\theta_0^2}{24}$$

To evaluate value of a small θ_1 in R.H.S. we using $\omega^2=p^2$ we get $\theta_1 = -\frac{\theta_0^3}{192}$

So we can see w.r.t. θ_0, θ_1 is so small.

So from the above discussion I have already showed that the restoring force is not properly proportional to θ rather we have seen its proportional to θ - $\frac{\theta^3}{6}$ .As θ and θ^3 are always of same sine , the restoring force is slightly decreased.

Acknowledgement

By this projetwork I come to know about a detailed discussion on my topic which helped me to build a depth on this topic.So,I give by regreat from inner heart to all the teacher of my Physics department and specially my supervisor.

Bibliography

1. Advanced acoustics by *D.P.Roy Chaudhury*

2. Harmonic oscillator by *Yilun Shang*

Hall Effect and Its Application

The project submitted, in partial fulfilment of the requirement for the assignments in ***PHSA CC-XI, PHSA CC-XII, PHSA DSE-I, PHSA DSE-II*** *Paper (Semester - V) in the Department of Physics*

Submitted by

Tathagata Das

Registration No : A01-1112-111-013-2019

Supervisor Teacher : Dr. Rajen Kundu

RAMAKRISHNA MISSION VIVEKANANDA
CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

Introduction

The science of today is the technology of tomorrow.

–Edward Teller–

The **Hall Effect** is well-known for over a hundred years and finds wide range of application for the past three decades. The first active application (excluding laboratory tests) was in the 1950s as a microwave power sensor. With the mass production of semiconductors, it became possible to apply Hall effect to in many different products. MICRO SWITCH Sensing and Control revolutionized the keyboard industry in 1968 by introducing the first solid state keyboard using the Hall effect. For the first time, the Hall Output sensor feature and associated electrical equipment are integrated into a single integrated location. Today, Hall effect devices are integrated into many products, from computers to sewing equipment, vehicles, aircraft to medical equipment.

In this project, we briefly introduce **Hall Effect** and the science behind it. A few **application of Hall Effect** which are useful in our daily life are also presented in the later part of this article.

Section - 1

Basic Idea of Hall-effect

Influx: It is named Hall Effect after the discoverer **Edwin Herbert Hall, American physicist**, who discovered it in 1879. Then he was a graduate student and trying to prove that a magnet directly affects current and not a wire bearing the current, as was believed at that time. He was experimenting with a metal (gold, silver, iron) leaf mounted on a glass plate (fig-1). An electric current was passed through the leaf, and a sensitive galvanometer was connected across the leaf at two nearly equipotential points. The leaf was placed between the poles of an electromagnet (fig-2). Experimenting with this arrangement, Hall discovered ‘a new action of the magnet on electric currents’. This action is now called the Hall effect.

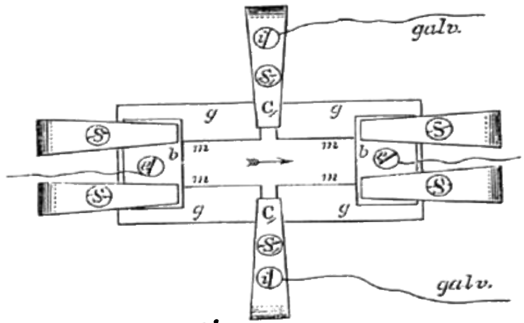


Fig : 1

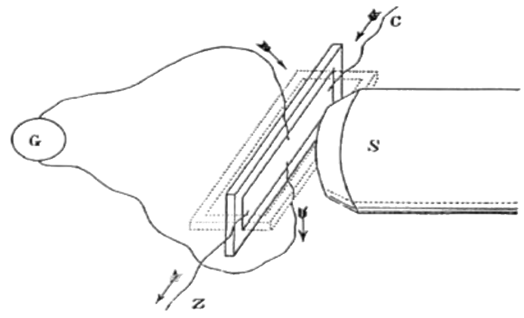


Fig : 2

Let's take a very thin film of the material, so that the electrons are basically moving in one single plane.

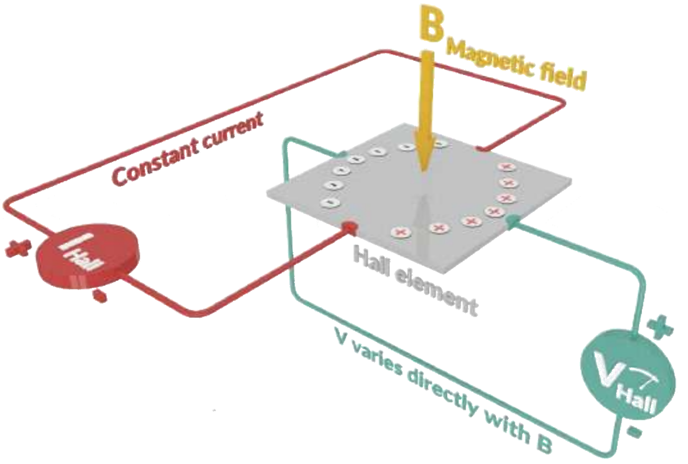


Fig : 3

Then let's apply a voltage across the film. Because of the voltage which is applies there will be a current 'I' which will start flowing through the thin film and as per convention the electrons (free) will drift in the opposite direction. And according to Ohm's law, there we shall get a voltage drop because of this current 'I'. Which we can calculate through the resistivity of the thin film. Then a magnetic field should be applied perpendicular to the surface of thin film.

Now apart from this longitudinal voltage which is in the direction of the flow of the current 'I', we shall get an additional transverse voltage perpendicular to the longitudinal flow of current 'I'. The transverse voltage is called "Hall Voltage". This effect of obtaining a measurable voltage is known as the Hall Effect.

Section - 2

Science Behind the Hall-Effect

Principle: The basic theory of Hall Effect is that when a current-carrying conductor or semiconductor is introduced in a perpendicular magnetic field, the voltage can be measured at a right angle in the current path.

How It Works: To know how Hall effect work, we have to take a thin film of conductor or semi-conductor.

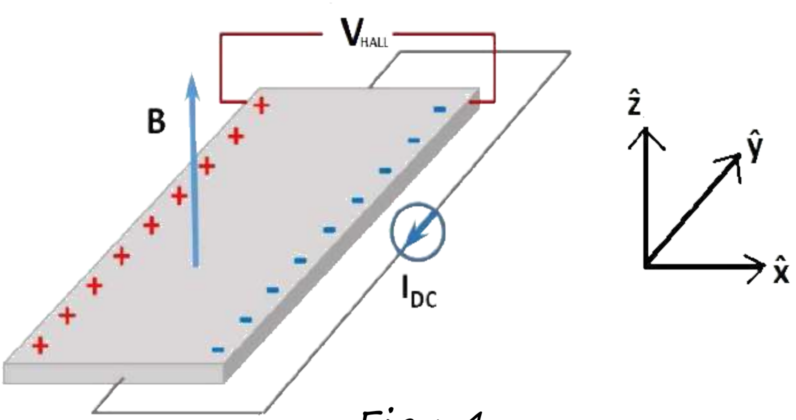


Fig : 4

Let's assume there is a current flow along $\hat{y}$ direction and electrons are flowing with a velocity $\bar{v}=v_0(-\hat{y})$ toward the negative $\hat{y}$ direction, and the applied magnetic field $\bar{B}=B_0\hat{k}$ is acting along the $\hat{z}$ direction.

So, the Lorentz Force on free electron $\vec{F}_L=-e(\bar{v}\times\bar{B})=-e(v_0 B_0)(-\hat{y}\times\hat{z})=e\,v_0 B_0\hat{x}$

As a result of this force, the electron get deflected toward $\hat{x}$ direction, resulting an electric field perpendicular to the flow of current.

Hall Electric Field vs Magnetic Field: To find how Hall Electric field depends on magnetic field, we have to know Drude's Model for a particle (here e^-) moving in an electric field along with a magnetic field

rate of change of momentum of free electron on thin film:

$$\frac{dp}{dt}=-\frac{P}{\tau}+\bar{f}(t) \quad \text{..............} \ (i)$$

Where, $\bar{P}$ is the momentum of free electron on the thin film, τ is the mean time interval between two successive collision experienced conducting electron and $\bar{f}(t)$ is electric & magnetic field acting on electrons.

Here, $\bar{f}(t)=-e\bar{E}-e(\bar{v}\times\bar{B})$, and as electric field & magnetic field not varying with time so is also constant and we are considering every thing on the system is under equilibrium.

So, change of momentum of free electron with time

$$\frac{d\bar{P}}{dt}=0$$

$$or,-\frac{\bar{P}}{\tau}-e\bar{E}-e(\bar{v}\times\bar{B})=0 \quad \text{............. (ii)}$$

We know that current can be written in term of velocity,

$$\bar{J}=-ne\bar{v}$$

$$or,\bar{v}=-\frac{\bar{J}}{ne}$$

and also current can be written in term of momentum,

$$\bar{J}=-\frac{ne}{m}\bar{P}$$

$$or,-\bar{P}=\frac{m}{ne}\bar{J}$$

Now, by substitution the value of $\bar{v}$ and $-\bar{P}$ in the equation (ii),

$$\frac{m}{ne\tau} - e\bar{E} + e\left(\frac{\bar{J}}{ne} \times \bar{B}\right) = 0$$

readjusting the above equation slightly,

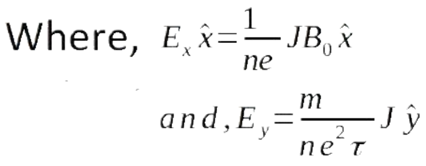
$$\bar{E} = \left(\frac{\bar{J} \times \bar{B}}{ne}\right) + \frac{m}{ne^2\tau}\bar{J}$$

Now, according to our configuration $\bar{J}$ is along $\hat{y}$ direction $\bar{J} = J\hat{y}$ and $\bar{B}$ is along $\hat{z}$ direction $\bar{B} = B_0\hat{z}$

So, rewriting the equation we get,

$$\bar{E} = \frac{1}{ne}JB_0\hat{x} + \frac{m}{ne^2\tau}J\hat{y}$$

$$or, \bar{E} = E_x\hat{x} + E_y\hat{y}$$



Where, $E_x\hat{x} = \frac{1}{ne}JB_0\hat{x}$

$and, E_y = \frac{m}{ne^2\tau}J\hat{y}$

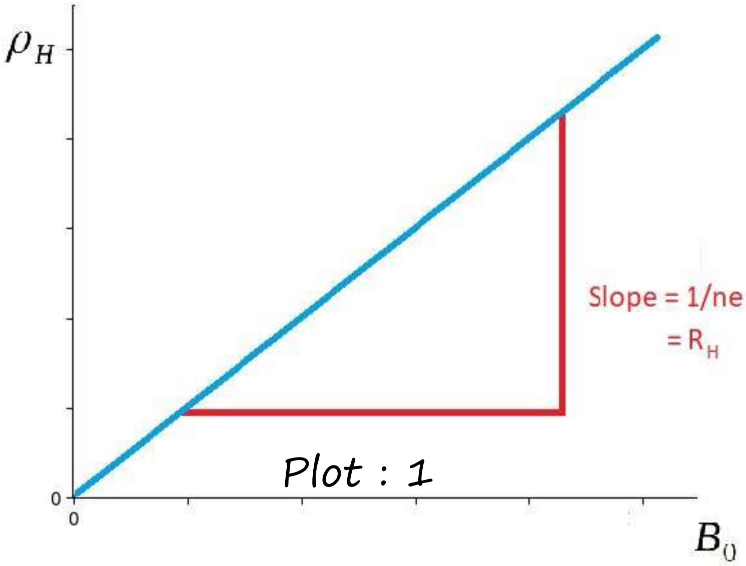
Here, along $\hat{x}$ direction there is an electric field and a corresponding resistivity to the transverse direction which is called Hall resistivity,

$$E_x = \frac{1}{ne} B_0 J$$
$$= \rho_H J$$

Where, $\rho_H = \frac{B_0}{ne}$ is Hall Resistivity

If one plot the Hall Resistivity as a function of magnetic field, this expression shows that hall resistivity is going to a straight line.

Where the slope (see plot-1) of the straight line is $\frac{1}{ne} = R_H$ (the quantity $\frac{1}{ne}$ is given a term R_H) which is called Hall Co-efficient.



The important aspect of that in a magnetic field if one measures ρ_H and if one know R_H of a material (R_H is constant for a material.) then one can measure magnetic field immedietly.

Section – 3

Applications of Hall-Effect

Hall effect is a widely used phenomena, as in science, its use in our daily life is far-reaching, so we divided the use of Hall effects into two parts. -

First, its direct application in the branch of science. And Second, its use in the daily life of ordinary people

1. Direct Application in the Branch of Science -

- Determine the Type of Semiconductor – By knowing the direction of the Hall Voltage, one can decide what type of semiconductor it is. Because Hall coefficient (R_H) is **positive** for **p-type** semiconductor while it is **negative** for **n-type** semiconductor.

- Finding the Carrier Concentration – The equation for the carrier density of electrons (n) and holes (p) in the term of Hall co-efficient (R_H) are given by

$$n=\frac{1}{q\,R_H} \quad \text{and} \quad p=\frac{1}{q\,R_H}$$

- Determine the Mobility (Hall Mobility) – Mobility equation for the holes (μ_p) and the electrons (μ_n

), expressed in term of Hall coefficient (R_H) is given by $\mu_p = \sigma_p R_H$ and $\mu_n = \sigma_n R_H$.

- Current sensors – Hall sensors can be used in ammeter to measure currents from milli-amperes to kilo-amperes. When the hall sensor is placed next to the current carrying wire, the output voltage generated is proportional to the magnitude of the field surrounding the wire. Since the field magnitude at a particular point is proportional to the current level as shown in fig-5.The analog voltage generated by the Hall sensor can be adjusted by adding a transistor for digital output. This type of configuration is usually used to measure relatively large current surges around high voltage lines or equipment found in electrical power plants.

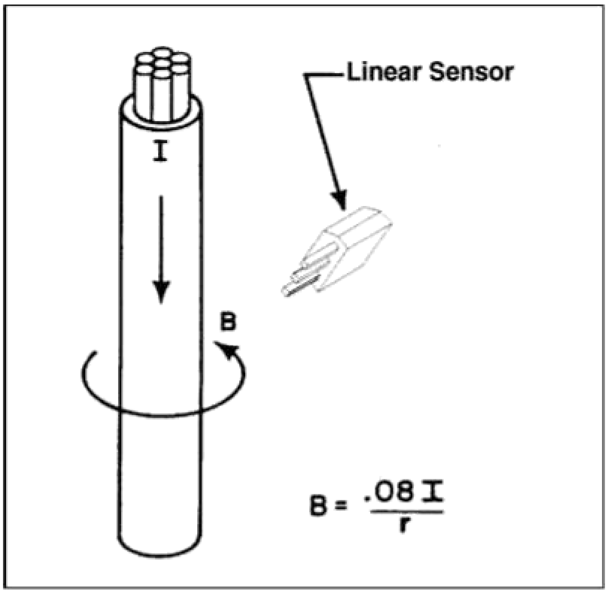


Fig : 5

2. Use in Daily Life -

- Automatically Mobile On and Off – The hall sensors are located inside phone and operate on the Hall effect system. According to the Hall effect, when a change in the magnetic field occurs across the piezoelectric crystal, there is a noticeable change in magnitude across the other two sides, on the contrary. The most important and common application is in the case of covering phones.

When one turn on or off your cover, the screen automatically turns on or off.

- <u>Automotive sensors</u> – a Fig-6 suggests many concepts where Hall effect sensors can be applied as monitoring, positioning or safety feedback devices for the automotive market. Both digital and linear output sensors are used in such applications as: flow meters, current sensors, position sensors, interlocks, pressure sensors, RPM sensors, etc.

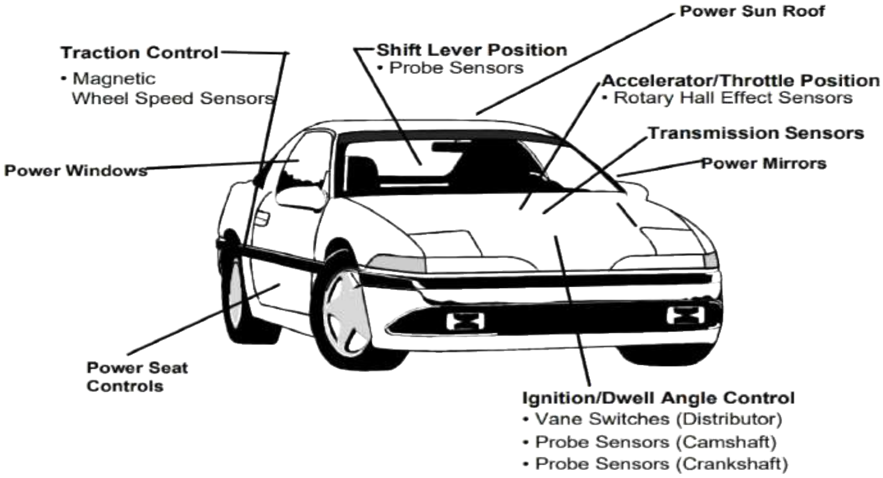


Fig : 6

Conclusion

In more than a hundred years of their history, Hall effect devices have been used to demonstrate the basic laws of physics, to study details of carrier transport phenomena in solids, to detect the presence of a magnet, and as measuring devices for magnetic fields. Not only that today, Hall effect magnetic sensors form the basis of a mature and important industrial activity. They are mostly used as key elements in contact-less sensors for linear position, angular position, velocity, rotation, electrical current, and so on. We estimated that more that two billion Hall magnetic sensors were sold worldwide in the year 2000. So we can say, science is much more advanced now, and the application of science today is also much more complex. But there is a simple concept hidden in these complex things. Once one understands these, it is easy to look at all these complex things in a simple way. after doing this project, I can understand how technologies are work, that we use in our every day life.

Acknowledgement

Inspiration and motivation have always played a key role to complete any venture.
Firstly, I like to express my gratitude to our institution **Ramakrishna Mission Vivekananda Centenary College, Rahara** and as well as to our revered **Principal Maharaj Swami Kamalasthananda, Ramakrishna Mission Vivekananda Centenary College, Rahara** for including the project as part of our academic curriculum.

Then I like to thanks **Dr. Rajen Kundu, Professor, Ramakrishna Mission Centenary College, Rahara** who suggested me the topic and encouraged to take up this project as an introduction to the subject.

I am also grateful to **my family** for their continuous support and motivation.
Last but not the least, I like to pay thanks to **Apache OpenOffice** for their free software which helped me a lot to decorate this project work.

Bibliography

For the completion of this project work, I have consulted several websites and books. I have also collected some images form these sites and books. The references are given below.

Sites:

1. https://www.melexis.com/en/articles/hall-effect

2.https://nationalmaglab.org/education/magnet-academy/watch-play/interactive/hall-effect

Books:

1. Hall Effect Devices, R S Popovic, Institute of Physics Publishing Bristol and Philadelphia

2. The Hall Effect and Its Applications, Colin M. Hurd (auth.), C. L. Chien, C. R. Westgate (eds.), Springer US

HISTORY AND EVOLUTION OF NUCLEAR PHYSICS

The project submitted, in partial fulfilment of the requirement for the assignments in (CC XI,CC XII,DSE-I, DSE-II Paper), (Semester V) in the Department of Physics.

Submitted by

ARANYA DAS

Registration No: A01-1112-111-014-2019

Supervisor Teacher : BHASKAR HALDER

RAMAKRISHNA MISSION VIVEKANANDA CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

Introduction

It is human nature to test, to observe, and to dream. The history of nuclear energy is the story of a centuries-old dream becoming a reality. Ancient Greek philosophers first developed the idea that all matter is composed of invisible particles called atoms. The word atom comes from the Greek word “atomos”, meaning indivisible. Scientists in the 18th and 19th centuries revised the concept based on their experiments. By 1900, physicists knew the atom contains large quantities of energy. British physicist Ernest Rutherford was called the father of nuclear science because of his contribution to the theory of atomic structure. In 1904 he wrote:

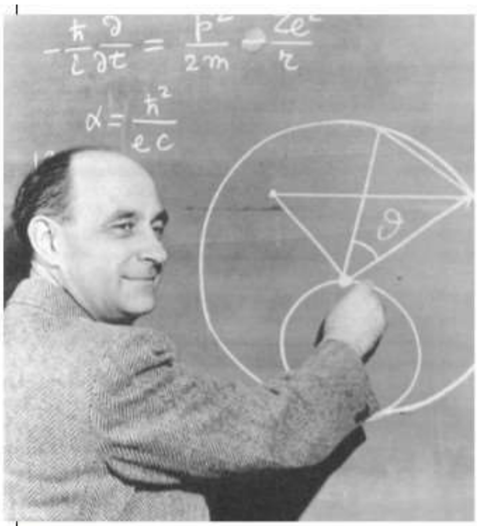
> *If it were ever possible to control at will the rate of disintegration of the radio elements, an enormous amount of energy could be obtained from a small amount of matter.*

Albert Einstein developed his theory of the relationship between mass and energy one year later. The mathematical formula is $E=mc^2$, or “energy equals mass times the speed of light squared.” It took almost 35 years for someone to prove Einstein’s theory.

Section 1

The Discovery Of Fission

In 1934, physicist Enrico Fermi conducted experiments in Rome that showed neutrons could split many kinds of atoms. The results surprised even Fermi himself. When he bombarded uranium with neutrons, he did not get the elements he expected. The elements were much lighter than uranium. In the fall of 1938, German scientists Otto Hahn and Fritz Strassman fired neutrons from a source containing the elements radium and beryllium into uranium (atomic number 92). They were surprised to find lighter elements, such as barium (atomic number 56), in the leftover materials. These elements had about half the atomic mass of uranium. In previous experiments, the leftover materials were only slightly lighter than uranium.



Enrico Fermi, an Italian physicist, led the team of scientists who created the first self-sustaining nuclear chain reaction.

Hahn and Strassman contacted Lise Meitner in Copenhagen before publicizing their discovery. She was an Austrian colleague who had been forced to flee Nazi Germany. She worked with Niels Bohr and her nephew, Otto R. Frisch. Meitner and Frisch thought the barium and other light elements in the leftover material resulted from the uranium splitting — or fissioning. However, when she added the atomic masses of the fission products, they did not total the uranium’s mass. Meitner used Einstein’s theory to show the lost mass changed to energy. This proved fission occurred and confirmed Einstein’s work.

Section 2

The First Self-Sustaining Chain Reaction

In 1939, Bohr came to America. He shared with Einstein the HahnStrassman-Meitner discoveries. Bohr also met Fermi at a conference on theoretical physics in Washington, D.C. They discussed the exciting possibility of a self-sustaining chain reaction. In such a process, atoms could be split to release large amounts of energy.

Leo Szilard

Scientists throughout the world began to believe a self-sustaining chain reaction might be possible. It would happen if enough uranium could be brought together under proper conditions. The amount of uranium needed to make a self-sustaining chain reaction is called a critical mass.

Fermi and his associate, Leo Szilard, suggested a possible design for a uranium chain reactor in 1941. Their model consisted of uranium placed in a stack of graphite to make a cube-like frame of fissionable material.

Early in 1942, a group of scientists led by Fermi gathered at the University of Chicago to develop their theories. By November 1942, they were ready for construction to begin on the world's first nuclear reactor, which became known as Chicago Pile-1. The pile was erected on the floor of a squash court beneath the University of Chicago's athletic stadium. In addition to uranium and graphite, it contained control rods made of cadmium. Cadmium is a metallic element that absorbs neutrons. When the rods were in the pile, there were fewer eutrons to fission uranium atoms. This slowed the chain reaction.

When the rods were pulled out, more neutrons were available to split atoms. The chain reaction sped up.

On the morning of December 2, 1942, the scientists were ready to begin a demonstration of Chicago Pile-1. Fermi ordered the control rods to be withdrawn a few inches at a time during the next several hours. Finally, at 3:25 p.m., Chicago time, the nuclear reaction became self-sustaining. Fermi and his group had successfully transformed scientific theory into technological reality. The world had entered the nuclear age.

Section 3

The Development Of Nuclear Energy For Peaceful Applications

The first nuclear reactor was only the beginning. Most early atomic research focused on developing an effective weapon for use in World War II. The work was done under the code name Manhattan Project.

However, some scientists worked on making breeder reactors, which would produce fissionable material in the chain reaction. Therefore, they would create more fissionable material than they would use.

After the war, the United States government encouraged the development of nuclear energy for peaceful civilian purposes. Congress created the Atomic Energy Commission (AEC) in 1946. The AEC authorized the construction of Experimental Breeder Reactor I at a site in Idaho. The reactor generated the first electricity from nuclear energy on December 20, 1951.

A major goal of nuclear research in the mid-1950s was to show that nuclear energy could produce electricity for commercial use. The first commercial electricity-generating plant powered by nuclear energy was located in Shippingport, Pennsylvania. It reached its full design power in 1957. Light-water reactors like Shippingport use ordinary water to cool the reactor core during the chain reaction. They were the best design then available for nuclear powerplants.

Lise Meitner and Otto R. Frisch

Enrico Fermi led a group of scientists in initiating the first self-sustaining nuclear chain reaction. The historic event, which occurred on December 2, 1942, in Chicago, is recreated in this painting.

Private industry became more and more involved in developing light-water reactors after Shippingport became operational. Federal nuclear energy programs shifted their focus to developing other reactor technologies.

The nuclear power industry in the U.S. grew rapidly in the 1960s. Utility companies saw this new form of electricity production as economical, environmentally clean, and safe. In the 1970s and 1980s, however, growth slowed. Demand for electricity decreased and concern grew over nuclear issues, such as reactor safety, waste disposal, and other environmental considerations.

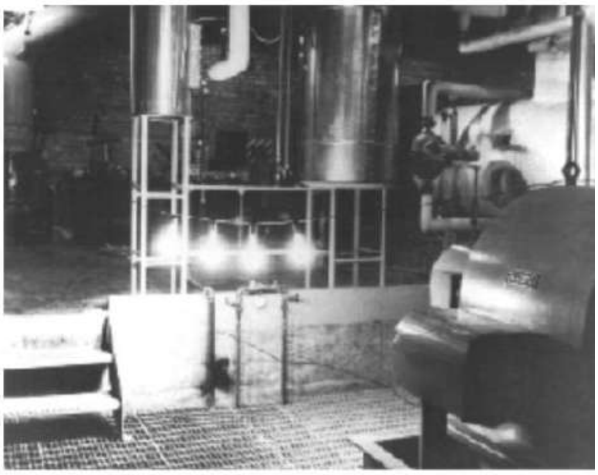
Still, the U.S. had twice as many operating nuclear powerplants as any other country in 1991. This was more than one-fourth of the world's operating plants. Nuclear energy supplied almost 22 percent of the electricity produced in the U.S.

At the end of 1991, 31 other countries also had nuclear powerplants in commercial operation or under construction. That is an impressive worldwide commitment to nuclear power technology.

During the 1990s, the U.S. faces several major energy issues and has developed several major goals for nuclear power, which are:

- To maintain exacting safety and design standards;
- To reduce economic risk;
- To reduce regulatory risk; and
- To establish an effective high-levelnuclear waste disposal program.

Several of these nuclear power goals were addressed in the Energy Policy Act of 1992, which was signed into law in October of that year.



The Experimental Breeder Reactor I generated electricity to light four 200-watt bulbs on December 20, 1951. This milestone symbolized the beginning of the nuclear power industry.

The U.S. is working to achieve these goals in a number of ways. For instance, the U.S. Department of Energy has undertaken a number of joint efforts with the nuclear industry to develop the next generation of nuclear powerplants. These plants are being designed to be safer and more efficient. There is also an effort under way to make nuclear plants easier to build by standardizing the design and simplifying the licensing requirements, without lessening safety standards.

In the area of waste management, engineers are developing new methods and places to store the radioactive waste produced by nuclear plants and other nuclear processes. Their goal is to keep

In Oak Ridge, Tennessee, workers package isotopes, which are commonly used in science, industry, and medicine.

the waste away from the environment and people for very
long periods of time.

Scientists are also studying the power of nuclear fusion. Fusion occurs when atoms join — or fuse — rather than split. Fusion is the energy that powers the sun. On earth, the most promising fusion fuel is deuterium, a form of hydrogen. It comes from water and is plentiful. It is also likely to create less radioactive waste than fission. However, scientists are still unable to produce useful amounts of power from fusion and are continuing their research.

Research in other nuclear areas is also continuing in the 1990s. Nuclear technology plays an important role in medicine, industry, science, and food and agriculture, as well as power generation. For example, doctors use radioisotopes to identify and investigate the causes of disease. They also use them to enhance traditional medical treatments. In industry, radioisotopes are used for measuring microscopic thicknesses, detecting irregularities in metal casings, and testing welds. Archaeologists use nuclear techniques to date prehistoric objects accurately and to locate structural defects in statues and buildings. Nuclear irradiation is used in preserving food. It causes less vitamin loss than canning, freezing, or drying. Nuclear research has benefited mankind in many ways. But today, the nuclear industry faces huge, very complex issues. How can we minimize the risk? What do we do with the waste? The future will depend on advanced engineering, scientific research, and the involvement of an enlightened citizenry.

Section 4

Chronology of Nuclear Research and Development

The '40s 1942 December 2. The first self-sustaining nuclear chain reaction occurs at the University of Chicago.

The '40s

1942 December 2. The first self-sustaining nuclear chain reaction occurs at the University of Chicago.

1945 July 16. The U.S. Army’s Manhattan Engineer District (MED) tests the first atomic bomb at Alamogordo, New Mexico, under the code name Manhattan Project.

1945 August 6. The atomic bomb nicknamed Little Boy is dropped on Hiroshima, Japan. Three days later, another bomb, Fat Man, is dropped on Nagasaki, Japan. Japan surrenders on August 15, ending World War II.

1946 August 1. The Atomic Energy Act of 1946 creates the Atomic Energy Commission (AEC) to control nuclear energy development and explore peaceful uses of nuclear energy.

1947 October 6. The AEC first investigates the possibility of peaceful uses of atomic energy, issuing a report the following year.

1949 March 1. The AEC announces the selection of a site in Idaho for the National Reactor Testing Station.

The '50s

1951 December 20. In Arco, Idaho, Experimental Breeder Reactor I produces the first electric power from nuclear energy, lighting four light bulbs.

1952 June 14. Keel for the Navy's first nuclear submarine, Nautilus, is laid at Groton, Connecticut.

1953 March 30. Nautilus starts its nuclear power units for the first time.

1953 December 8. President Eisenhower delivers his "Atoms for Peace" speech before the United Nations. He calls for greater international cooper-aton in the development of atomic energy for peaceful purposes.

1954 August 30. President Eisenhower signs The Atomic Energy Act of 1954, the first major amendment of the original Atomic Energy Act, giving the civilian nuclear power program further access to nuclear technology.

1955 January 10. The AEC announces the Power Demonstration Reactor Program. Under the program, AEC and industry will cooperate in constructing and operating experimental nuclear power reactors. **1955 July 17.** Arco, Idaho, population 1,000, becomes the first town powered by a nuclear powerplant, the experimental boiling water reactor BORAX III.

1955 August 8-20. Geneva, Switzerland, hosts the first United Nations International Conference on the Peaceful Uses of Atomic Energy. **1957 July 12.** The first power from a civilian nuclear unit is generated by the Sodium Reactor Experiment at Santa Susana, California. The unit provided power until 1966.

The Nautilus-the First Atomic-Powered Sub

1957 September 2. The Price-Anderson Act provides financial protection to the public and AEC licensees and contractors if a major accident occurs at a nuclear powerplant.

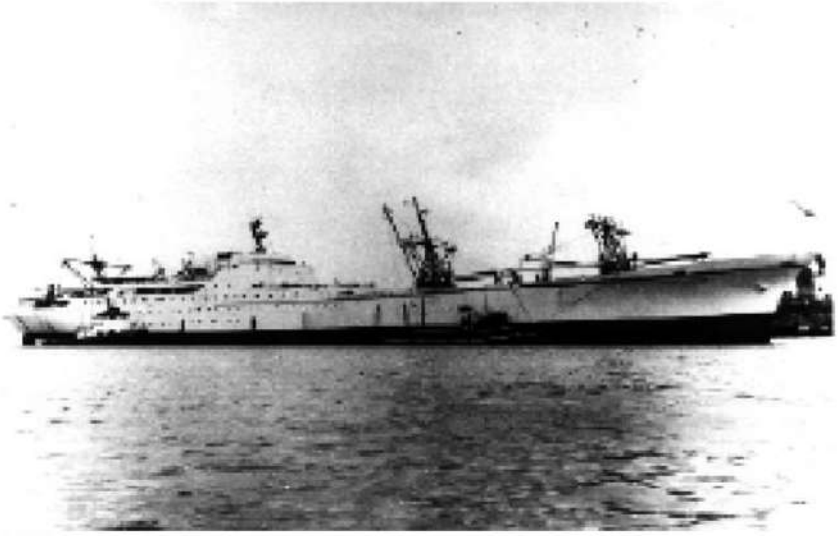
1957 October 1. The United Nations creates the International Atomic Energy Agency (IAEA) in Vienna, Austria, to promote the peaceful use of nuclear energy and prevent the spread of nuclear weapons around the world.

1957 December 2. The world’s first large-scale nuclear powerplant begins operation in Shipping port, Pennsylvania. The plant reaches full power three weeks later and supplies electricity to the Pittsburgh area.

1958 May 22. Construction begins on the world's first nuclear-powered merchant ship, the N.S. Savannah, in Camden, New Jersey. The ship is launched July 21, 1959.

1959 October 15. Dresden-1 Nuclear Power Station in Illinois, the first

U.S. nuclear plant built entirely without government funding, achieves a self-sustaining nuclear reaction.



NS Savannah

The '60s

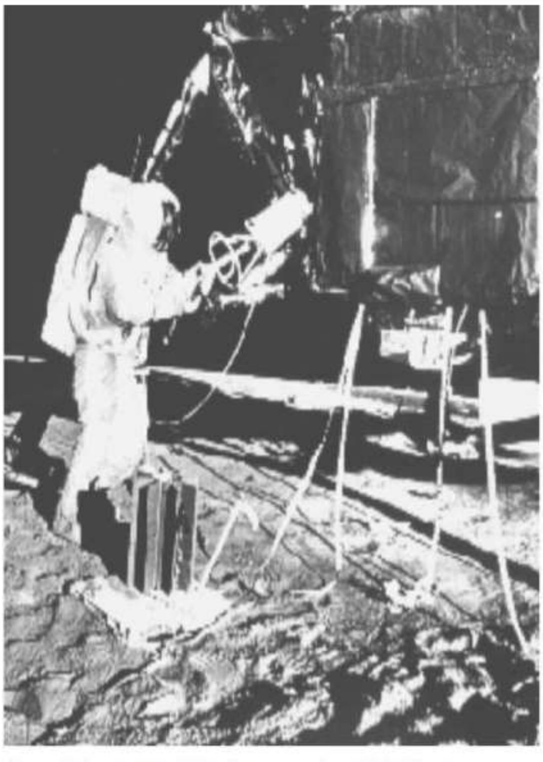
1960 **August 19.** The third U.S. nuclear powerplant, Yankee Rowe Nuclear Power Station, achieves a selfsustaining nuclear reactor.

Early 1960s. Small nuclear-power generators are first used in remote areas to power weather stations and to light buoys for sea navigation.

1961 **November 22.** The U.S. Navy commissions the world's largest ship, the U.S.S. Enterprise. It is a nuclear-owered aircraft carrier with the ability to operate at speeds up to 30 knots for distances up to 400,000 miles (740,800 kilometers) without refueling.

1962 **August 26.** President Lyndon B. Johnson signs the Private Ownership of Special Nuclear Materials Act, which allows the nuclear power industry to own the fuel for its units. After June 30, 1973, private ownership of the uranium fuel is mandatory.

1963 **December 12.** Jersey Central Power and Light Company announces its commitment for the Oyster Creek nuclear powerplant, the first time a nuclear plant is ordered as an



An atomic battery operated on the moon continuously for three years. Nuclear electric power arrived on the moon for the first time on November 19, 1969, when the Apollo 12 astronauts deployed the AEC's SNAP-27 nuclear generator on the lunar surface.

economic alternative to a fossil-fuel plan.

1964 **October 3.** Three nuclear-powered surface ships, the Enterprise, Long Beach, and Bainbridge, complete "Operation Sea Orbit," an around-the-world cruise.

1965 **April 3.** The first nuclear reactor in space (SNAP-10A) is launched by the United States. SNAP stands for Systems for Nuclear Auxiliary Power.

The '70s

1970 March 5. The United States, United Kingdom, Soviet Union, and 45 other nations ratify the Treaty for Non- roliferation of Nuclear weapons. 1971 Twenty-two commercial nuclear powerplants are in full operation in the United States. They produce 2.4 percent of U.S. electricity at this time. 1973 U.S. utilities order 41nuclear powerplants, a one-year record. 1974 The first 1,000- megawatt-electric nuclear powerplant goes into service – Commonwealth dison's Zion 1 Plant.

1974 October 11. The Energy Reorganization Act of 1974 divides AEC functions between two new agencies — the Energy Research and Development Administration (ERDA), to carry out research and development, and the Nuclear Regulatory Commission (NRC), to regulate nuclear power.

1977 April 7. President Jimmy Carterannounces the United States will defer indefinitely plans for reprocessing spent nuclear fuel.

1977 August 4. President Carter signs the Department of Energy Organization Act, which transfers ERDA functions to the new Department of Energy (DOE).

1977 October 1. DOE begins operations.

1979 March 28. The worst accident in U.S. commercial reactor history occurs at the Three Mile Island nuclear power station near Harrisburg, Pennsylvania. The accident is caused by a loss of coolant from the reactor core due to a combination of mechanical malfunction and human error. No one is injured, and no overexposure to radiation results from the accident. Later in the year, the NRC imposes stricter reactor safety regulations and more rigid inspection procedures to improve the safety of reactor operations. 1979 Seventy-two licensed reactors generate 12 percent of the electricity produced commercially in the United States

The '80s

1980 March 26. DOE initiates the Three Mile Island research and development program to develop technology for disassembling and de-fueling the damaged reactor. The program will continue for 10 years and make significant advances in developing new nuclear safety technology.

1982 October 1. After 25 years of service, the Shippingport Power Station is shut down. Decommissioning would be completed in 1989. **1983 January 7.** The Nuclear Waste Policy Act (NWPA) establishes a program to site a repository for the disposal of high-level radioactive waste, including spent fuel from nuclear powerplants. It also establishes fees for owners and generators of radioactive waste and spent fuel, who pay the costs of the program. 1983 Nuclear power generates more electricity than natural gas. 1984 The atom overtakes hydropower to become the second largest source of electricity, after coal. Eighty-three nuclear power reactors provide about 14 percent of the electricity produced in the United States. 1985 The Institute of Nuclear Power Operations forms a national academy to accredit every nuclear powerplant's training program. 1986 The Perry power Plant in Ohio becomes the 100th U.S. nuclear powerplant in operation.

1986 April 26. Operator error causes two explosions at the Chernobyl No. 4 nuclear powerplant in the former Soviet Union. The reactor has an inadequate containment building, and large amounts of radiation escape. A plant of such design would not be licensed in the United States.

1987 December 22. The Nuclear Waste Policy Act (NWPA) is amended. Congress directs DOE to study only the potential of the Yucca Mountain, Nevada, site for disposal of high-level radioactive waste.

1988 U.S. electricity demand is 50 percent higher than in 1973. 1989 One hundred and nine nuclear powerplants provide 19 percent of the electricity used in the U.S.; 46 units have entered service during the decade.

1988 U.S. electricity demand is 50 percent higher than in 1973. **1989** One hundred and nine nuclear powerplants provide 19 percent of the electricity used in the U.S.; 46 units have entered service during the decade.

1989 April 18. The NRC proposes a plan for reactor design certification, early site permits, and combined construction and operating licenses. #

The '90s

1990 March. DOE launches a joint initiative to improve operational safety practices at civilian nuclear powerplants in the former Soviet Union.

1990 America's 110 nuclear powerplants set a record for the amount of electricity generated, surpassing all fuel sources combined in 1956.

The Omaha Public District Fort Calhoun Nuclear Power Station located at Fort Calhoun, Nebraska

1990 April 19. The final shipment of damaged fuel from the Three Mile Island nuclear plant arrives at a DOE facility in Idaho for research and interim storage. This ends DOE's 10-year Three Mile Island research and development program.

1991 One hundred and eleven nuclear powerplants operate in theUnited States with a combined capacity of 99,673 megawatts. They produce almost 22 percent of the electricity generated commercially in the United States.

1992 One hundred and ten nuclear powerplants account for nearly 22percent of all electricity used in the U.S.

1992 February 26. DOE signs a cooperative agreement with the nuclear industry to co-fund the development of standard designs for advanced light-water reactors.

1992 October 24. The Energy Policy Act of 1992 is signed into law. The Act makes several important changes in the licensing process for nuclear powerplants.

1992 December 2. The 50th anniversary of the historic Fermi experiment is observed worldwide.

1993 March 30. The U.S. nuclear utility consortium, the Advanced Reactor Corporation (ARC), signs a contract with Westinghouse Electric Corporation to perform engineering work for an advanced, standardized 600-megawatt pressurized water reactor. Funding for this next-generation plant comes from ARC, Westinghouse, and DOE. **1993 September 6.** The U.S. nuclear utility consortium, ARC, signs a contract with General Electric Company for cost-shared, detailed engineering of a standardized design for a large, advanced nuclear powerplant. The engineering is being funded under a joint program among utilities, General Electric, and DOE.

Conclusion

The concept of the atom has existed for many centuries. But we only recently began to understand the enormous power contained in the tiny mass. In the years just before and during World War II, nuclear research focused mainly on the development of defense weapons. Later, scientists concentrated on peaceful applications of nuclear technology. An important use of nuclear energy is the generation of electricity. After years of research, scientists have successfully applied nuclear technology to many other scientific, medical, and industrial purposes. This pamphlet traces the history of our discoveries about atoms. We begin with the ideas of the Greek philosophers. Then we follow the path to the early scientists who discovered radioactivity. Finally, we reach modern-day use of atoms as a valuable source of energy.

Acknowledgement

Inspiration and motivation have always played a key role to complete any venture.
Firstly, I would like to express my gratitude to our institution **Ramakrishna Mission Vivekananda Centenary College, Rahara** for including the project as part of our academic curriculum.

Then I like to thanks **Prof. Bhaskar Halder, Professor, Ramakrishna Mission Centenary College, Rahara** who suggested me the topic and encouraged to take up this project as an introduction to the subject.

I also grateful to **my friends** for their continuous support and motivation.
Last but not the least, I would say thank to **Apache OpenOffice** for their free software which helped me a lot.

Bibliography

For the completion of this project work, I have consulted several websites and books.
I have also collected some images form these sites and books.
The references are given below.

Sites:

1. https://www.britannica.com/science/physics-science/Nuclear-physics

2. http://www.world-nuclear.org/information-library/current-and-futuregeneration/outline-history-of-nuclear-energy.aspx

Books:

1. **Modern Atomic and Nuclear Physics** Book by J**oseph H. Hamilton** and **Yang Fujia**

2. **Modern Nuclear Physics: From Fundamentals to Frontiers** Book by **Alexandre Obertelli and Hiroyuki Sagawa**

SIR MODEL OF COVID-19

The project submitted, in partial fulfilment of the requirement for the assignments in (CC XI,CC XII,DSE-I, DSE-II Paper), (Semester V) in the Department of Physics.

Submitted by
SAYANTAN BOSE

Registration No: A01-1112-111-015-2019

Supervisor Teacher : Dr. KALYAN CHATTERJEE

RAMAKRISHNA MISSION VIVEKANANDA CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

ACKNOWLEDGEMENT

The preparation and completion of this project was made possible largely because of the following people/sources. And I am obliged to express my gratitude to those whose assistance was present throughout.

To begin with, I am thankful to our professor Dr.Kalyan Chatterjee for assigning me this topic. Their proffered encouragement was of great aid. Additionally, working on this project has helped me to obtain knowledge and experience.

Following I would like to express my sincerest of gratitude to the owners of those sources that I have utilized. This paper has been augmented in its value because of all the details I was able to gather from these websites. It has succeeded with the understanding of facts and explanation broached and its credits are to their owners.

At the end, I am thankful to the owners of the images I have used to furnish my paper with. It has been enlivening and seems polished greatly because of them.

A SIR model assumption for the spread of COVID-19 in different communities

ABSTRACT

In this paper, we study the effectiveness of the modelling approach on the pandemic due to the spreading of the novel COVID-19 disease and develop a susceptible-infected-removed (SIR) model that provides a theoretical framework to investigate its spread within a community. Here, the model is based upon the well-known susceptible-infected-removed (SIR) model with the difference that a total population is not defined or kept constant per se and the number of susceptible individuals does not decline monotonically. To the contrary, as we show herein, it can be increased in surge periods! In particular, we investigate the time evolution of different populations and monitor diverse significant parameters for the spread of the disease in various communities, represented by China, South Korea, India, Australia, USA, Italy and the state of Texas in the USA. The SIR model can provide us with insights and predictions of the spread of the virus in communities that the recorded data alone cannot. Our work shows the importance of modelling the spread of COVID-19 by the SIR model that we propose here, as it can help to assess the impact of the disease by offering valuable predictions. Our analysis takes into account data from January to June, 2020, the period that contains the data before and during the implementation of strict and control measures. We propose predictions on various parameters related to the spread of COVID-19 and on the number of susceptible, infected and removed populations until September 2020. By comparing the recorded data with the data from our modelling approaches, we deduce that the spread of COVID-19 can be under control in all communities considered, if proper restrictions and strong policies are implemented to control the infection rates early from the spread of the disease.

Introduction

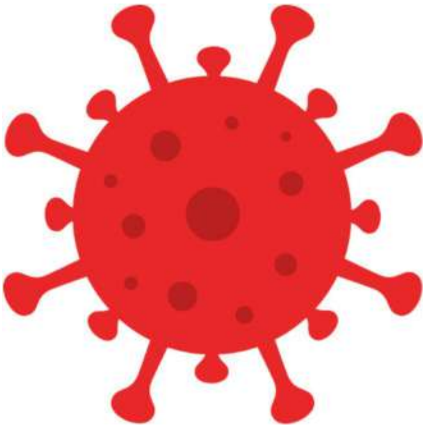
In December 2019, a novel strand of Coronavirus (SARS-CoV-2) was identified in Wuhan, Hubei Province, China causing a severe and potentially fatal respiratory syndrome, i.e., COVID-19. Since then, it has become a pandemic declared by World Health Organization (WHO) on March 11, which has spread around the globe. WHO published in its website preliminary guidelines with public health care for the countries to deal with the pandemic. Since then, the infectious disease has become a public health threat. Italy and USA are severely affected by COVID-19. Millions of people are forced by national governments to stay in self-isolation and in difficult conditions. The disease is growing fast in many countries around the world. In the absence of availability of a proper medicine or vaccine, currently social distancing, self-quarantine and wearing a face mask have been emerged as the most widely-used strategy for the mitigation and control of the pandemic.

In this context, mathematical models are required to estimate disease transmission, recovery, deaths and other significant parameters separately for various countries, that is for different, specific regions of high to low reported cases of COVID-19. Different countries have already taken precise and differentiated measures that are important to control the spread of the disease. However, still now, important factors such as population density, insufficient evidence for different symptoms, transmission mechanism and unavailability of a proper vaccine, makes it difficult to deal with such a highly infectious and deadly disease, especially in high population density countries such as India Recently, many research articles have adopted the modelling approach, using real incidence datasets from affected countries and, have investigated different characteristics as a function of various parameters of the outbreak and the effects of intervention strategies in different countries, respective to their current situations.

It is imperative that mathematical models are developed to provide insights and make predictions about the pandemic, to plan effective control strategies and policies. Modelling approaches are helpful to understand and predict the possibility and severity of the disease outbreak and, provide key information to determine the intensity of COVID-19 disease intervention. The susceptible-infected-removed (SIR) model and its extended modifications, such as the extended-susceptible-infected-removed (eSIR) mathematical model in various forms have been used in previous studies to model the spread of COVID-19 within communities.

Here, we propose the use of a novel SIR model with different characteristics. One of the major assumptions of the classic SIR model is that there is a homogeneous mixing of the infected and susceptible populations and that the total population is constant in time. In the classic SIR model, the susceptible population decreases monotonically towards zero. However, these assumptions are not valid in the case of the spread of the COVID-19 virus, since new epicentres spring up around the globe at different times. To account for this, the SIR model that we propose here does not consider the total population and takes the susceptible population as a variable that can be adjusted at various times to account for new infected individuals spreading throughout a community, resulting in an increase in the susceptible population, i.e., to the so-called surges. The SIR model we introduce here is given by the same simple system of three ordinary differential equations (ODEs) with the classic SIR model and can be used to gain a better understanding of how the virus spreads within a community of variable populations in time, when surges occur. Importantly, it can be used to make predictions of the number of infections and deaths that may occur in the future and provide an estimate of the time scale for the duration of the virus within a community. It also provides us with insights on how we might lessen the impact of the virus, what is nearly impossible to discern from the recorded data alone! Consequently, our SIR model can provide a theoretical framework and predictions that can be used by government authorities to control the spread of COVID-19.

In our study, we used COVID-19 datasets in the form of time-series, spanning January to June, 2020. In particular, the time series are composed of three columns which represent the total cases Itotd, active cases Id and Deaths Dd in time (rows). These datasets were used to update parameters of the SIR model to understand the effects and estimate the trend of the disease in various communities, represented by China, South Korea, India, Australia, USA, Italy and the state of Texas in the USA. This allowed us to estimate the development of COVID-19 spread in these communities by obtaining estimates for the number of deaths D, susceptible S, infected I and removed Rm populations in time. Consequently, we have been able to estimate its characteristics for these communities and assess the effectiveness of modelling the disease.



MODEL INFORMATION

In this section of manuscript, we formulate our new model for NCOVID−19 in the form of following system (1). We take whole population N(t) into three classes S(t), I(t) and R(t), which represent Susceptible, Infected and Recovered compartment in the form of differential equations given below

$$\begin{aligned}
\frac{dS(t)}{dt} &= b - k(1 - \alpha S(t)I(t)) - \alpha k\beta S(t)I(t) - \mu S(t) \\
\frac{dI(t)}{dt} &= k(1 - \alpha S(t)I(t)) + \alpha k\beta S(t)I(t) - (d_0 + \gamma + \mu)I(t) \\
\frac{dR(t)}{dt} &= \gamma I(t) - \mu R(t).
\end{aligned}$$

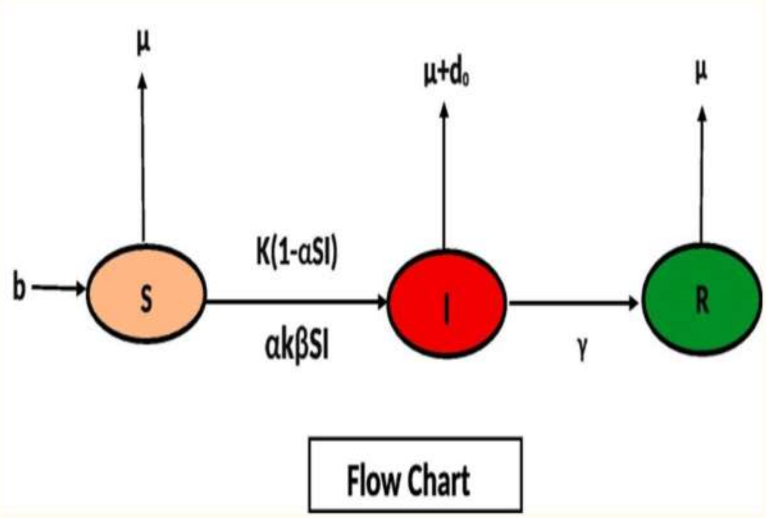
For above system (1) is presented in the form of flow chart as.

In Table 1 , we describe parameters used in system (1). In system (1), add all equations, implies

$$dN(t)/dt = -(\mu N(t)+d_0 t\ (t)-b).$$

TABLE 1

Physical Interpretation of parameters of the system.



Parameters	The physical Description
S(t)	Susceptible compartment
I(t)	Infected compartment
R(t)	Recovered compartment
d_0	Death due to corona
μ	Natural death
b	Birth rate
β	Protection rate
k	Constant rate
α	Isolation rate
γ	Recovery rate

Section – 1

- The SIR model that can accommodate surges in the susceptible population:

The world around us is highly complicated. For example, how a virus spreads, including the novel strand of Coronavirus (SARS-CoV-2) that was identified in Wuhan, Hubei Province, China, depends upon many factors, among which some of them are considered by the classic SIR model, which is rather simplistic and cannot take into consideration surges in the number of susceptible individuals. Here, we propose the use of a modified SIR model with characteristics, based upon the classic SIR model. In particular, one of the major assumptions of the classic SIR model is that there is a homogeneous mixing of the infected I and susceptible S populations and that the total population N is constant in time. Also, in the SIR model, the susceptible population S decreases monotonically towards zero. These assumptions however are not valid in the case of the spread of the COVID-19 virus, since new epicentres spring up around the globe at different times. To account for this, we introduce here a SIR model that does not consider the total population N, but rather, takes the susceptible population S as a variable that can be adjusted at various times to account for new infected individuals spreading throughout a community, resulting in its increase. Thus, our model is able to accommodate surges in the number of susceptible individuals in time, whenever these occur and as evidenced by published data, such as those in that we consider here.

Our SIR model is given by the same, simple system of three ordinary differential equations (ODEs) with the classic SIR model that can be easily implemented and used to gain a better understanding of how the COVID-19 virus spreads within communities of variable populations in time, including the possibility of surges in the susceptible populations. Thus, the SIR model here is designed to remove many of the complexities associated with the real-time evolution of the spread of the virus, in a way that is useful both quantitatively and qualitatively. It is a dynamical system that is given by three coupled ODEs that describe the time evolution of the following three populations:

1. *Susceptible individuals, $S(t)$*: These are those individuals who are not infected, however, could become infected. A susceptible individual may become infected or remain susceptible. As the virus spreads from its source or new sources occur, more individuals will become infected,

thus the susceptible population will increase for a period of time (surge period).

2. *Infected individuals,* $I(t)$: These are those individuals who have already been infected by the virus and can transmit it to those individuals who are susceptible. An infected individual may remain infected, and can be removed from the infected population to recover or die.
3. *Removed individuals,* $R_m(t)$: These are those individuals who have recovered from the virus and are assumed to be immune, $R_m(t)$ or have died, $D(t)$.

Furthermore, it is assumed that the time scale of the SIR model is short enough so that births and deaths (other than deaths caused by the virus) can be neglected and that the number of deaths from the virus is small compared with the living population.

Based on these assumptions and concepts, the rates of change of the three populations are governed by the following system of ODEs, what constitutes the SIR model used in this study

$dS(t)dt = -aS(t)I(t), dI(t)dt = aS(t)I(t) - bI(t), dR_m(t)dt = bI(t)$ --------------------- (1)

where a and b are real, positive, parameters of the initial exponential growth and final exponential decay of the infected population I.

It has been observed that in many communities, a spike in the number of infected individuals, I, may occur, which results in a surge in the susceptible population, S, recorded in the COVID-19 datasets, what amounts to a secondary wave of infections. To account for such a possibility, S in the SIR model, can be reset to S_{surge} at any time t_s that a surge occurs, and thus it can accommodate multiple such surges if recorded in the published data in, what distinguishes it from the classic SIR model.

The evolution of the infected population I is governed by the second ODE in system (1), where a is the transmission rate constant and b the removal rate constant. We can define the basic effective reproductive rate $R_e = \frac{aS(t)}{b}$,as the fate of the evolution of the disease depends upon it. If R_e is smaller than one, the infected population I will decrease monotonically to zero and if greater than one, it will increase, i.e., if $dI(t)dt < 0 \Rightarrow R_e < 1$and if $dI(t)dt > 0 \Rightarrow R_e > 1$. Thus, the effective reproductive rate $R_e$acts as a threshold that determines whether an infectious disease will die out quickly or will lead to an epidemic.

At the start of an epidemic, when $R_e > 1$ and $S \approx 1$, the rate of infected population is described by the approximation $dI(t)dt \approx (a - b)I(t)$and thus, the infected population I will initially increase exponentially according to I(t)=I(0)e(a−b)t. The infected population will reach a peak when the rate of change of the infected population is zero, $\frac{dI(t)}{dt} = 0$, and this occurs when $R_e = 1$. After the peak, the infected population will start to decrease exponentially, following I(t)∝e−bt. Thus, eventually (for $t \to \infty$), the system will approach $S \to 0$ and $I \to 0$. Interestingly, the existence of a threshold for infection is not obvious from the recorded data, however can be discerned from the model. This is crucial in identifying a possible second wave where a sudden increase in the susceptible population Swill result in $R_e > 1$, and to another exponential growth of the number of infections I.

Section – 2

- ## Methodology:

The data for China, South Korea, India, Australia, USA, Italy and the state of Texas (communities) are organised in the form of time-series where the rows are recordings in time (from January to June, 2020), and the three columns are, the total cases Itotd(first column), number of infected individuals I^d (second column) and deaths D^d (third column). Consequently, the number of removals can be estimated from the data by

$$R_m^d = I_{tot}^d - I^d - D^d.$$

Since we want to adjust the numerical solutions to our proposed SIR model (1) to the recorded data, for each dataset (community), we consider initial conditions in the interval [0,1] and scale them by a scaling factor f to fit the recorded data by visual inspection. In particular, the initial conditions for the three populations are set such that S(0)=1(i.e., all individuals are considered susceptible initially), $I(0) = R_m(0) = I_{max}^d/f < 1$, where I_{max}^d is the maximum number of infected individuals I^d. Consequently, the parameters a, b, f and I_{max}^d are adjusted manually to fit the recorded data as best as possible, based on a trial-and-error approach and visual inspections. A preliminary analysis using non-linear fittings to fit the model to the published data provided at best inferior results to those obtained in this paper using our trial-and-error approach with visual inspections, in the sense that the model solutions did not follow as close the published data, what justifies our approach in the paper. A prime reason for this is that the published data are data from different countries that follow different methodologies to record them, with not all infected individuals or deaths accounted for.

In this context, S, I and $R_m \geq 0$ at any $t \geq 0$. System can be solved numerically to find how the scaled (by f) susceptible S, infected I and removed R_m populations (what we call model solutions) evolve with time, in good agreement with the recorded data. In particular, since this system is simple with well-behaved solutions, we used the first-order Euler integration method to solve it numerically, and a time step h=200/5000=0.04that corresponds to a final integration time t_f of 200 days since January, 2020. This amounts to double the time interval in the recorded data and allows for predictions for up to 100 days after January, 2020.

Obviously, what is important when studying the spread of a virus is the number of deaths D and recoveries R in time. As these numbers are not provided directly by the SIR model , we estimated them by first, plotting the data for deaths D^d vs the removals R_m^d , where $R_m^d = D^d + R^d = I_{tot}^d - I^d$ then fitting the plotted data with the nonlinear function

$$D = D_0(1-e^{-kRm})$$

where D_0 and k are constants estimated by the non-linear fitting. The function is expressed in terms of only model values and is fitted to the curve of the data. Thus, having obtained D from the non-linear fitting, the number of recoveries R can be described in time by the simple observation that it is given by the scaled removals, R_m from the SIR model , less the number of deaths, D .

$$\mathbf{R = R_m - D}$$

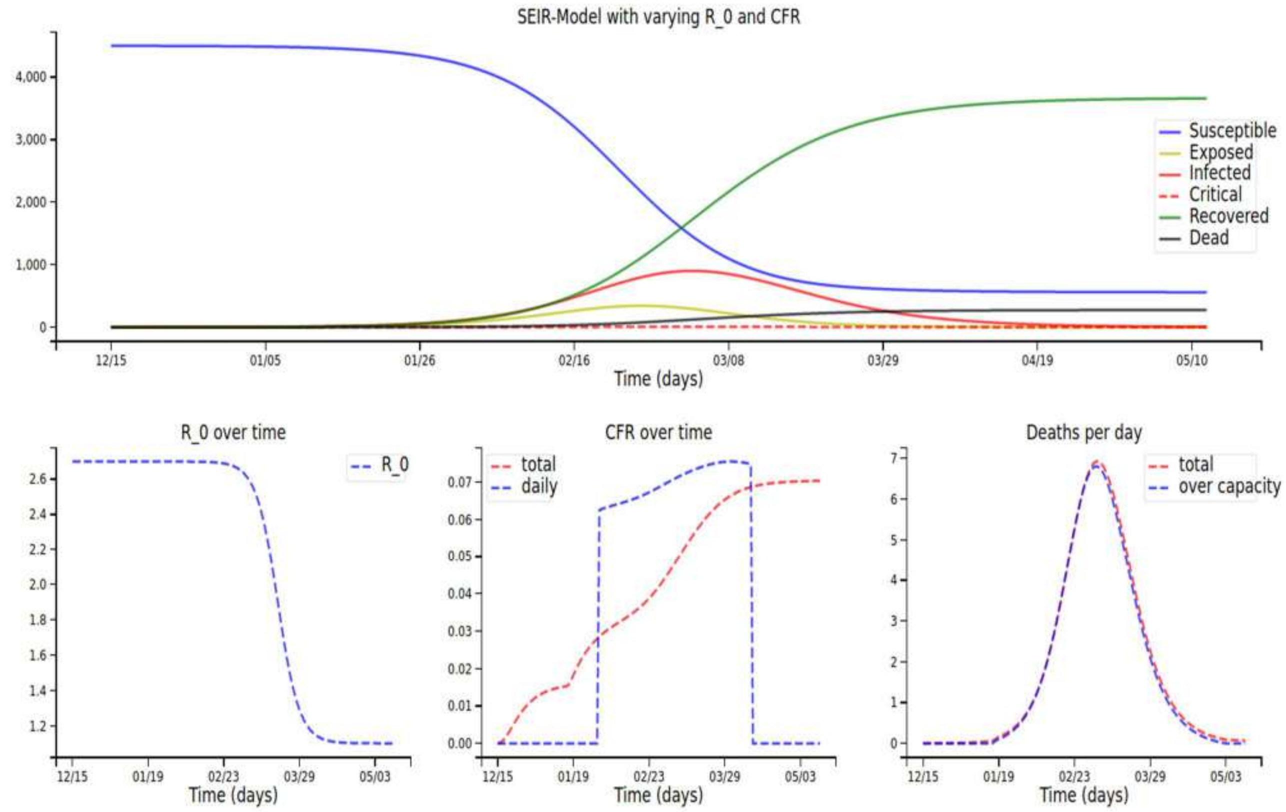
Section – 3

- ## Results:

The rate of increase in the number of infections depends on the product of the number of infected and susceptible individuals. An understanding of the system of Eqs. (1) explains the staggering increase in the infection rate around the world. Infected people traveling around the world has led to the increase in infected numbers and this results in a further increase in the susceptible population. This gives rise to a positive feedback loop leading to a very rapid rise in the number of active infected cases. Thus, during a surge period, the number of susceptible individuals increases and as a result, the number of infected individuals increases as well. For example, as of 1 March, 2020, there were 88,590 infected individuals and by 3 April, 2020, this number had grown to a staggering 1,015,877.

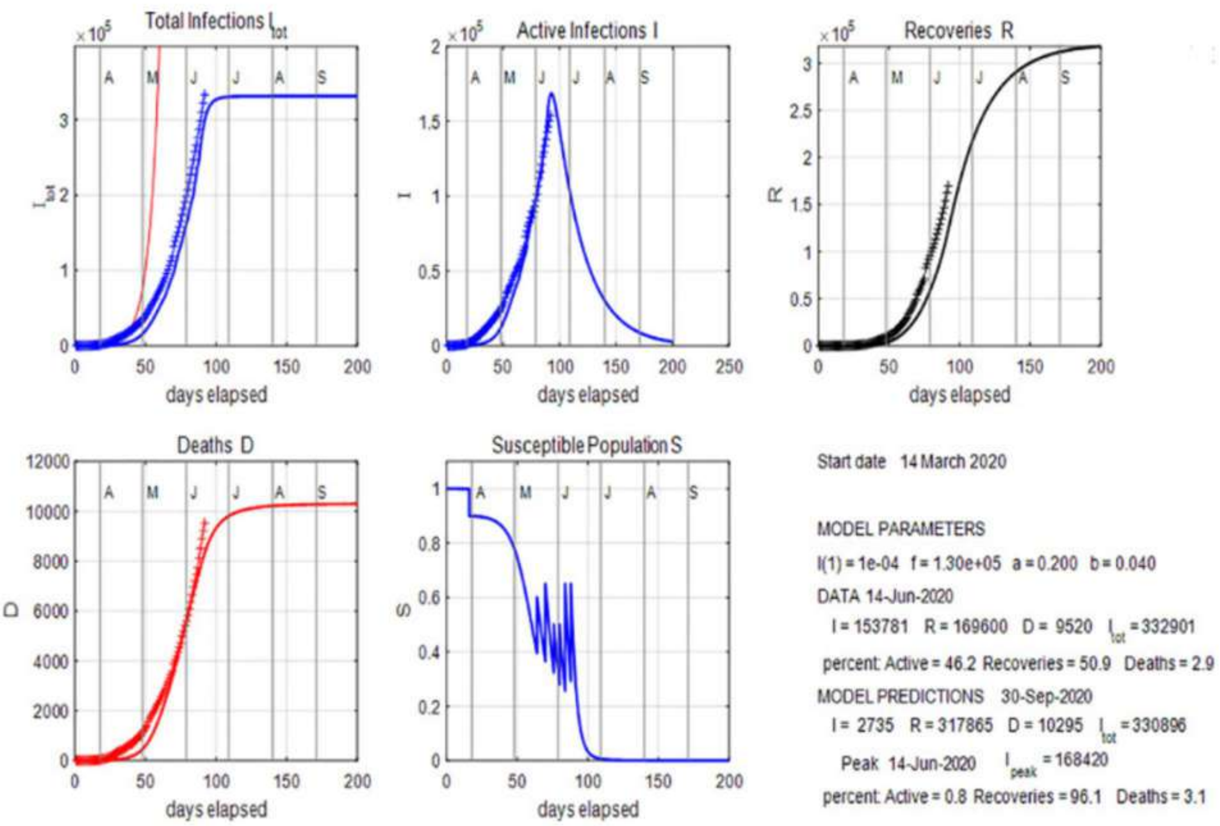
Understanding the implications of what the system tells us, the only conclusion to be drawn using scientific principles is that drastic action needs to be taken as early as possible, while the numbers are still low, before the exponential increase in infections starts kicking in. For example, if we consider the results of policies introduced in the UK to mitigate the spread of the disease, there were 267,240 total infections and 37,460 deaths by 27 May and in the USA, 1,755,803 and 102,107, total infections and deaths, respectively. Thus, even if one starts with low numbers of infected individuals, the number of infections will at first grow slowly and then, increase approximately exponentially, then taper off until a peak is reached. Comparing these results for the UK and USA with those for South Korea, where steps were taken immediately to reduce the susceptible population, there were 11,344 total infections and 269 deaths by 27 May. The number of infections in China reached a peak about 16 February, 2020. The government took extreme actions with closures, confinement, social distancing, and people wearing masks. This type of action produces a decline in the number of infections and susceptible individuals. If the number of susceptible individuals does not decrease, then the number of infections just gets increased rapidly. As at this moment, there is no effective vaccine developed, the only way to reduce the number of infections is to reduce the number of individuals that are susceptible to the disease. Consequently, the rate of infection tends to zero only if the susceptible population goes to zero.

Here, we have applied the SIR model considering data from various countries and the state of Texas in the USA. Assuming the published data are reliable, the SIR model can be applied to assess the spread of the COVID-19 disease and predict the number of infected, removed and recovered populations and deaths in the communities, accommodating at the same time possible surges in the number of susceptible individuals. The time evolution of the cumulative total infections Itot, current infected individuals, I, recovered individuals, R, dead individuals, D, and normalized susceptible populations, S for China, South Korea, India, Australia, USA, Italy and Texas in the USA, respectively. The crosses show the published data and the smooth lines, solutions and predictions from the SIR model. The cumulative total infections plots also show a curve for the initial exponential increase in the number of infections, where the number of infections doubles every five days. The figures also show predictions, and a summary of the SIR model parameters in and published data for easy comparisons.



INDIA

To match the recorded data from India with predictions from the SIR model , it is necessary to include a number of surge periods. This is because the SIR model cannot predict accurately the peak number of infections, if the actual numbers in the infected population have not peaked in time. It is most likely the spread of the virus as of early June, 2020 is not contained and there will be an increasing number of total infections. However, by adding new surge periods, a higher and delayed peak can be predicted and compared with future data. A consequence of the surge periods is that the peak is delayed and higher than if no surge periods were applied. The model predictions for the 30 September, 2020 including the surges are: 330,000 total infections, 700 active infections and 7,500 deaths, whereas if there were no surge periods, there would be 130,000 total infections, 700 active infections and 6,300 deaths, with the peak of 60,000, which is about 40% of the current number of active cases occuring around 20 May 2020. Thus, the model can still give a rough estimate of future infections and deaths, as well as the time it may take for the number of infections to drop to safer levels, at which time restrictions can be eased, even without an accurate prediction in the peak in active infections



Start date 14 March 2020

MODEL PARAMETERS
I(1) = 1e-04 f = 1.30e+05 a = 0.200 b = 0.040
DATA 14-Jun-2020
I = 153781 R = 169600 D = 9520 I_{tot} = 332901
percent: Active = 46.2 Recoveries = 50.9 Deaths = 2.9
MODEL PREDICTIONS 30-Sep-2020
I = 2735 R = 317865 D = 10295 I_{tot} = 330896
Peak 14-Jun-2020 I_{peak} = 168420
percent: Active = 0.8 Recoveries = 96.1 Deaths = 3.1

CONCLUSION:-

Mathematical modelling theories are effective tools to deal with the time evolution and patterns of disease outbreaks. They provide us with useful predictions in the context of the impact of intervention in decreasing the number of infected-susceptible incidence rates .

In this work, we have augmented the classic SIR model with the ability to accommodate surges in the number of susceptible individuals, supplemented by recorded data from China, South Korea, India, Australia, USA, Italy and the state of Texas in the USA to provide insights into the spread of COVID-19 in communities. In all cases, the model predictions could be fitted to the published data reasonably well, with some fits better than others. For China, the actual number of infections fell more rapidly than the model prediction, which is an indication of the success of the measures implemented by the Chinese government. There was a jump in the number of deaths reported in mid-April in China, which results in a less robust estimate of the number of deaths predicted by the SIR model. The susceptible population dropped to zero very quickly in South Korea showing that the government was quick to act in controlling the spread of the virus. As of the beginning of June, 2020, the peak number of infections in India has not yet been reached. Therefore, the model predictions give only minimum estimates of the duration of the pandemic in the country, the total cumulative number of infections and deaths. The case study of the virus in Australia shows the importance of including a surge where the number of susceptible individuals can be increased. This surge can be linked to the arrival of infected individuals from overseas and infected people from the Ruby Princess cruise ship. The data from USA is an interesting example, since there are multiple epicentres of the virus that arise at different times. This makes it more difficult to select appropriate model parameters and surges where the susceptible population is adjusted. The results for Texas show that the model can be applied to communities other than countries. Italy provides an example where there is excellent agreement between the published data and model predictions.

Thus, our SIR model provides a theoretical framework to investigate the spread of the COVID-19 virus within communities. The model can give insights into the time evolution of the spread of the virus that the data alone does not. In this context, it can be applied to communities, given reliable data are available. Its power also lies to the fact that, as new data are added to the model, it is easy to adjust its parameters and provide with best-fit curves between the data and the predictions from the model. It is in this context then, it can provide with estimates of the number of likely deaths in the future and time scales for decline in the number of infections in communities. Our results show that the SIR model is more suitable to predict the epidemic trend due to the spread of the disease as it can accommodate surges and be adjusted to the recorded data. By comparing the published data with predictions, it is possible to predict the success of government interventions. The considered data are taken in between January and June, 2020 that contains the datasets before and during the implementation of strict and control measures. Our analysis also confirms the success and failures in some countries in the control measures taken.

Strict, adequate measures have to be implemented to further prevent and control the spread of COVID-19. Countries around the world have taken steps to decrease the number of infected citizens, such as lock-down measures, awareness programs promoted via media, hand sanitization campaigns, etc. to slow down the transmission of the disease. Additional measures, including early detection approaches and isolation of susceptible individuals to avoid mixing them with no-symptoms and self-quarantine individuals, traffic restrictions, and medical treatment have shown they can help to prevent the increase in the number of infected individuals. Strong lockdown policies can be implemented, in different areas, if possible. In line with this, necessary public health policies have to be implemented in countries with high rates of COVID-19 cases as early as possible to control its spread. The SIR model used here is only a simple one and thus, the predictions that come out might not be accurate enough, something that also depends on the published data and their trustworthiness. However, as the model data show, one thing that is certain is that COVID-19 is not going to go way quickly or easily.

REFERENCES:

1. World Health organization, coronavirus disease (COVID-19) outbreak. https://www.who.int/emergencies/diseases/novel-coronavirus-2019.

2. Wu J.T., Leung K., Leung G.M. Nowcasting and forecasting the potential domestic and international spread of the 2019-ncov outbreak originating in wuhan, china: a modelling study. *The Lancet.* 2020;395(10225):689–697. [PMC free article] [PubMed] [Google Scholar]

3. Novel coronavirus (COVID-19) cases, provided by JHU CSSE. https://github.com/CSSEGISandData/COVID-19.

4. Tang B., Wang X., Li Q., Bragazzi N.L., Tang S., Xiao Y., Wu J. Estimation of the transmission risk of the 2019-ncov and its implication for public health interventions. *Journal of clinical medicine.* 2020;9(2):462. [PMC free article] [PubMed] [Google Scholar]

5. Kraemer M.U., Yang C.H., Gutierrez B., Wu C.H., Klein B., Pigott D.M., Brownstein J.S. The effect of human mobility and control measures on the COVID-19 epidemic in china. *Science.* 2020;368(6490):493–497. [PMC free article] [PubMed] [Google Scholar]

6. https://www.ncbi.nlm.nih.gov/

ANTI-LOCK BRAKING SYSTEM

The project submitted, in partial fulfilment of the requirement for the assignments in ***PHSA CC-XI, PHSA CC-XII,PHSA DSE-I,PHSA DSE-II*** *Paper (Semester 5th) in the Department of Physics*

Submitted by

Ritam Dey

Registration No: A01-1112-111-016-2019

Supervisor Teacher: Prof. Anjan Kumar Chandra

RAMAKRISHNA MISSION VIVEKANANDA CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

INDEX

AIM: IN Today's World, Braking is one of the most important system in an Automobile. For Mechanical Brake, applying brakes on a slippery surface is too hard and can cause the wheels to get lock. When wheels get lock, steering control is lost and in most cases it produces longer Stopping distances.

Anti-lock Braking System **(ABS)** Prevents Wheel Locking and Skidding, No Matter how hard Brakes are applied, or how slippery the road surface. Steering stays under control and Stopping Distances are generally reduced.

In this Project I will try to Briefly Describe (1) Working Components used in ABS, (2) How does ABS Work on a car and (3) Advantages Of ABS

HISTORICAL BACKGROUND*:*

(i) 1929 - ABS was first developed for aircraft by the French automobile and aircraft pioneer Gabriel Voisin, as threshold braking on airplanes is nearly impossible.

(ii) 1985: First ABS installed on US vehicles.

(iii) 2000: 6 of 10 new cars on the road were ABS equipped.

(iv) 2003: 100M Bosch ABS installed

(v) Nowadays:- Almost all new cars have ABS.

Figure 1: Left Car Uses Mechanical Brake and Right Card Uses ABS

From the above picture **(Fig. 1)** Since the **Left car** have not equipped ABS, The Inability to steer Leads to collision with obstacles. But the **Right Car** Can Avoid Collisions with Obstacles using Steering Wheel as it has Equipped ABS

ROLE OF FRICTION IN BRAKE

When a body is rolling without slipping on a rough horizontal surface, the sum of **Translational** and **Rotational** velocities at the contact point should be zero (**Fig. 2**). *(Concept of Rolling)*

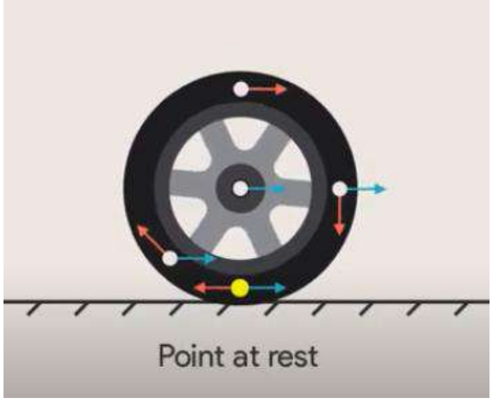


Figure 2: Contact Point of a Wheel While Rolling without Slipping is zero

These are the **Key Points** on how friction comes into action in Brakes

- When the wheels are locked and skidding, braking is by *kinetic friction*
- When the wheels are rotating, braking is by *static friction*
- The coefficient of static friction is greater than that of kinetic friction
- The larger the coefficient of friction, the greater the frictional force from braking. A larger frictional force means the car stops in less time and less distance.
- Both coefficients of friction for rubber and concrete are reduced when it is raining and the car is wet. This means that in the rain, the driving will be more slippery and it will take longer to stop.
- Since driving in the rain is more slippery, you would like the largest frictional force possible to minimize your braking distance and maximize your ability to keep the car under control.
- Anti-lock brakes prevent the wheels from locking up and skidding so that the braking force is from static friction instead of kinetic friction

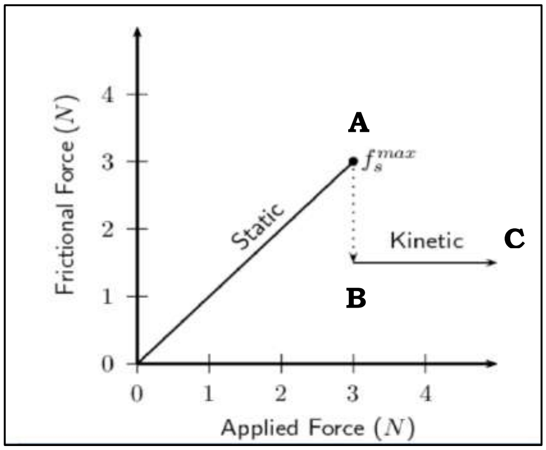


Figure 3: Graph of Friction Force and Applied Force

A Graph between the Applied Force and force of Friction is shown in the above figure (**Fig. 3).** Point A represents the Limiting Friction. Beyond A the force of friction is seen to decrease slightly. The Portion BC Represents Kinetic Friction Which is Constant. And it is Also Clear from the Graph That Kinetic Friction Is Always Smaller than Static Friction.

OVERVIEW OF COMPONENTS IN ABS

Typical ABS components include: There are four main components of ABS as mentioned in Fig. 4.

COMPONENTS

- **wheel speed sensors (up to 4)**
- **brake master cylinder**
- **Modulator unit with pump and valves**
- **Electronic control unit (ECU)**

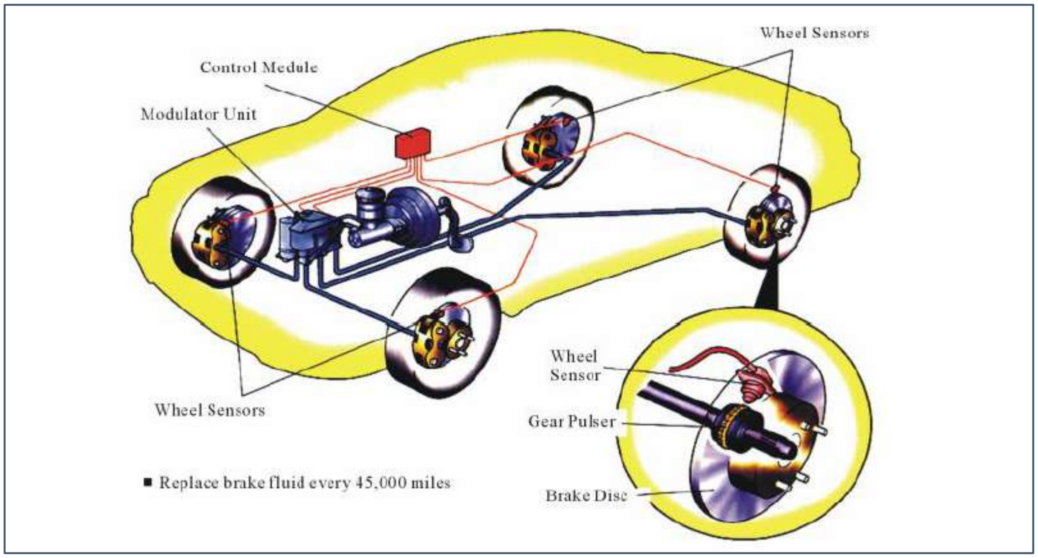


Figure 4: Components of Typical ABS system

Wheel Speed Sensor (WSS)

Teeth on the sensor ring rotate past the magnetic sensor, causing a reversal of the magnetic field polarity, resulting in a signal with frequency related to the angular velocity of the axle. Some components of WSS are shown in **Fig. 5**

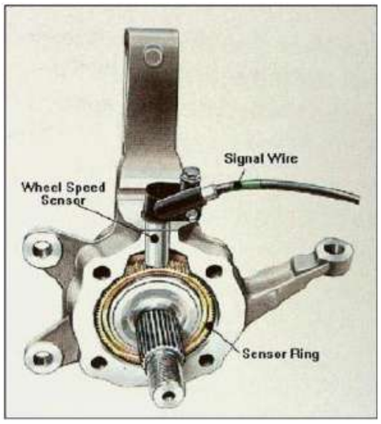


Figure 5: Picture Of WSS

Electronic Control Unit

A controller is a unit in the ABS which receives the information from the individual wheel ABS speed sensor. When an individual wheel loses the traction, a signal is sent to the controller. The controller will then limit the brake force and activate the ABS modulator (**Fig. 6**).

Figure 6: Picture Of WSS

Hydraulic Modulator Unit

The hydraulic modulator unit contains the ABS pump as well as solenoid valves for each brake line. The fifth line - far right - is from the brake master cylinder, which is connected to the brake pedal (**Fig.7**).

Figure 7: Picture of Hydraulic Modular Unit

Valves and Brakes

The valves modulate the brake pressure up to 20 times per second, effectively realizing the ideal tire slip percentage. ABS 'pumps' the brakes much faster than any driver could (**Fig. 8**).

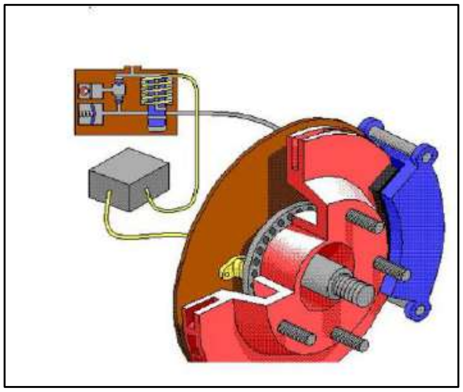


Figure 8: Picture of Valves and Brakes

How does ABS WORK?

Basically, there are sensors at each of the four wheels (or in the case of the less sophisticated three-channel system, one on each of the fronts and only one for the pair of rears). These sensors **watch the rotation of the wheels**. When any one of the wheels stops rotating due to too much brake application, the sensors **indicate the car's computer**, which then releases some of the brake line pressure that you've applied - allowing the wheel to turn again. Then, just as fast as it released the **pressure**, the computer allows the pressure to be applied again - which stops the rotation of the wheel again. Then it releases it again. And so on. With most ABS, this releasing and re-application - or pulsing - of the brake pressure happens 20 or more times per second. Practically speaking, this keeps the wheel just at the limit - the threshold - before locking up and skidding. ABS prevents you from ever locking up the brakes and skidding - no matter how hard you apply the brakes. Obviously, this is going to mean much more steering control (Fig. 9).

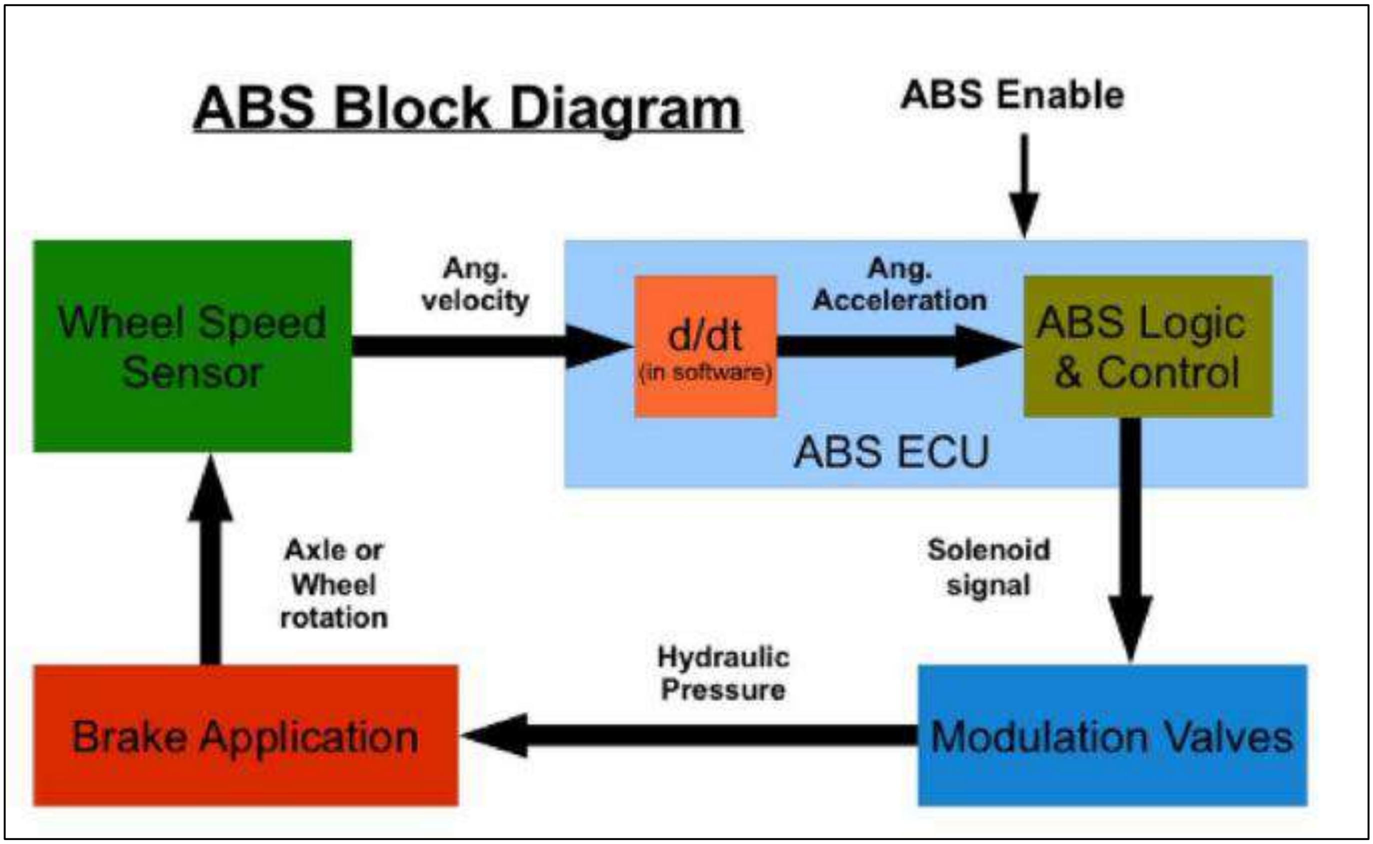


Figure 9: Block Diagram of Step Wise Working Principle in ABS System.

Limitations

Three points should be obvious, but don't appear to when looking at the type of crashes some drivers have with ABS-equipped vehicles. Contrary to popular belief, ABS:-does not allow you to drive faster; does not allow you to brake later; and does not allow you to corner faster.

Conclusion

We have to understand that ABS does not raise the traction limits of your vehicle. It only helps to stay within the limits Period. So, do not get caught up in believing that ABS will allow you to drive any faster, brake later or go around a corner any faster - or allow you to safely follow any closer.

It is clear from **Fig. 10** that **under hard braking**, an ideal braking system should provide the shortest stopping distances on all surfaces maintain vehicle stability and steer ability Anti-lock braking systems were developed to best meet these needs.

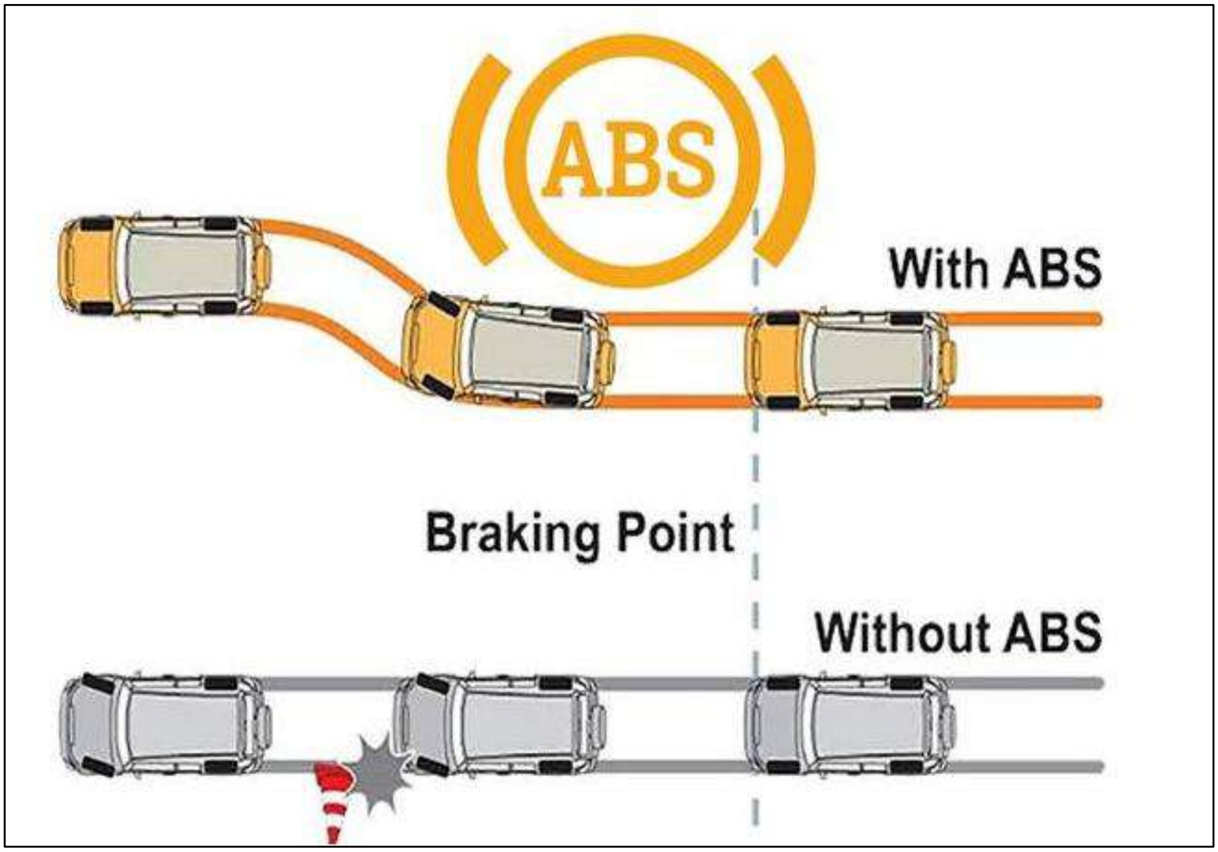


Figure 10: Upper One Represent Using ABS .Lower One Represent without ABS

Acknowledgement

The success and final outcome of this project required a lot of guidance and assistance from many people and I am extremely privileged to have got this all along the completion of my project. All that I have done is only due to such supervision and assistance and I would not forget to thank them.

I respect and thank our Principal, **Swami Kamalasthananda** and Vice Principal, **Swami Vedanuragananda** for providing me an opportunity to do the project work. I am extremely thankful to the Head of the Department of Physics, Prof**. Asok Kumar Pal** for providing such a nice support and guidance although he had a very busy schedule.

I owe my deep gratitude to our project guide Prof. **Anjan Kumar Chandra**, who took keen interest on my project work and guided me all along, till the completion of the project work by providing all the necessary information for developing a good system.

I heartily thank all my friends and classmates for their encouragement and more over for their timely support and guidance till the completion of my project work.

I am thankful to and fortunate enough to get constant encouragement, support and guidance from all teaching staffs of RKMVCC Physics Department which helped me in successful completion of my project work.

Ritam Dey, Physics Dept.

Sources

1) https://en.wikipedia.org/wiki/Anti-lock_braking_system
2) https://www.automotive-technology.com/articles/anti-lock-braking-system-main-components-and-advantages
3) https://www.youtube.com/watch?v=98DXe3uKwfc
4) https://auto.howstuffworks.com/auto-parts/brakes/brake-types/anti-lock-brake.htm

X

Stellar Evolution

The project submitted, in partial fulfilment of the requirement for the assignments in CC-11, CC-12, DSE-1, DSE-2 Papers (Semester V) in the Department of Physics

Submitted by
Snehasish Paul

Registration No:
A01-1112-111-018-2019

Supervisor Teacher:
Dr. Palash Nath

RAMAKRISHNA MISSION VIVEKANANDA CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

Table Of Contents

Introduction

This is an assignment on stellar evolution, in the following text we shall see the journey of a star from its birth till its death. We will see how stars are formed, how they continue to exist, what makes them tick and how they cease to exist.

Classification of stars

The generally accepted system of stellar classification is a combination of two classification schemes, the Harvard system, which classifies stars according to their surface temperature, and the MK system, which is a classification based on a star's luminosity.

The Harvard system:
In this scheme the stars types are classified by letters according to their surface temperature. The order is: O, B, A, F, G, K, M according to decreasing order of surface temperature. Additionally, numbers from 0 to 9 are used to subdivide each type, with higher numbers designating cooler stars. Additional letters have been used to classify novas and other less common types of stars. After the discovery of brown dwarf stars, the classification has been expanded to include types L, T, and Y.

- **Class O** stars are bluish white in colour with surface temperature generally in the range of 25,000-50,000 K (there are a few O-type stars with vastly greater temperature), their spectra contains lines of ionized helium.
- **Class B** stars are also bluish white in colour but have temperatures ranging from 10,000 K to 25,000 K, these show neutral helium lines.
- **Class A** stars are white in colour with surface temperature in range of 7,400 K to 10,000 K, the hydrogen line is prominent in these.
- **Class F** stars are yellow white, with temperatures 6,000-7,400 K and display many spectral lines of metals.
- **Class G** stars are yellow with temperatures 5,000-6,000 K. The sun is a G-type star.
- **Class K** stars are yellow to orange in colour with surface temperature in range of 3,500 K to 5,000 K.
- **Class M** stars are red with temperature of about 3,000 K, these show prominent lines of titanium oxide in their spectra.
- **Class L** are brown dwarf stars with temperatures 1,500 K to 2,500 K and show spectral lines due to alkali metals such as rubidium and sodium and metallic compounds like iron hydride.
- **Class T** brown dwarfs have temperatures between 800 K to 1,500 K and show distinct methane absorption in their spectra.
- **Class Y** brown dwarfs are cooler than 800 K and have spectral lines resulting from ammonia and water.

The MK system:
This method of classification is based on two sets of parameters, a refined version of the Harvard O-M scale, and a luminosity scale of grades I (for supergiants), II (bright giants), III (normal giants), IV (subgiants), and V (main sequence, or dwarf, stars); further specifications may be used, such as a grade Ia for bright supergiants and grades VI and VII for subdwarfs and white dwarfs, respectively. One can determine the luminosity Ls of a star if the amount of energy reaching Earth, as given by the apparent brightness of the star, and its distance are measured. Also, the temperature of the stellar atmosphere can be estimated from the distribution of energy in the spectrum of the star; it is often specified in terms of the effective temperature T_{eff}, defined such that

$$L_{eff}=4\pi\sigma T_{eff}^{4}R^{2}$$

where σ is the Stefan- Boltzmann constant.

Hertzsprung-Russell diagram

A diagram plotting the photographic magnitude and colour indices or spectral types. This was first done by the Danish astronomer E. Hertzsprung and American astronomer H. N. Russell. The term colour-magnitude diagram is also used alternatively.

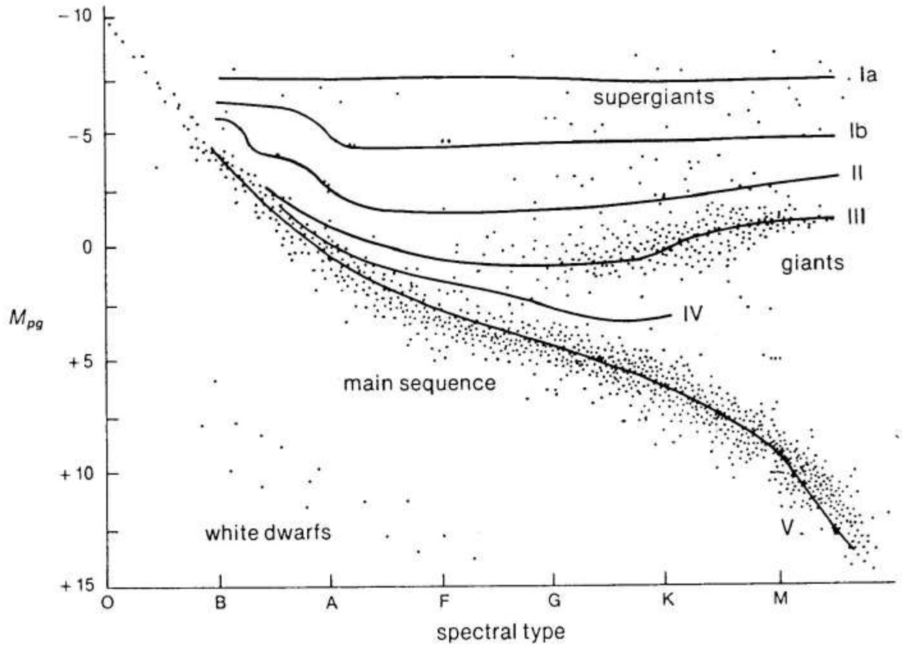


Fig 1:Hertzsprung-Russell diagram for stars in the solar vicinity, showing absolute photographic magnitude M_{pg}*(based on measurements on photographic plates)versus spectral type. Solid lines indicate the approximate definition of the luminosity classes.*

As we can see from the diagram, most of the stars are concentrated in the main sequence, with luminosity steeply increasing with increasing temperature. These stars correspond to luminosity class V. In addition there is a number of stars concentrated in the giant branch, of luminosity class III, and far fewer stars with even higher luminosities. Below the main sequence there is a small number of very faint and relatively hot stars, the white dwarfs

Birth of stars

Stars are born in molecular gaseous clouds. These gaseous structures majorly consists of hydrogen and dust grains with masses10^5 to 10^6solar mass. They are of the order of 150 light years in length. These molecular clouds have very low temperature (10-30 K). the stars start to form when a dense region inside the cloud starts to clump together due to gravitation pull. These lump of clouds are called protostars starts to heat up as it contracts, also since the individual molecules forming the protostar have some resultant angular momentum, the accumulated cloud begins to spin, and as it contracts it spins faster. If a protostar spins too fast it might fragment into a binary star
At this stage the random up and down motion of gaseous particle die down due to collision against each other, essentially flattening up the cloud around the protostars into a disk like shape. Often hot gases or jets are driven from the protostars.

The gravity causes the centre of the gas cloud to collapse into a dense core, which greatly heats up the core.

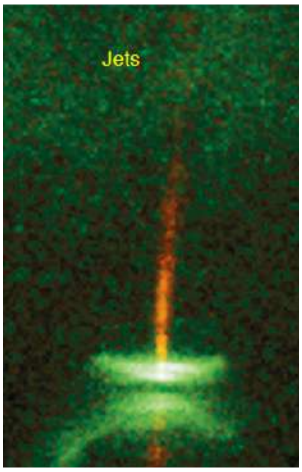


Fig 2: jets coming out of protostars

Source of energy

When the core reaches a certain temperature nuclear fusion starts which resists the protostar from shrinking. This contraction ends when the energy released due to hydrogen fusion balances out the energy radiated from the surface

In this initial fusion phase the star fuses hydrogen to form helium atoms, this is called the main sequence. It takes 50 million years for stars like Sun to reach this stage since its birth. Almost 90% of the lifetime of a star is spent in this state. In this phase energy is generated stably.

$$4\,{}^{1}H \rightarrow {}^{4}He+2\,e^{+1}+2\,v_{e}$$

For stars like the Sun, once the hydrogen starts depleting from the core, an inert core of helium is formed with hydrogen shell surrounding the core. The inert core doesn't reach enough temperature yet to start undergoing fusion, and the fusion occurs is the hydrogen shell. This produces more energy than the main sequence phase. In this state more helium is dumped into the inert core, which due to higher mass of helium contracts further, and the hydrogen burning accelerates with time. During this period the outer layers of the sun expands up to 100 times the previous size and is known as a red giant.

$$3\,{}^{4}He \rightarrow {}^{12}C$$

$${}^{4}He+{}^{12}C \rightarrow {}^{16}O$$

Eventually after the red giant stage the core becomes hot enough to fuse helium in its core to form carbon and further oxygen, this phase is marked with steady energy generation. In this phase the star becomes smaller and hotter

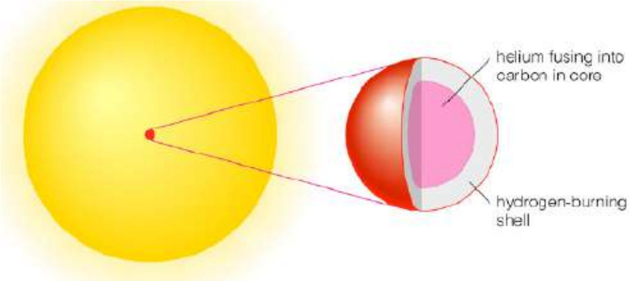
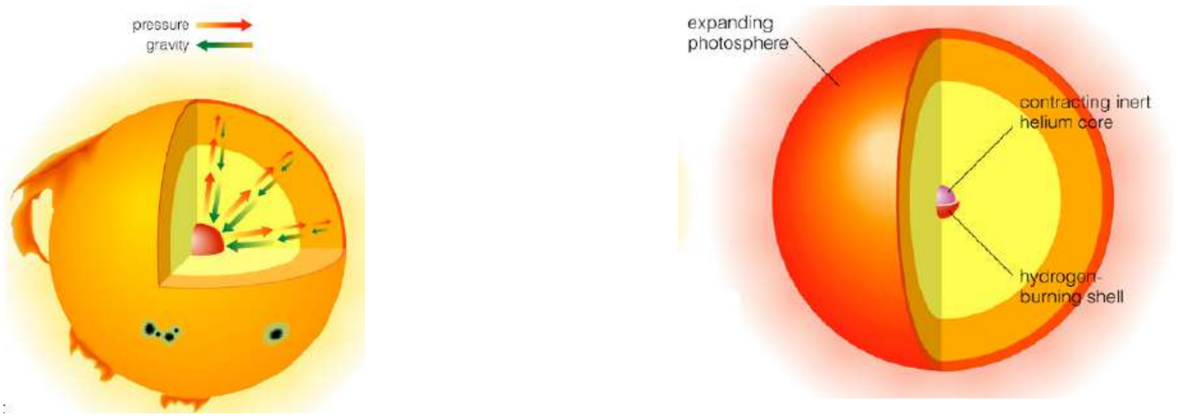


Fig 3: hydrogen burning shell

Dynamical equilibrium

Inside a star there are two forces at play, the outward force due to nuclear fusion and the inward force due to gravitational attraction. For a stable star to form these two forces must balance each other. If the nuclear force becomes greater than the gravitation then the star will shed off its mass, and if the gravitation dominates the nuclear force the star might collapse upon itself.
If the mass of the sun is greater than 0.08 time the mass of the sun, then the energy generated by fusion provides thermal pressure to stop the collapse, however if the mass of the star is less than 0.08 solar mass then it is the degeneracy pressure which stops the gravitational collapse, this prevents the core from being hot enough to initiate hydrogen fusion and the star becomes a brown dwarf.
After a certain time the hydrogen in the core is used up and the star is left with an inert helium core. The core shrinks in size along with its inner layers. This results in the hydrogen shell surrounding the core to get hot enough to begin fusion. This H-burning shell produces more energy than main sequence phase, as a result the star expands. This is called the red giant phase. H-burning shell produces more helium which is accumulated in the inert core, thus the core contracts further due to added mass, increasing the pressure. This phenomenon accelerates with time. The star grows massively in size during this. During this stage and after the outer surface of the star is not very tightly bound and is prone to loss due to stellar winds.
Eventually after the red giant phase the core becomes hot enough to initiate helium fusion and the star starts to fuse helium to form carbon with a surrounding hydrogen burning shell. In this phase the energy generation is steady. The star becomes smaller and hotter. Afterwards we can see a repetition of previous phenomenon as an inert core of carbon forms surrounded by shells of helium and hydrogen.



(a) (b)

Fig 4(a): equilibrium between pressure and gravity. Fig 4(b): expansion of photosphere

Death and decay

Low mass stars such as our sun never reach the temperature required for fusion beyond carbon to take place and no additional heat and gas pressure is generated. Thus the core begins to collapse again. This shrinking is stopped by electron degeneracy pressure before it gets hot enough to fuse carbon and oxygen into magnesium and silicon. The outer layer of the star begins to fall apart, producing a planetary nebula. The planetary nebula's material slowly begins to drift away, leaving a dense hot core of the star. This leftover remains of the star, entirely supported by electron degeneracy pressure is called a white dwarf star. Newly formed, it is quite hot and looks white oar blue, however with time it cools down enough and becomes red.
For high mass stars, whose mass are in the range $2-3\,M_{sun}$the nuclear reactions proceed until the core becomes mainly iron, beyond this the density becomes so high that the neutron degeneracy pressure comes into play and stops further shrinking. The neutrons star die in a truly magnificent way. The gravitational forces crushes its core, releasing extreme gravitational energy which leads to the explosion of the star as a supernova. If the iron core of the collapsing star has mass larger than $3\,M_{sun}$it becomes a black hole.

(a) (b)

Fig 5(a): planetary nebula. Fig 5(b): supernova explosion

Conclusion

We see that the journey of a star from its birth to death is a marvellous phenomenon of nature. This journey of a star is not very much unlike that of a living being, it starts its life, matures and dies, albeit in a time scale which is humongous when compared to that of any living thing we know of. Studying such matter one cannot help but come face to face with the realisation that compared to these how small and insignificant our lives are. Yet, as grand as these cosmic entities be, they too do not exist forever. So in a sense these grand objects are as mortal as we are, only that they exist for a timescale which is much larger than ours. However, the inevitability of death harbours the promise of creation. When large star dies in a supernova explosion or a small star sheds its planetary nebula, it spreads throughout the space the materials of creation, without which the universe would remain barren. These materials form other cosmic objects such as planets and other stars. This grand dance of cosmic birth and death enables beings from the third planet of a medium sized star to direct its gaze towards the sky and marvel at the cosmic objects, because without this dance, neither of these would have existed.

Acknowledgement

I would like to thank Dr. Palash Nath for guiding me throughout this assignment. I am thankful to Dr. Asok kumar Pal, Head of the department of physics. I am also indebted to Swami Kamalasthananda, Principal of Ramakrishna Mission Vivekananda Centenary College, Rahara.

References

Figure 1: Lecture notes on Stellar Structure and Evolution, Jørgen Christensen-Dalsgaard (Institut for Fysik og Astronomi, Aarhus Universitet)
Figure 2, 3, 4(a), 4(b): The life cycle of stars, Shadia Habbal (Institute for Astronomy, University of Hawaii)
Figure 5(a): Hubble telescope, NASA
Figure 5(b): supernova 1987 A, Anglo-Australian observatory

Lecture notes on Stellar Structure and Evolution, Jørgen Christensen-Dalsgaard (Institut for Fysik og Astronomi, Aarhus Universitet)
The life cycle of stars, Shadia Habbal (Institute for Astronomy, University of Hawaii)
Stellar Evolution: Birth, Life and Death of Stars, John R. Percy(International Astronomical Union, University of Toronto, Canada)
Astronomy a beginner’s guide to the universe, Chaisson, Mcmillan
Encyclopedia Britannica

Steller Evolution

The project submitted, in partial fulfilment of the requirement for the assignments in ***PHSA CC-XI,PHSA CC-XII,PHSA DSE-I,PHSA DSE-II****. Paper (Semester 5th) in the Department of Physics*

Submitted by

Anubhab Jana

Registration No: A01-1152-111-019-2019

Supervisor Teacher: Prof.Sankhasubhra Nag

STELLAR EVOLUTION

Abstract:

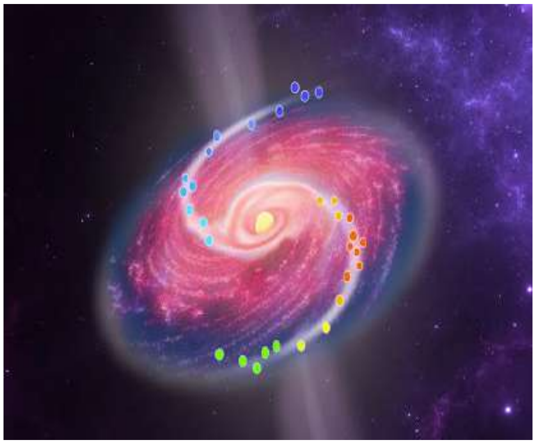
This contribution is meant as a first brief introduction to stellar physics. First I shortly describe the main physical processes active in stellar structures then I summarize the most important features during the stellar life-cycle

Stellar evolution is a description of the way that **stars** change with time. On human timescales, most stars do not appear to change at all, but if we were to look for billions of years, we would see how stars are born, how they age, and finally how they die.

The primary factor determining how a **star** evolves is its **mass** as it reaches the main sequence. The following is a brief outline tracing the evolution of a low-mass and a high-mass star.

The life of a star

- Stars are born out of the gravitational collapse of cool, dense **molecular clouds**. As the cloud collapses, it fragments into smaller regions, which themselves contract to form stellar cores. These protostars rotate faster and increase in temperature as they condense, and are surrounded by a protoplanetary disk out of which **planets** may later form.
- The central temperature of the contracting **protostar** increases to the point where nuclear reactions begin. At this point, **hydrogen** is converted into **helium** in the core and the star is born onto the main sequence. For about 90% of its life, the star will continue to burn hydrogen into helium and will remain a main sequence star.



.

Fig: **Protostar**

Source: physics-astronomy@uiowa.edu

- Once the hydrogen in the core has all been burned to helium, energy generation stops and the core begins to contract. This raises the internal temperature of the star and ignites a shell of hydrogen burning around the inert core. Meanwhile, the helium core continues to contract and increase in temperature, which leads to an increased energy generation rate in the hydrogen shell. This causes the star to expand enormously and increase in **luminosity** – the star becomes a **red giant**.

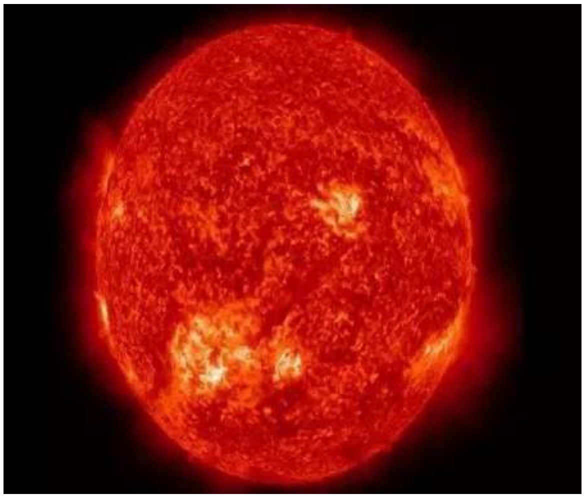


Fig: Red Giant

Source: 5. https://hubblesite.org/

- Eventually, the core reaches temperatures high enough to burn helium into carbon. If the mass of the star is less than about 2.2 **solar masses**, the entire core ignites suddenly in a helium core flash. If the star is more massive than this, the ignition of the core is more gentle. At the same time, the star continues to burn hydrogen in a shell around the core.

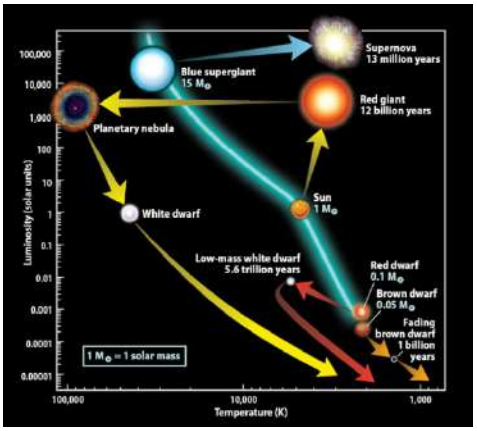


Fig: Luminosity vs Temperature

Source: http://www.esa.int/

- The star burns helium into carbon in its core for a much shorter time than it burned hydrogen. Once the helium has all been converted, the inert carbon core begins to contract and increase in temperature. This ignites a helium burning shell just above the core, which in turn is surrounded by a hydrogen burning shell.

What happens next depends on the mass of the star

  - The inert carbon core continues to contract but never reaches temperatures sufficient to initiate carbon burning. However, the existence of two burning **shells** leads to a thermally unstable situation in which hydrogen and helium burning occur out of phase with each other. This thermal pulsing is characteristic of asymptotic giant branch stars.

- The carbon core continues to contract until it is supported by **electron degeneracy pressure**. No further contraction is possible (the core is now supported by the pressure of **electrons**, not gas pressure), and the core has formed a **white dwarf**.

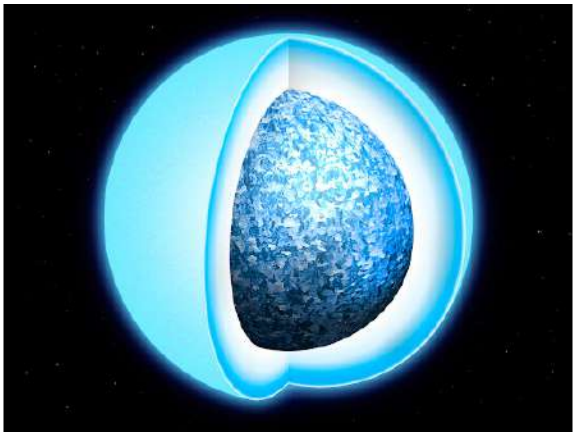


Fig:White Dwarf

Source: . https://hubblesite.org/

- Meanwhile, each thermal pulse causes the outer layers of the star to expand, resulting in a **period** of mass loss. Eventually, the outer layers of the star are ejected completely and ionised by the white dwarf to form a planetary **nebula**.

Fig: Planetary nebula

Source: https://en.wikipedia.org/wiki/Radio-quiet_neutron_star

- The contracting core will reach the temperature for carbon ignition, and begin to burn to neon. This process of core burning followed by core contraction and shell burning, is repeated in a series of nuclear reactions producing successively heavier elements until iron is formed in the core.

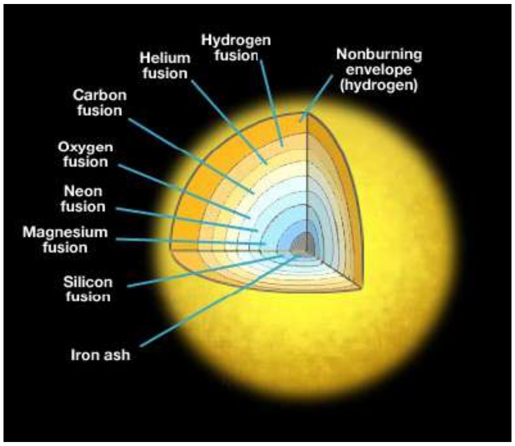


Fig: Core of a star

Source: https://en.wikipedia.org/wiki/Radio-quiet_neutron_star

- Iron cannot be burned to heavier elements as this reaction does not generate energy – it requires an input of energy to proceed. The star has therefore finally run out of fuel and collapses under its own **gravity**.
- The mass of the core of the star dictates what happens next. If the core has a mass less than about 3 times that of our **Sun**, the **collapse of the core** may be halted by the pressure of neutrons (this is an even more extreme state than the electron pressure that supports **white dwarfs**!). In this case, the core becomes a **neutron star**.

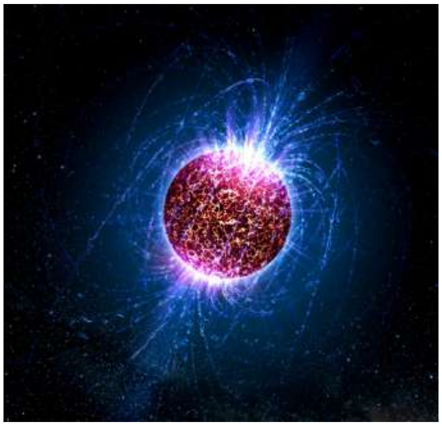


Fig: Neutron Star

Source: https://hubblesite.org/

- The sudden halt in the contraction of the core produces a shock wave which propagates back out through the outer layers of the star, blowing it apart in a core-collapse **supernova** explosion.

- If the core has a mass greater than about 3 solar masses, even neutron pressure is not sufficient to withstand gravity, and it will collapse further into a **stellar black hole**.

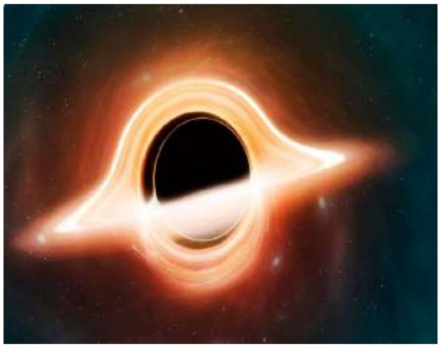
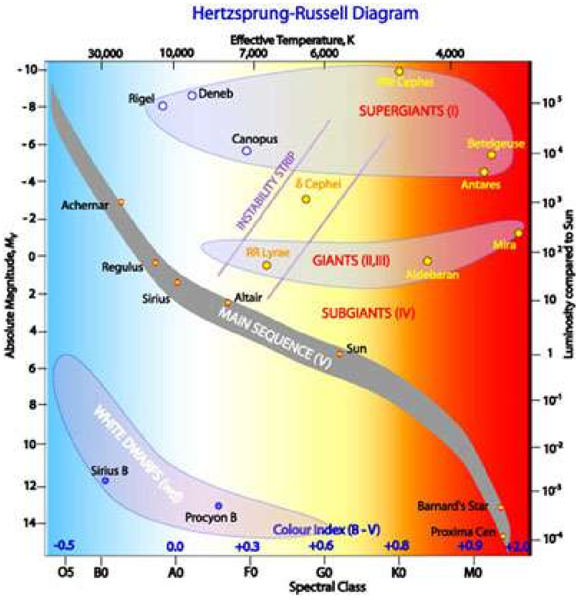


Fig: Black Hole

Source: : physics-astronomy@uiowa.edu

- The ejected gas expands into the interstellar medium, enriching it with all the elements synthesised during the star's lifetime and in the explosion itself. These supernova remnants are the chemical distribution centres of the **Universe**.

An important tool in the study of stellar evolution is the **Hertzsprung-Russell diagram** (**HR diagram**), which plots the absolute magnitudes of stars against their **spectral type** (or alternatively, stellar luminosity versus **effective temperature**).



Source: https://astronomy.com/

As a star evolves, it moves to specific regions in the HR diagram, following a characteristic path that depends on the star's mass and **chemical composition**.

Acknowledgment:

I am really really grateful to my supervisor **Dr. Sankhasubhra Nag** for advising me and introducing the project to me in a easy to understand the phenomenon which has helped me to complete my project easily and effectively on time.

.

Conclusion:

One of the key concepts in astronomy is that stars change over time -- they're born from clouds of interstellar gas and dust, they shine by their own light created through nuclear fusion of hydrogen in their cores, and eventually they run out of fuel and die, returning some of their mass back to interstellar space. Their remains can then be taken up into new generations of stars, starting the process over again. The process of change that a star undergoes during its lifetime is called stellar evolution. But this process can take millions or billions of years for a star, much longer than we can hope to observe directly.

Reference:

1.Astrophysics for Physicists , Cambridge University.

2.Astrosurf, ResearchGate, The Virtual Telescope Project.

3.Wikipedia, Hubble Heritage Project.

Title of the Project

Diamagnetic behavior of solids with reference to Len’s law of electromagnetic induction

The project submitted, in partial fulfilment of the requirement for the assignments in {CC-XI,CC-XII,DSE-I &DSE-II} Paper (Semester V) in the Department of Physics

Submitted by

Proyas Dutta

Registration No: A01-1112-111-020-2019

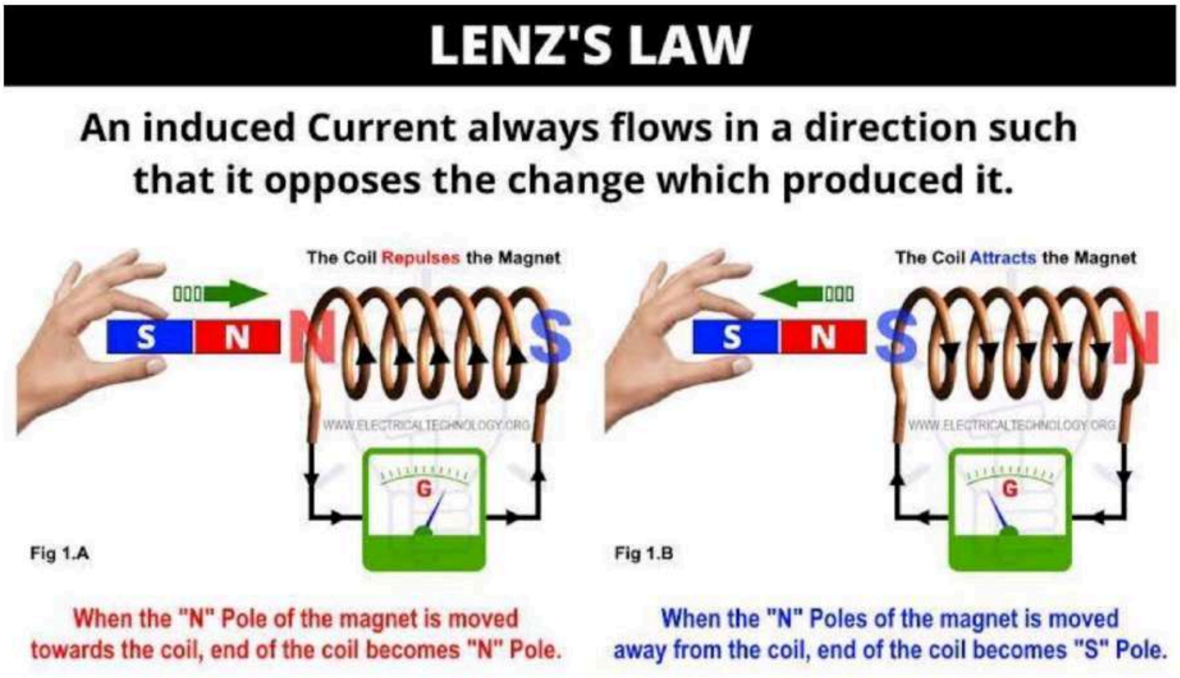
Supervisor Teacher: Dr. Asok Kumar Pal

RAMAKRISHNA MISSION VIVEKANANDA CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

AIM OF THE PROJRCT:

In this project I want to discuss the diamagnetic behavior of a solid with reference to Lenz law.By this project we give a brief description of this topic and developed my skill to do something on my own.I try to cover all the theoretical explanation of this topic that will grow my skills to write something.i go through several books to gather a idea about this topic and I illustrate my thinking in the next pages.i hope ith will be helpful for other readers.

Lenz's Law: This law stating that the direction of an induced current is always such as to oppose the change in the circuit or the magnetic field that produces it.



LENZ'S LAW

An induced Current always flows in a direction such that it opposes the change which produced it.

When the "N" Pole of the magnet is moved towards the coil, end of the coil becomes "N" Pole.

When the "N" Poles of the magnet is moved away from the coil, end of the coil becomes "S" Pole.

This law was deduced in 1834 by the Russian physicist Henrich Friedrich Emil Lenz. Lenz's law the general principle of the conservation of energy.

$$\epsilon = -N\frac{\partial \Phi_B}{\partial t}$$

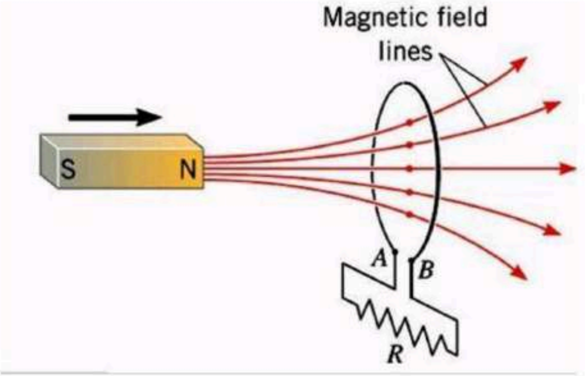
ϵ = induced emf
N = number of turns in coil
$\partial \Phi_B$ = change in magnetic flux
∂t = change in time

Fig: Henrich Friedrich Emil Lenz

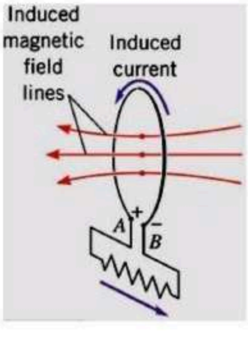
Lenz's Law & Law of conservation of energy: According to lenz's law,the induced emf opposes the change that produces of.It is the opposition against which we perform mechanical energy work in causing the charge in magnetive flux.Therefore mechanical energy is converted into electrical energy.Thus lenz's law in accordance with law of conversion of energy. If however the reverse would happen(i.e. the reduced emf does not oppose or aids the change),then a little charge in magnet flux would produce an induced current which would help the change of flux further thereby producing more current.The increased emf would then cause further change of flux and it would further increase the current and so on these moncrute energy out of nothing which would violate thelavel of conservation of energy.

a) Conservation of energy

Lenz's law is statement of energy conservation



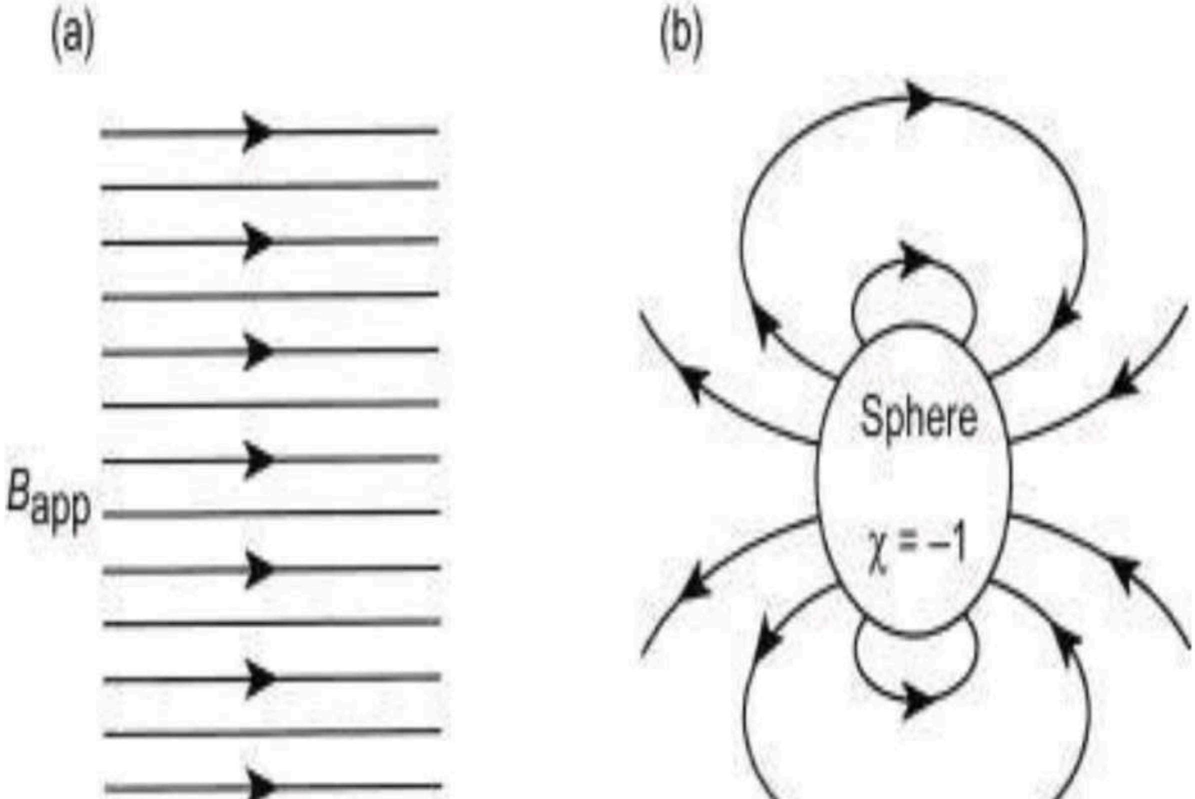
Flux increases



Induced field repels magnet; work required to produce current

EXPLANATION OF DIAMAGNETISM BY LENZ'S LAW

Diamagnetism of a substance has it’s origin in the change of the orbital motionof the atoms electrons due to the applied magnetic field .Due to this precession of the electronic orbit in the applied field ,there is an induced magnetic moment opposite to the direction of B jiust like lenz’s law.Extra electronic motion due to change of the motion of electronic orbit produce a constituent a current in such a degresing that opposes the field.Sosuccectable X is always less than zero.



DIAMAGNETISM

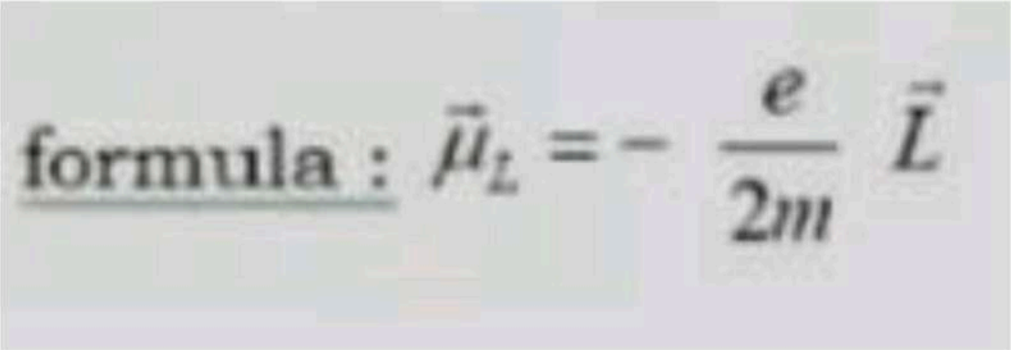
Diamagnetism was discovered and named in September 1845 by Michael Faraday. This is the weakest form of magnetism that is displayed in the presence of an external magnetic field. The orbital motion of electrons changes due to the external magnetic field. This induced magnetic moment is extremely small and in a direction opposite to that of the applied field. When placed within a strong electromagnet, diamagnetic materials are attracted toward regions where the magnetic field is weak (162).

Diamagnetism is found in all materials; however, because it is so weak it can only be observed in materials that do not exhibit other forms of magnetism. Diamagnetic substances are composed of atoms that have no net magnetic moments (i.e., all the orbital shells are filled and there are no unpaired electrons). However, when subjected to a field, a negative magnetization is produced and thus the susceptibility is negative (2).

An exception to the ‘weak’ nature of diamagnetism occurs when a number of materials become superconducting. Superconductors are ideal diamagnets; when positioned in an external magnetic field, they expel the field lines from their interiors (depending on field intensity and temperature). Superconducting magnets are the foremost elements of most magnetic resonance imaging (MRI) systems and are among the most important applications of diamagnetism. Bismuth, which is used in guns, displays the strongest diamagnetism. Bismuth can be melted down and molded to efficiently capture any diamagnetic properties. Pyrolytic graphite is also an unusually strongly diamagnetic material that can be stably floated on a magnetic field

DESCRIPTION OF DIAMAGNETISM

Magnetic moment of an electron due to its orbital motion is related to its orbital angular momentum by larmor's formula.



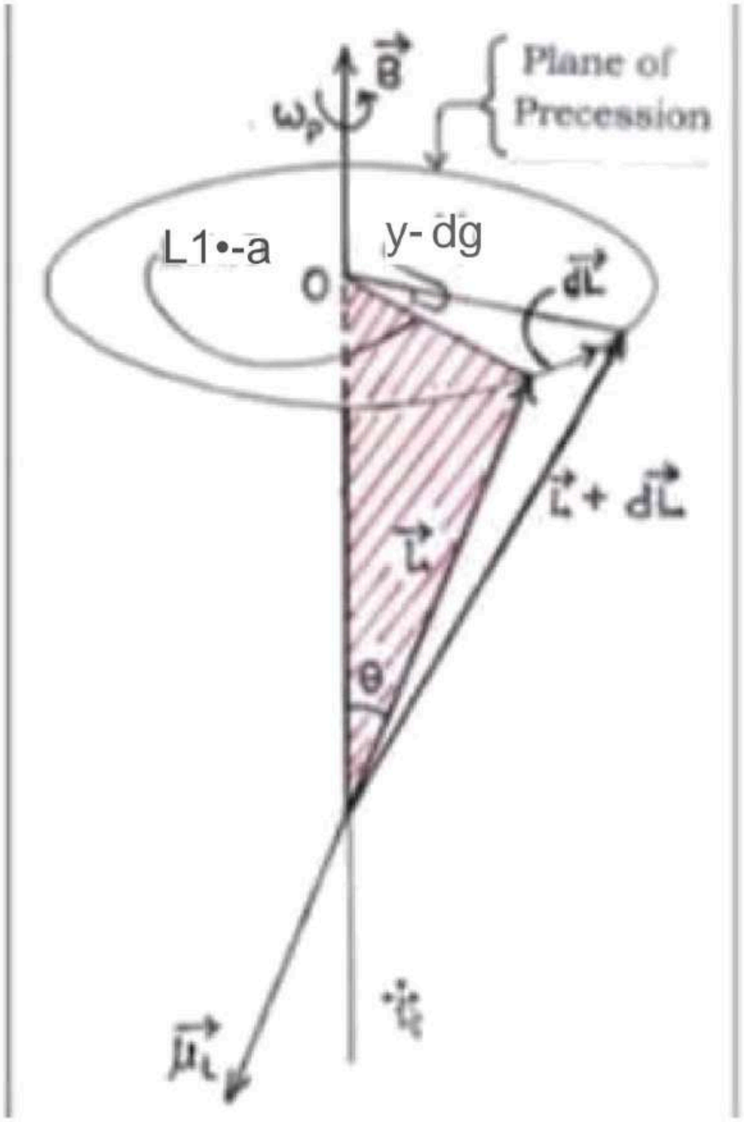
formula : $\vec{\mu}_L = -\frac{e}{2m}\vec{L}$

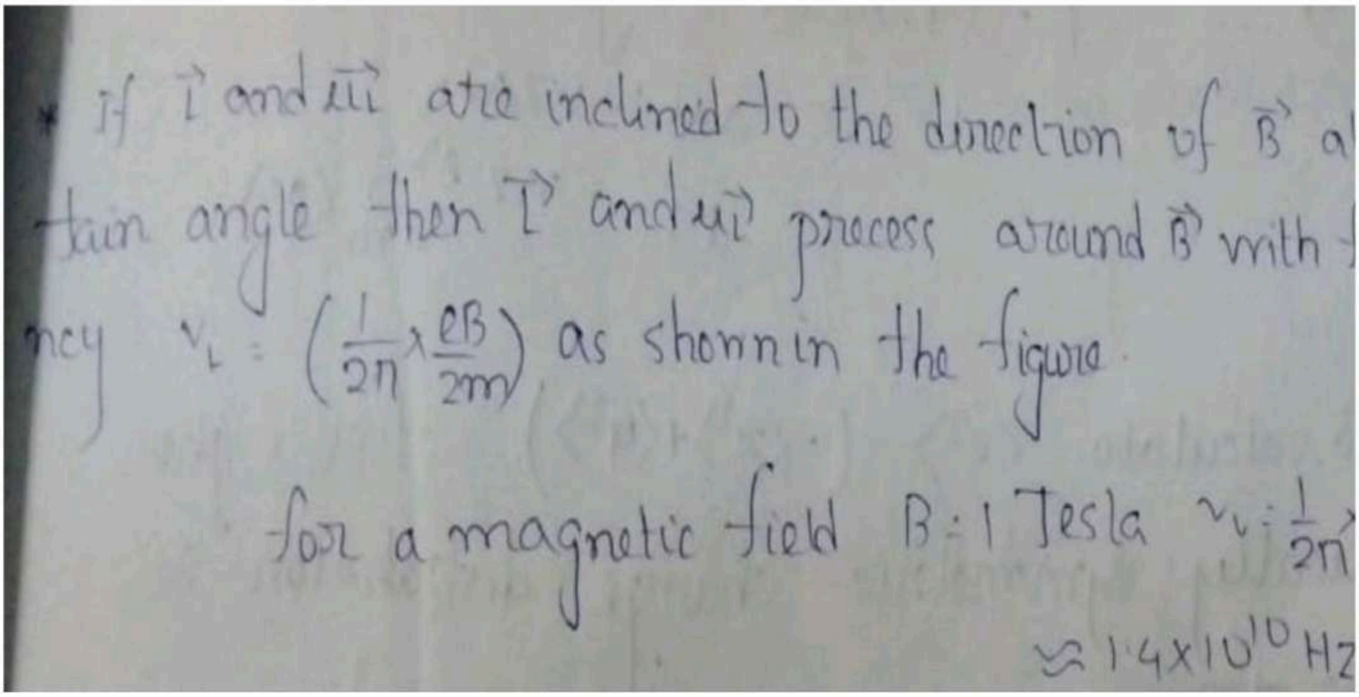
$\vec{\mu}_{L}$=magnetic moment

$\vec{L}$= Angular momentum

e= Charge of the electron

m=Mass of the electron


$\vec{B}$
ω_P
Plane of Precession
L1•-a
y- dg
O
$\vec{dL}$
$\vec{L} + \vec{dL}$
$\vec{L}$
θ
$\vec{\mu}_L$



* If $\vec{l}$ and $\vec{\mu_l}$ are inclined to the direction of $\vec{B}$ a

tain angle then $\vec{l}$ and $\vec{\mu_l}$ process around $\vec{B}$ with

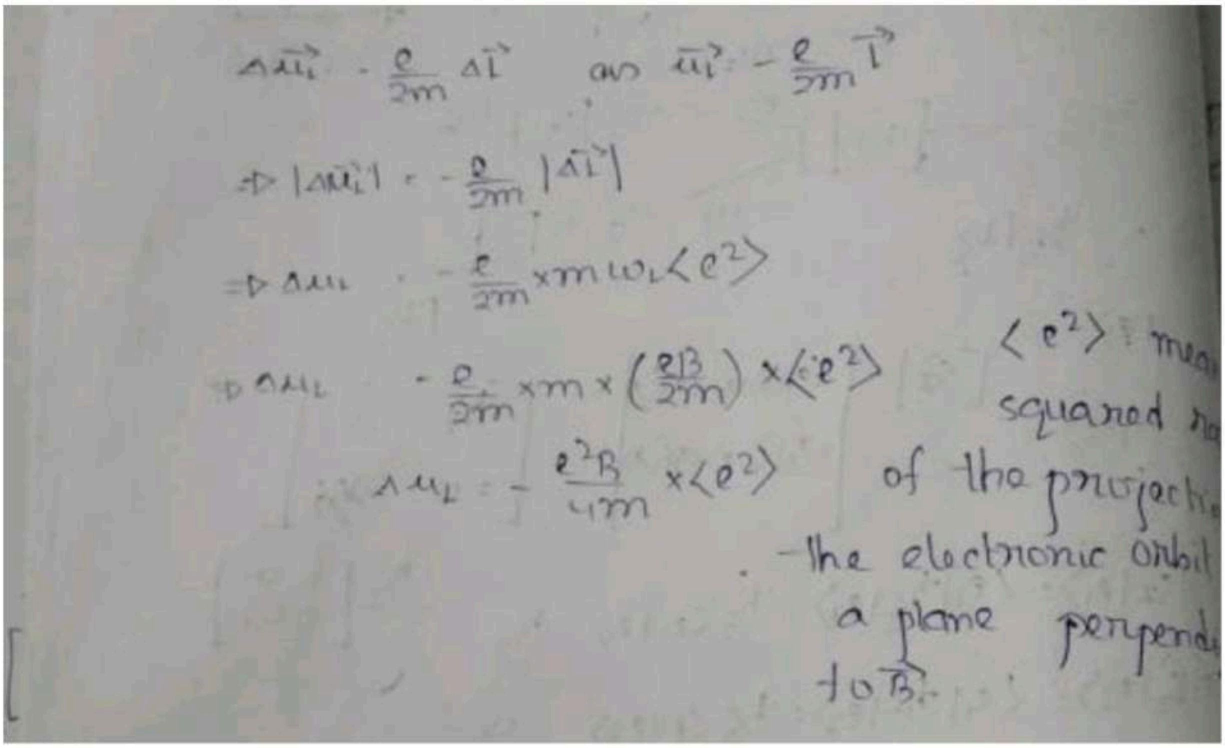
ncy $\nu_L = \left(\frac{1}{2\pi} \times \frac{eB}{2m}\right)$ as shown in the figure.

for a magnetic field $B = 1$ Tesla $\nu_L = \frac{1}{2\pi}$

$\approx 1.4 \times 10^{10}$ Hz

This is small compared to the frequency of rotation of an atomic electron in the orbit v0≈10^{13} to 10^{14} HZ.

The pressicion of the electronic attraction to the applied field, there is an induced magnetic current opposite to thew direction of B given by-



$$\Delta\vec{\mu_l} = -\frac{e}{2m}\Delta\vec{L} \quad \text{as} \quad \vec{\mu_l} = -\frac{e}{2m}\vec{L}$$

$$\Rightarrow |\Delta\vec{\mu_l}| = -\frac{e}{2m}|\Delta\vec{L}|$$

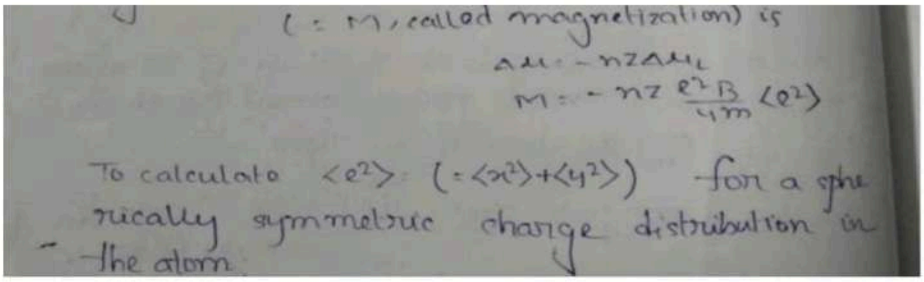
$$\Rightarrow \Delta\mu_L = -\frac{e}{2m}\times m\omega_L\langle e^2\rangle$$

$$\Rightarrow \Delta\mu_L = -\frac{e}{2m}\times m\times\left(\frac{eB}{2m}\right)\times\langle e^2\rangle$$

$$\therefore \Delta\mu_L = -\frac{e^2B}{4m}\times\langle e^2\rangle$$

$\langle e^2\rangle$ = mea squared ra of the projectio the electronic orbit a plane perpend to $\vec{B}$.

If there are n atom in unit volume and there are z electrons per atom,then the total induced magnetic moment per unit volume is du.



(= M, called magnetization) is

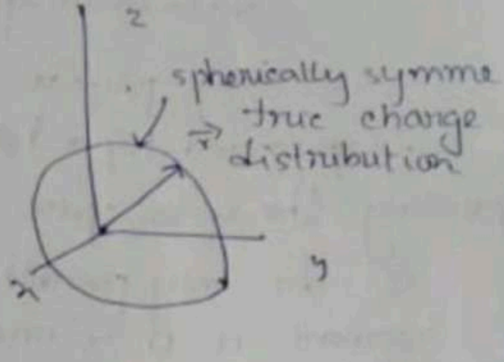
$$\Delta\mu = -nZ\Delta\mu_L$$

$$M = -nZ\frac{e^2B}{4m}\langle e^2\rangle$$

To calculate $\langle e^2\rangle = (=\langle x^2\rangle+\langle y^2\rangle)$ for a spherically symmetric charge distribution in the atom

take B=Bz and say (x,y,z) are the carfesian components of the position vector so the charge distribution at any point on the sphere.

$\langle r^2 \rangle$ = mean squared dictance of electrum from the centre of the atom



projection of the electron orbit on the $\cdot (x-y)$ plane is

$$\langle \rho^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle$$

$$\langle \rho^2 \rangle = \frac{1}{3}\langle r^2 \rangle + \frac{1}{3}\langle r^2 \rangle$$

$$= \frac{2}{3}\langle r^2 \rangle$$

Diamagnetic succeptibility'
we have magnetization

$$M = -nZ\frac{e^2B}{4m}\langle \rho^2 \rangle$$

$$= -nZ\frac{e^2B}{4m} \times \frac{2}{3}\langle r^2 \rangle$$

$$\therefore M = -nZ\frac{e^2}{6m}\langle r^2 \rangle B \quad \text{as } B = \mu_0 H$$

$$\frac{M}{H} = -\mu_0 nZ\left(\frac{e^2}{6m}\right)\langle r^2 \rangle$$

$$\chi_{dia} = -\mu_0 nZ\left(\frac{e^2}{6m}\right)\langle r^2 \rangle$$ χ_{dia} diamagnetic susceptibility

So, $\chi_{dia} = -ve$ and it is independent of temparature.

lalar succeptibility

- $\chi_{dia}^{molar} = \chi_{dia} \times$ vol of 1 mole

$$\chi_{dia}^{molar} = \chi_{dia} \times \frac{N_A}{n}$$

$$\chi_{dia}^{molar} = -\mu_0 nZ\left(\frac{e^2}{6m}\right)\langle r^2 \rangle \times \frac{N_A}{n}$$

$$\chi_{dia}^{molar} = -\mu_0 N_A Z\left(\frac{e^2}{6m}\right)\langle r^2 \rangle$$

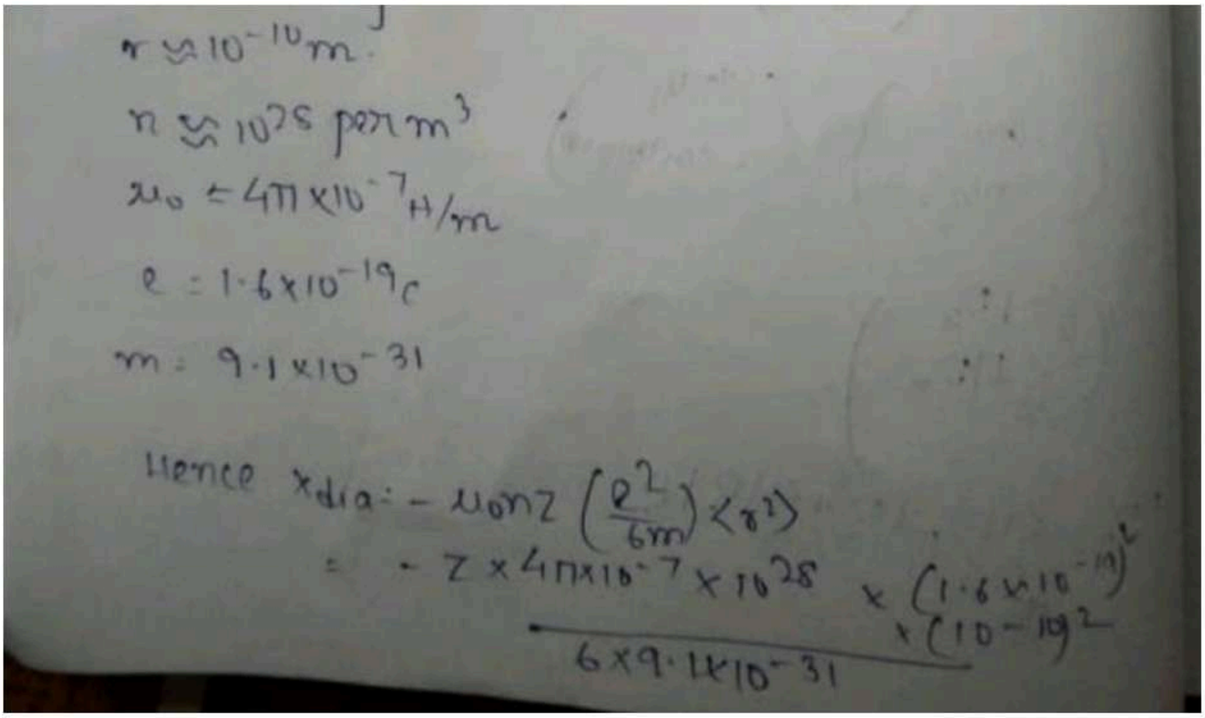
Where Avogadro number is $=6.023*10^{23}$ for 1gm/mol

$=6.032*10^{26}$ for 1kg/mol.

The problem of calculating the diamagnetic susceptibility of an isolated atom is reduced to the calculation of $< r^2>$ for the electron distribution within the atom.The distribution can be calculated.

The term diamagnetism is applied to the case of negative succeptibility i.e. cases in which the induced magnetic moment is opposite to the applied field.Diamagnetism is an universal phenomenon,affecting all atoms.However it is typically much weaker than paramagnetism. And is therefore obsewrved mainly in atoms with even number of electrons where paramagnetism is absent.

Theoritical estimation of x_{dia} and comparison with experiment-



$r \simeq 10^{-10} m$.

$n \simeq 10^{28}$ per m^3

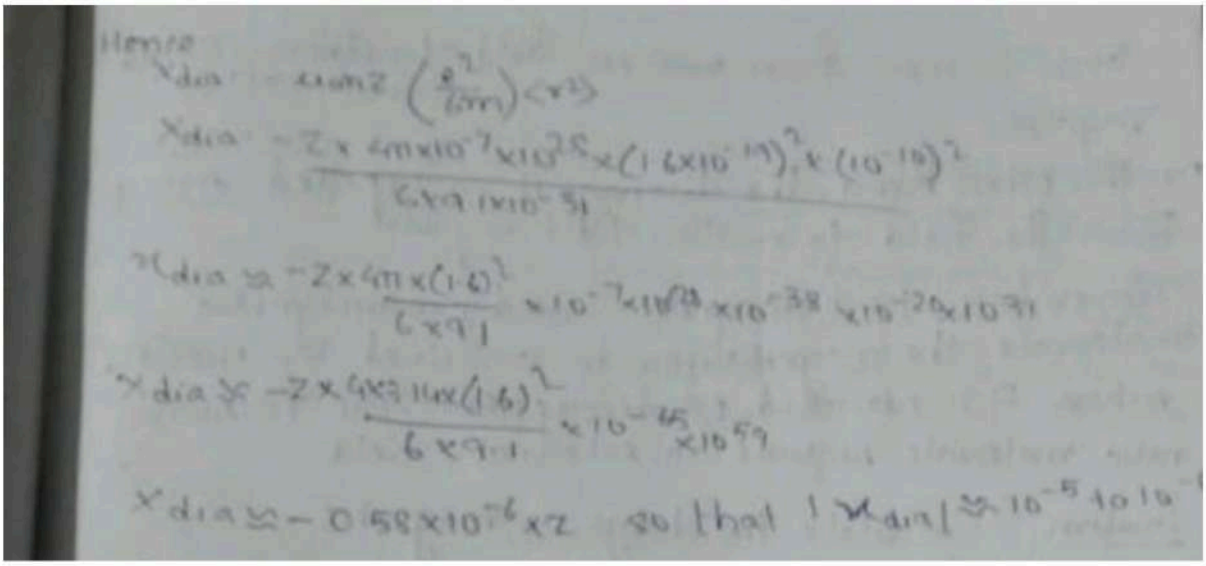
$\mu_0 = 4\pi \times 10^{-7} H/m$

$e = 1.6 \times 10^{-19} c$

$m = 9.1 \times 10^{-31}$

Hence $x_{dia} = -\mu_0 n Z \left(\frac{e^2}{6m}\right) \langle r^2 \rangle$

$$= \frac{-Z \times 4\pi \times 10^{-7} \times 10^{28} \times (1.6 \times 10^{-19})^2 \times (10^{-10})^2}{6 \times 9.1 \times 10^{-31}}$$



Hence

$\chi_{dia} = -\mu_0 n Z \left(\frac{e^2}{6m}\right)\langle r^2\rangle$

$\chi_{dia} = -\frac{Z\times 4\pi\times10^{-7}\times10^{28}\times(1.6\times10^{-19})^2\times(10^{-10})^2}{6\times9.1\times10^{-31}}$

$\chi_{dia} \simeq -Z\times\frac{4\pi\times(1.6)^2}{6\times9.1}\times10^{-7}\times10^{28}\times10^{-38}\times10^{-20}\times10^{31}$

$\chi_{dia} \simeq -Z\times\frac{4\times3.14\times(1.6)^2}{6\times9.1}\times10^{-45}\times10^{39}$

$\chi_{dia} \simeq -0.58\times10^{-6}\times Z$ so that $|\chi_{dia}| \simeq 10^{-5}$ to 10

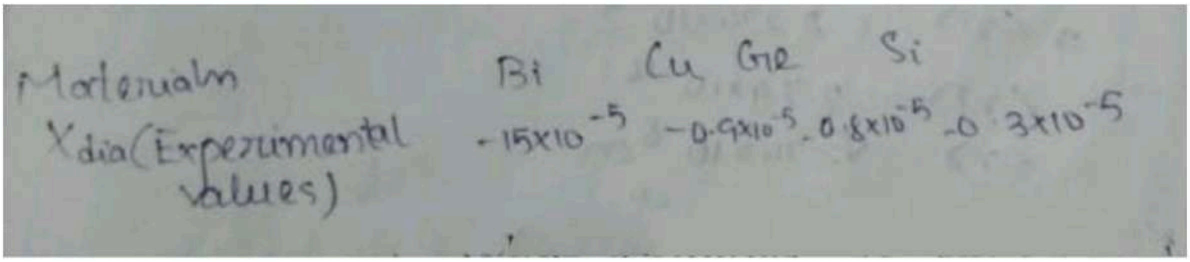
For a solid we conclude that diamagnetic succeptibilities are typically of order

10^{-5}-10^{-6} that is M is minute compared to H.

Example for helium $x_{dia}\approx0.98*10^{-6}$ or $x_{dia}\approx-1.14*10^{-6}$

Experimental value for helium $x_{dia}\approx-1.9*10^{-6}$

Experimental results agree fairly well with the theoretically calculated values particularly for light elements and rare gases.whereas large deviation exist for ions containing a very large number of electrons.



Materials	Bi	Cu	Ge	Si
χ_{dia}(Experimental values)	-15×10^{-5}	-0.9×10^{-5}	-0.8×10^{-5}	-0.3×10^{-5}

CONCLUSION:

This is a vast topic to be realized .This is expressed in terms of vector Atom model and on behalf of classical physics.I hope this can be expressed quantum mechanically as well. This is a vast topic for research.

This can be expressed as well as from classical electrodynamics.i wish to think about the topic in detail in future.

ACKNOWLEDGEMENT:

I would like to express my special thanks of gratitude to my professor Dr.Asok kumar Pal for his special guidance and support
in completing project .I would also like to extend my gratitude to
Topic and he also encourage me for this project work.

REFERENCE

- **ATOMIC PHYSICS**- DR. S.N GHOSHAL
- **Solid State Physics-A J DEKKER**
- **Introduction to Solid State Physics**-CHARLES KITTEL

Analog Electronics : Transistor

The project submitted, in partial fulfilment of the requirement for the assignments in (CC-XI,CC-XII,DSE-I,DSE-Ii) Paper (Semester V) in the Department of Physics

Submitted by

(Prasanta Basak)

Registration No: A01-1112-111-021-2019

Supervisor Teacher: Prof. Dr. Chandan Kumar Das

RAMAKRISHNA MISSION VIVEKANANDA
CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

1. History of Transistor :

(a)Early use of vacumn tubes: *During the period 1904 to 1947 , the* ***vacumn tube*** *was the electronic device of interest and development .In 1904 the vacumn tube diode was introduced by.* ***J.A. Fleming****. The theory behind the operation of vacumn tube is based on a concept known as* ***thermionic emission****.*

The concept of vacuum tubes used the idea that a heated element in a vacuum emitted electrons that would normally remain in the vicinity of this heated element because of the charge attraction. If a second electrode was placed into the vacuum and a high positive potential placed on it, then the electrons would be attracted away from the heated element towards this element with a high potential. As a result a current would flow in this direction.

As electrons were unable to travel in the reverse direction, this simple valve or vacuum tube acts as a diode. Vacumn diode has two elements – ***cathod filament*** *and* ***anode plate.***

If a third element, called ***control grid****, is added ,then the vacumn tube is known as triode. The control grid is placed between cathod and anode . By applying a potential to the grid, it is possible to repel or attract the electrons being emitted from the cathode and in this way affect the flow between cathode and anode of the triode vacuum tube.*

There are also many varients of the vacumn tube like ***pentode , tetrode*** *etc.*

Early vacumn tubes were used to ***amplify*** *signals for radio and other audio devices. It is also used as* ***switches****.*

(b) Disadvantages of using vacumn tubes:

- ***Large size***
- ***High cost***
- ***Power consumption and heat wastage***
- ***Low voltage gain and high input impedance etc…***

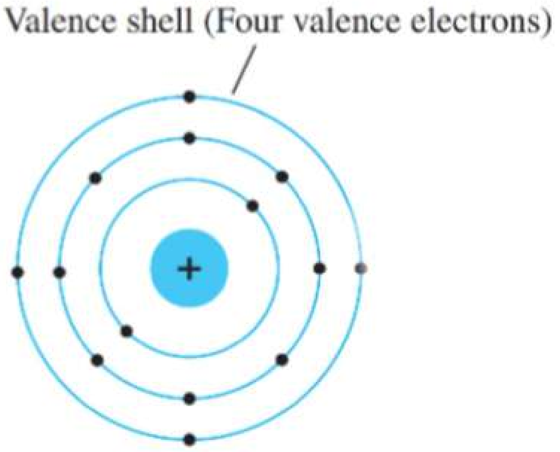
(c) Developement of semiconductor devices:

To overcome the difficulties associated with vacumn tubes , reserachers were searching for other type of devices. It is the start of ***semiconductor devices****.*

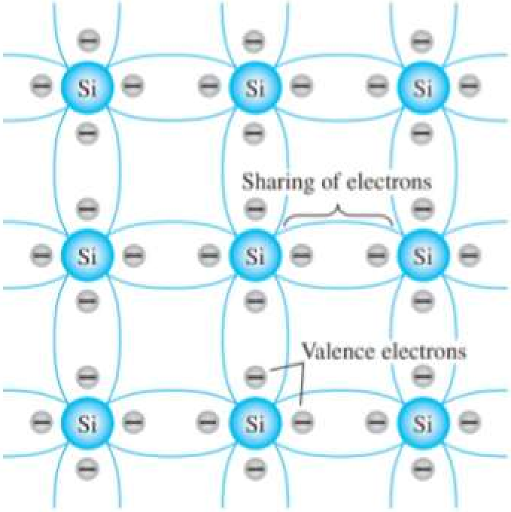
Semiconductors are a special class of elements having a ***conductivity*** *between that of a good* ***conductor*** *and that of an* ***insulator****.* ***Germanium*** *was the first semiconductor used to build semiconductor devices* ***(But silicon is the widely used semiconductor).***

- ***Properties of semiconductor material:***

(i)Atomic structure and covalent bonding:



Picture: Atomic structure of silicon



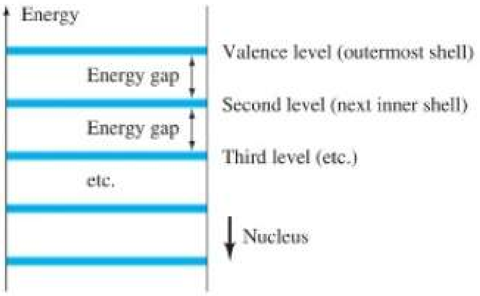
Picture : Covalent bonding of silicon

In a pure silicon crystal the four valence electrons of one atom form a bonding arrangement with four adjoining atoms .

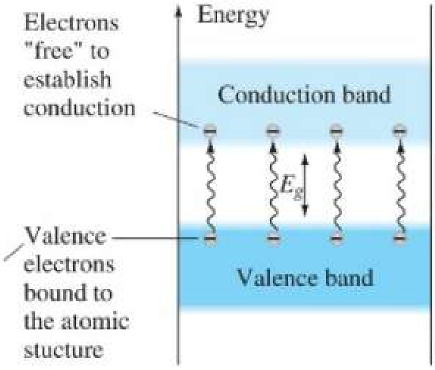
This bonding of atoms, strengthened by the sharing of electrons, is called covalent bonding.

(ii)*Energy levels:*

Within the atomic structure of each and every isolated atom there are specific energy levels associated with each shell and orbiting electron . The energy levels associated with each shell will be different for every element. However, in general: The farther an electron is from the nucleus, the higher is the energy state, and any electron that has left its parent atom has a higher energy state than any electron in the atomic structure.



Picture: Discrete energy levels in isolated atomic structure



Picture :Conduction and valence bands of silicon (Eg=1.1ev)

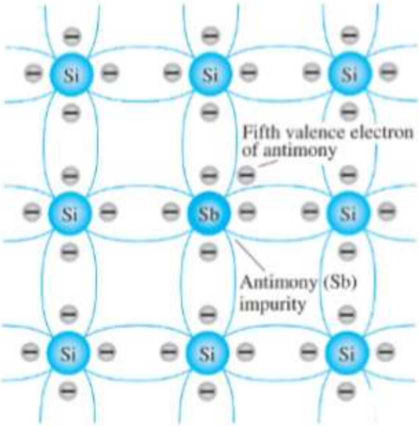
Only specific energy levels can exist for the electrons in the atomic structure of an isolated atom. The result is a series of gaps between allowed energy levels where carriers are not permitted. However, as the atoms of a material are brought closer together to form the crystal lattice structure, there is an interaction between atoms, which will result in the electrons of a particular shell of an atom having slightly different energy levels from electrons in the same orbit of an adjoining atom. The result is an expansion of the fixed, discrete energy levels of the valence electrons. In other words, the valence electrons in a silicon material can have varying energy levels as long as they fall within the band . There is a minimum energy level associated with electrons in the conduction band and a maximum energy level of electrons bound to the valence shell of the atom. Between the two is an energy gap that the electron in the valence band must overcome to become a free carrier.

- ***Doping process:***

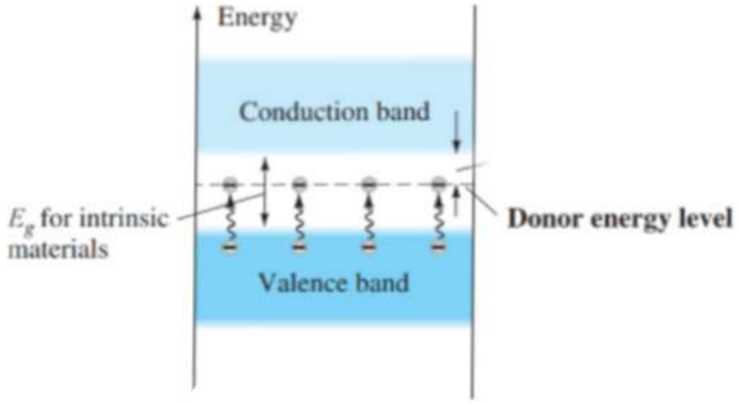
Characteristics of a semiconductor material can be altered significantly by the addition of specific impurity atoms to the relatively pure semiconductor material. This process is called **doping.** These impurities, although only added at 1 part in 10 million, can alter the band structure sufficiently to totally change the electrical properties of the material. A semiconductor material that has been subjected to the doping process is called an extrinsic material. There are two extrinsic materials of immeasureable importance to semiconductor device fabrication: n -type and p -type materials.

(i)N-type material: *An n -type material is created by introducing impurity elements that have five valence electrons (pentavalent), such as antimony , arsenic , and phosphorus. Each is a member of a subset group of elements in the Periodic Table of Elements referred to as Group V because each has five valence electrons. Diffused impurities with five valence electrons are called donor atoms.*

The effect of this doping process on the relative conductivity can best be described through the use of the energy-band diagram . Discrete energy level (called the donor level) appears in the forbidden band with an Eg significantly less than that of the intrinsic material. Those free electrons due to the added impurity sit at this energy level and have less difficulty absorbing a sufficient measure of thermal energy to move into the conduction band at room temperature. The result is that at room temperature, there are a large number of carriers (electrons) in the conduction level, and the conductivity of the material increases significantly. At room temperature in an intrinsic Si material there is about one free electron for every 10^12 atoms. If the dosage level is 1 in 10 million (10^7), the ratio 10^12/10^7 = 10^5 indicates that the carrier concentration has increased by a ratio of 100,000:1.

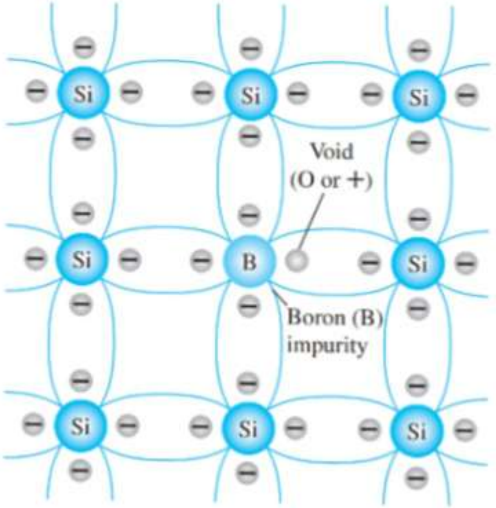


Picture : Antimony impurity in n-type material



Picture: Effect of donor impurities on the energy band structure

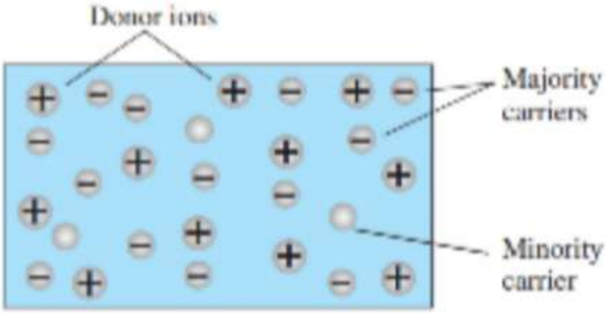
(ii)P-type material: *The p -type material is formed by doping a pure germanium or silicon crystal with impurity atoms having three valence electrons. The elements most frequently used for this purpose are boron , gallium , and indium . Each is a member of a subset group of elements in the Periodic Table of Elements referred to as Group III because each has three valence electrons. there is now an insufficient number of electrons to complete the covalent bonds of the newly formed lattice. The resulting vacancy is called a hole and is represented by a small circle or a plus sign, indicating the absence of a negative charge. Since the resulting vacancy will readily accept a free electron: The diffused impurities with three valence electrons are called acceptor atoms.*



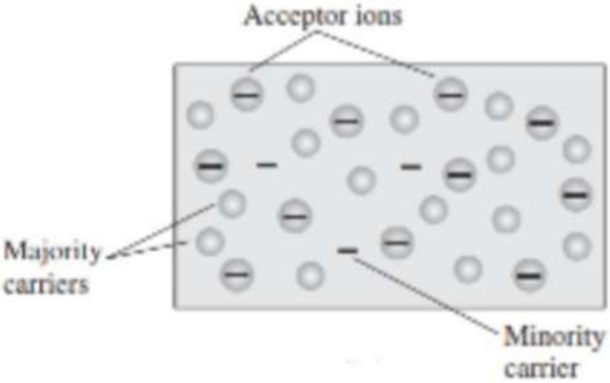
Picture: Boron impurity in p-type material

If a valence electron acquires sufficient kinetic energy to break its covalent bond and fills the void created by a hole, then a vacancy, or hole, will be created in the covalent bond that released the electron. There is, therefore, a transfer of holes to the left and electrons to the right.

In an n-type material , the electron is called the majority carrier and the hole the minority carrier. In a p-type material the hole is the majority carrier and the electron is the minority carrier.



Picture: n-type material

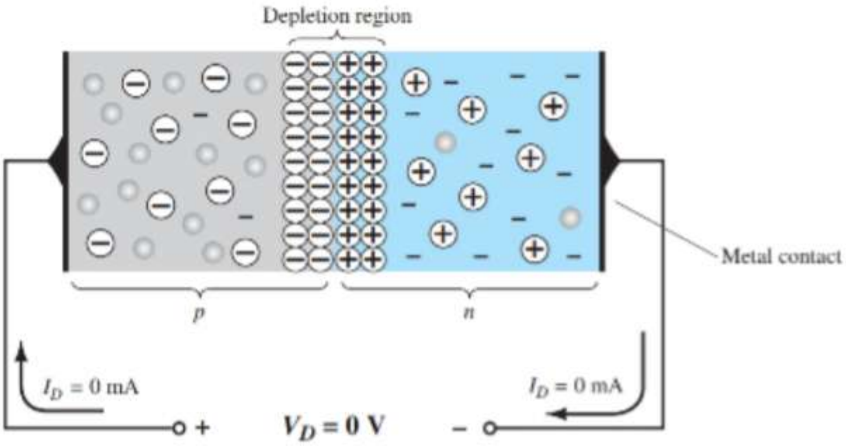


Picture: p-type material

(d) Invention of semiconductor diode: *The semiconductor diode is created by simply joining an n -type and a p -type material together. The joining of one material with a majority carrier of electrons to one with a majority carrier of holes.*

- **Diode biasing:** The term bias refers to the application of an external voltage across the two terminals of the device to extract a response.

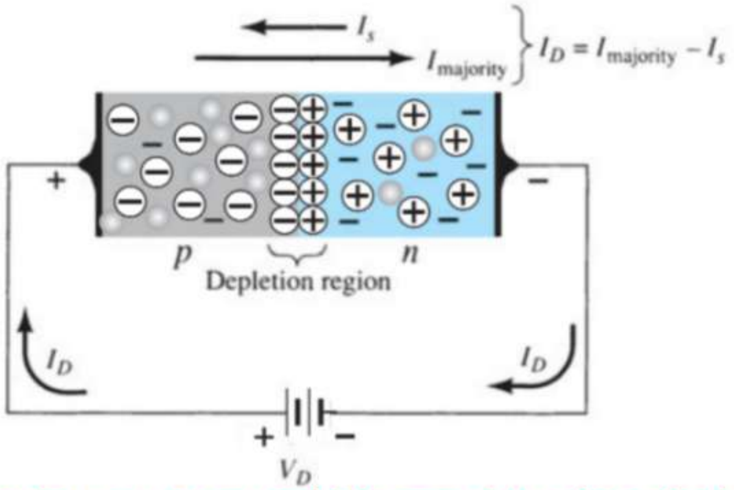
(i)No applied bias (V=0): In the absence of an applied bias across a semiconductor diode, the net flow of charge in one direction is zero.



Picture: Diode under no applied bias

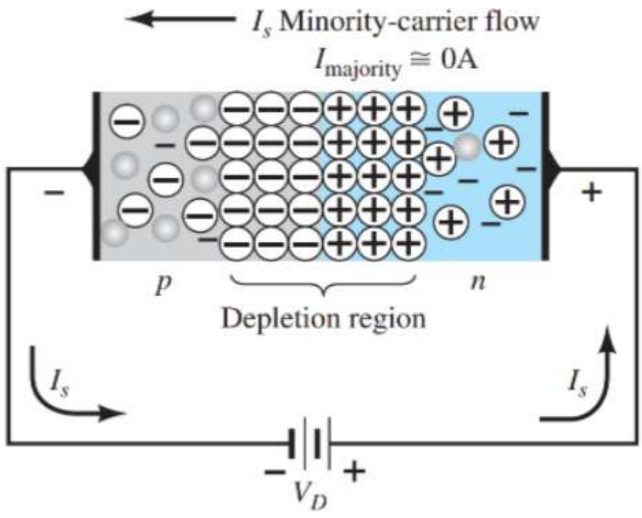
(ii) Forward-Bias Condition (VD> 0 V):

A forward-bias or “on” condition is established by applying the positive potential to the p -type material and the negative potential to the n -type material. The application of a forward-bias potential VD will “pressure” electrons in the n -type material and holes in the p -type material to recombine with the ions near the boundary and reduce the width of the depletion region. the reduction in the width of the depletion region has resulted in a heavy majority flow across the junction.



Picture: Forward-bias pn junction diode

(iii) Reverse-Bias Condition (VD<0): *If an external potential of V volts is applied across the p – n junction such that the positive terminal is connected to the n -type material and the negative terminal is connected to the p -type material , the number of uncovered positive ions in the depletion region of the n -type material will increase due to the large number of free electrons drawn to the positive potential of the applied voltage. For similar reasons, the number of uncovered negative ions will increase in the p -type material. The net effect, therefore, is a widening of the depletion region. This widening of the depletion region will establish too great a barrier for the majority carriers to overcome, effectively reducing the majority carrier flow to zero*



Picture: Reverse-bias pn junction diode

Development of bipolar junction transistor technology :

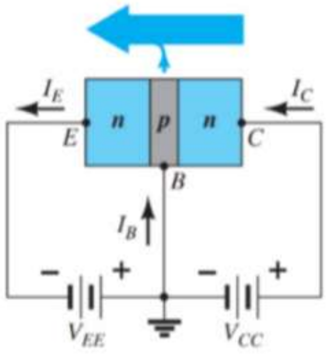
The bipolar junction transistor is a three-layer semiconductor device consisting of either two n - and one p -type layers of material or two p - and one n -type layers of material. The former is called an npn transistor , and the latter is called a pnp transistor .A transistor has three parts – Emitter(E) , Collector(C) and Base (B) . The emitter layer is heavily doped, with the base and collector only lightly doped.

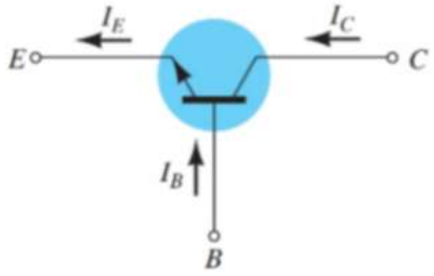
There are mainly three configurations of bipolar junction transistor –

- *Common base*
- *Common emitter and*
- *Common collector*

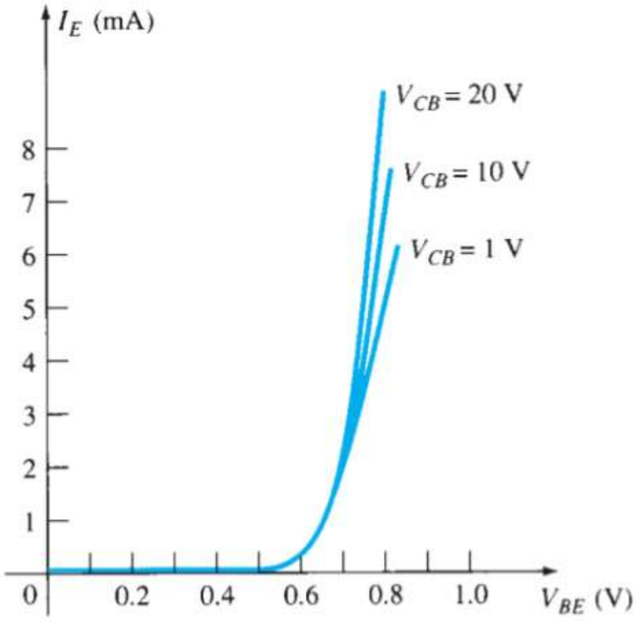
(i)Common base configuration: *The common-base terminology is derived from the fact that the base is common to both the input and output sides of the configuration.*

- ### Notation and symbols :



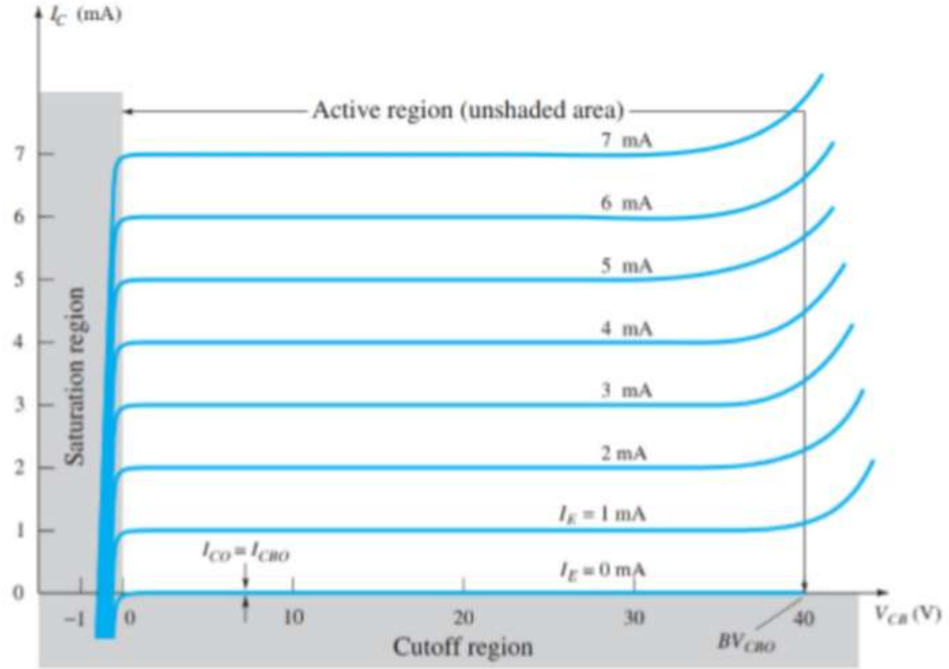


Input characteristics:



Picture : Input characteristics for common base configuration

Output characteristics:



Picture : Output characteristics for common base configuration

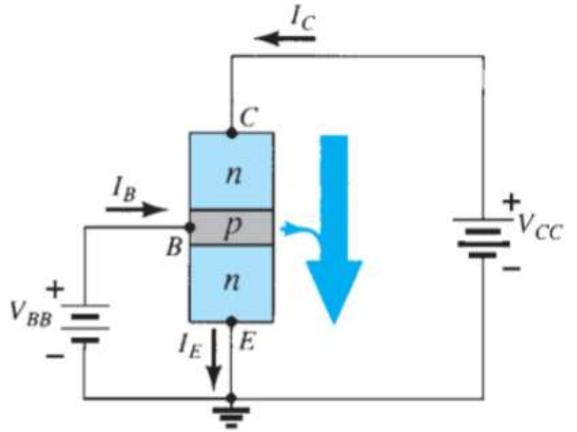
In the cutoff region the base–emitter and collector–base junctions of a transistor are both reverse-biased.

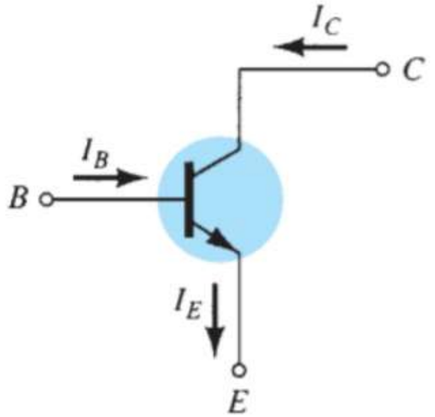
In the active region the base–emitter junction is forward-biased, whereas the collector–base junction is reverse-biased.

In the saturation region the base–emitter and collector–base junctions are forward-biased.

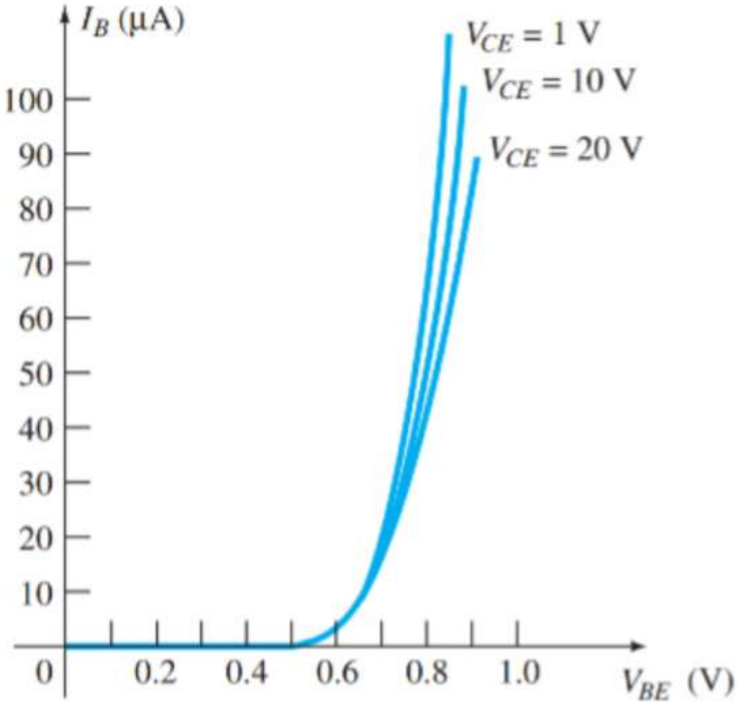
(ii) Common emitter configuration: *The emitter is common to both the input and output terminals (in this case common to both the base and collector terminals). Two sets of characteristics are necessary to describe fully the behavior of the common-emitter configuration: one for the input or base–emitter circuit and one for the output or collector–emitter circuit.*

- **Notation and symbols :**



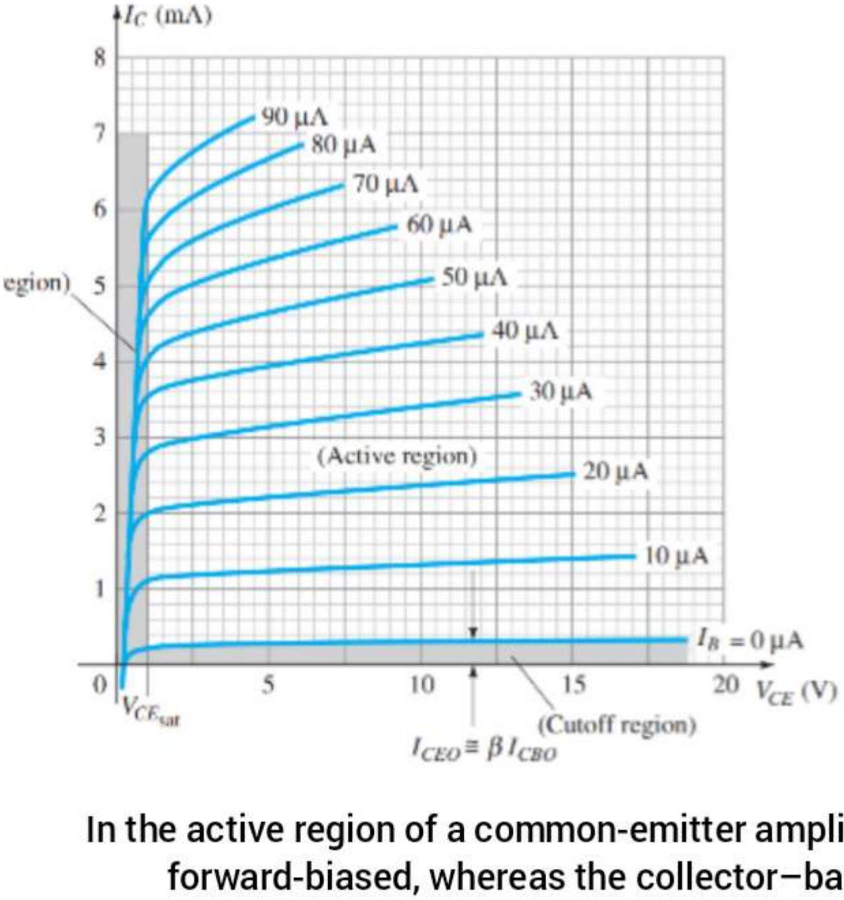


Input characteristics:



Picture : Input characteristics for common emitter configuration

Output characteristics:

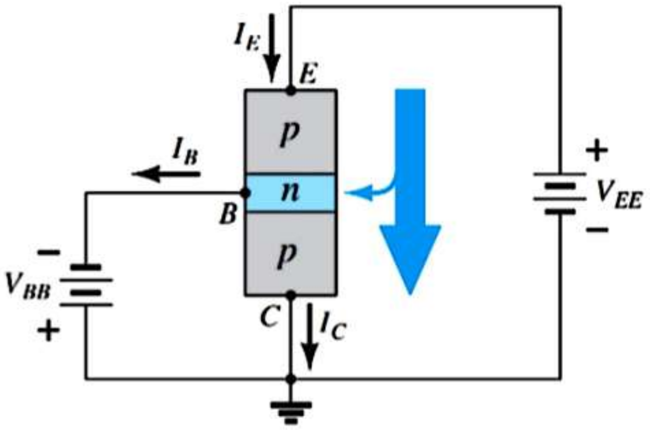


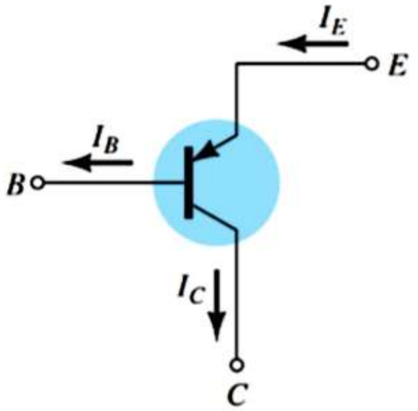
Picture : Output characteristics for common emitter configuration

In the active region of a common-emitter amplifier, the base–emitter junction is forward-biased, whereas the collector–base junction is reverse-biased.

(iii) Common collector configuration : In this configuration , collector is common to both input and output terminals .

- **Notation and symbols :**





There is no need for a set of common-collector characteristics to choose the parameters of the circuit . It can be designed using the common-emitter characteristics . For all practical purposes, the output characteristics of the common-collector configuration are the same as for the common-emitter configuration. For the common-collector configuration the output characteristics are a plot of IE versus V CE for a range of values of IB . The input current, therefore, is the same for both the common-emitter and common-collector characteristics. The horizontal voltage axis for the common-collector configuration is obtained by simply changing the sign of the collector-to-emitter voltage of the common-emitter characteristics.

Current gain in bipolar junction transistor:

Current gain in bipolar junction transistor is defined by the ratio of output current and input current or change in output current and change in input current .

For common base configuration **Current gain is** *defined by alpha*

$$\alpha_{\mathrm{dc}} = \frac{I_C}{I_E} \qquad \alpha_{\mathrm{ac}} = \left.\frac{\Delta I_C}{\Delta I_E}\right|_{V_{CB=\mathrm{constant}}}$$

For common emitter configuration *Current gain is defined by beta*

$$\beta_{\mathrm{dc}} = \frac{I_C}{I_B} \qquad \beta_{\mathrm{ac}} = \left.\frac{\Delta I_C}{\Delta I_B}\right|_{V_{CE=\mathrm{constant}}}$$

For common collector configuration *Current gain is defined by gamma*

$$\mathrm{Y}_{\mathrm{dc}} = \frac{I_E}{I_B} \qquad \mathrm{Y}_{\mathrm{ac}} = \frac{\Delta I_E}{\Delta I_B}$$

In any circuit, the current gain of a bipolar transistor will be of paramount importance. Whether the circuit is common emitter, common collector, etc, and whether it uses NPN transistors or PNP transistors. Although other parameters of these semiconductor devices are also important, the current gain is particularly key because the bipolar transistor is a current operated device. When designing any transistor circuit, it is necessary to ensure there is sufficient gain to enable the circuit to operate correctly. Normally current gain specifications for transistors normally have a very wide tolerance, and therefore circuits need to be able to accommodate this. However the minimum transistor gain must be sufficient to support the correct operation.

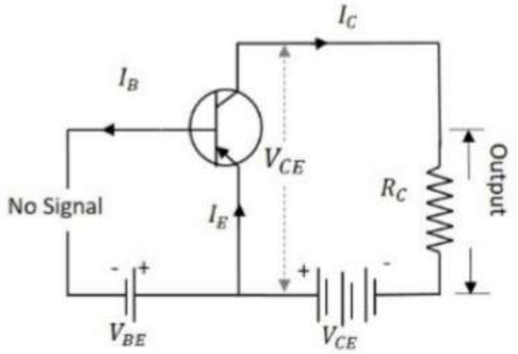
Load line analysis: *When a value for the maximum possible collector current is considered, that point will be present on the Y-axis, which is nothing but the **saturation point**. As well, when a value for the maximum possible collector emitter voltage is considered, that point will be present on the X-axis, which is the **cutoff point**.*

*When a line is drawn joining these two points, such a line can be called as **Load line**. This is called so as it symbolizes the output at the load. This line, when drawn over the output characteristic curve, makes contact at a point called as **Operating point**.*

*This operating point is also called as **quiescent point** or simply **Q-point**.*

Dc load line:

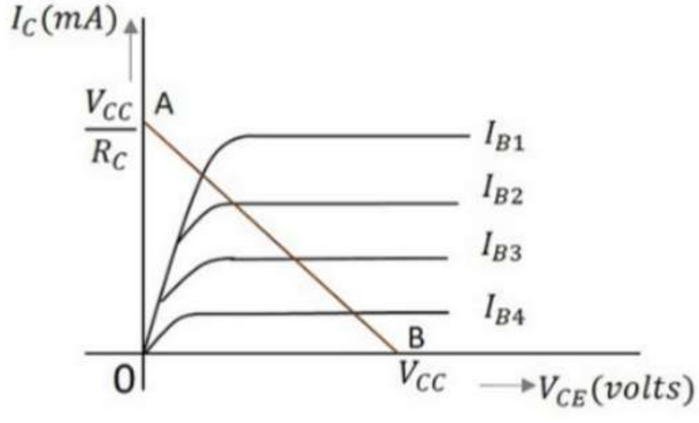
When the transistor is given the bias and no signal is applied at its input, the load line drawn at such condition, can be understood as **DC** condition. Here there will be no amplification as the signal is absent. The circuit will be as shown below.



[Load line is locus of operating point of the transistor. It is graph plotted between Ic and Vce for any configuration. It helps to decide respective values of collector current and collector to emitter voltage to operate transistor in any particular mode or region.]

$$V_{CE}=V_{CC}-I_CR_C$$

To obtain load line we have to get intersection of load line at y-axis(Ic) and x-axis(Vce). When collector emitter voltage $V_{CE} = 0$, the collector current is maximum and is equal to V_{CC}/R_C. When the collector current IC = 0, then collector emitter voltage is maximum and will be equal to the VCC.



Evolution of transistor technology:

Year	Technology	Organization
1947	Point contact	Bell labs
1948	Grown junction	Bell labs
1951	Alloy junction	General electric
1953	Surface barrier	Philco
1953	JFET	Bell labs
1954	Diffused base	Bell labs
1954	Mesa	Bell labs
1959	Planar	Fairchild
1959	Mosfet	Bell labs

Acknowledgement

The success and final outcome of this project required a lot of guidance and assistance from many people and I am extremely privileged to have got this all along the completion of my project. All that I have done is only due to such supervision and assistance and I would not forget to thank them.

I respect and thank our Principal, **Swami Kamalasthananda** and Vice Principal, **Swami Vedanuragananda** for providing me an opportunity to do the project work. I am extremely thankful to **Prof. Dr . Chandan kumar Das** for providing such a nice support and guidance although he had a very busy schedule.

I owe my deep gratitude to our project guide **Prof. Dr . Chandan kumar Das** who took keen interest on my project work and guided me all along, till the completion of the project work by providing all the necessary information for developing a good system. I heartily thank all my friends and classmates for their encouragement and more over for their timely support and guidance till the completion of my project work.
I am thankful to and fortunate enough to get constant encouragement, support and guidance from all Teaching staffs of RKMVCC Physics Department which helped me in successfully completing my project work.

Prasanta Basak , Physics Dept.

BIBLIOGRAPHY

https://www.elprocus.com/bipolar-junction-transistor-working/

https://www.ecstuff4u.com/2018/05/advantages-and-disadvantages-of-bjt.html?m=1

https://www.electronics-tutorials.ws/transistor/tran_5.html

https://encyclopedia2.thefreedictionary.com/surface-barrier+diode

https://en-academic.com/dic.nsf/enwiki/1668800

Electronic devices and circuit theory – Robert L. Boylested , Louis Nashelsky

Quantum Tunneling

The project submitted, in partial fulfilment of the requirement for the assignments in **PHSA CC-XI, PHSA CC-XII, PHSA DSE-I, PHSA DSE-II** *Paper (Semester V) in the Department of Physics*

Submitted by

Devindra Singh

Registration No: A01-1112-111-022-2019

Supervisor Teacher: Dr. Atisdipankar Chakrabarti

RAMAKRISHNA MISSION VIVEKANANDA CENTENARY COLLEGE

P.O. RAHARA, KOLKATA-700118, WEST BENGAL, INDIA

Contents:

1.Introduction:

A particle without the energy to pass over a potential barrier may still tunnel through it.

In classical mechanics if we have a particle with energy **E** and have a barrier of height V0 then the **Classical expectation** is: If E<V0, **(**the particle will totally reflect from the barrier**)**

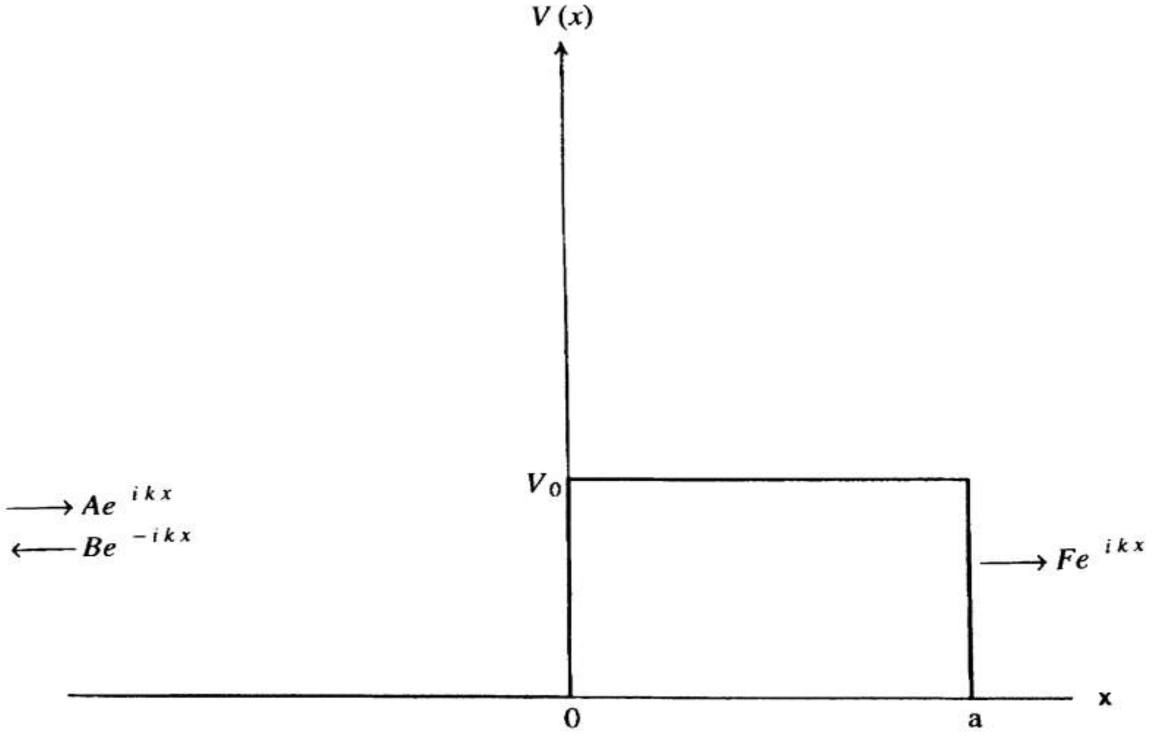
If E>V0, **(**the particle will totally transmit over the barrier**)**

But in quantum mechanics we have a different scenario. In quantum mechanics if a particle that strikes a potential barrier of height V0, then even if the energy of the particle E<V0, we will find is that the particle has a certain probability not necessarily great, but not zero either of passing through the barrier and emerging on the other side which is **classically impossible** for the particle. The particle lacks the energy to go over the barrier but it can nevertheless **tunnel** through it **(**which is really surprising from the point of view of classical mechanics).

Quantum tunneling was first noticed in 1927 by Friedrich Hund while he was calculating the ground state of the double-well potential. Leonid Mandelstam and Mikhail Leontovich discovered it independently in the same year.

Quantum mechanics and classical mechanics differ in their treatment of this scenario of tunneling. The reason for this difference comes from treating matter as having properties of waves and particles. One interpretation of this duality involves the Heisenberg uncertainty principle, which defines a limit on how precisely the position and the momentum of a particle can be simultaneously known. This implies that no solutions have a probability of exactly zero (or one).

2.The Rectangular Potential barrier (Calculation):



The reflection and transmission of a wave by a rectangular potential barrier

We now proceed to study the motion of a particle in a potential V(x) which has the form of the rectangular barrier shown in the figure above.

V(x) = 0 when x<0
V0 when 0< x <a
0 when x>a

<u>We consider the case when E (the energy of the particle) < V0(potential height of the barrier</u>:

For such a case the solutions of the Schrodinger equation in the three regions are given by:

$$\psi = Ae^{ikx} + \ B\, e^{-ikx} \quad x < 0$$

$$= C\, e^{\text{К}x} + De^{-\text{К}x} \quad 0 < x < a \quad \text{where } k = (\frac{2mE}{\hbar^2})^{1/2}$$

$$= Fe^{ikx} + Ge^{-ikx} \quad x > a \quad \text{and } \text{К} = \left(\frac{2m(Vo-E)}{\hbar^2}\right)^{1/2}$$

We set G = 0 because there cannot be a wave propagating in the -x direction in the region x>a.

Continuity of ψ and dψ/dx at x=0 and at x=a gives us

$$A + B = C + D$$

$$\frac{ik}{К}(A - B) = C - D$$

$$C = \frac{1}{2}\left(1 + \frac{ik}{К}\right)A + \frac{1}{2}\left(1 - \frac{ik}{К}\right)\mathrm{B} \qquad (1)$$

$$D = \frac{1}{2}\left(1 - \frac{ik}{К}\right)A + \frac{1}{2}\left(1 + \frac{ik}{К}\right)\mathrm{B} \qquad (2)$$

Continuity conditions at x=a gives us

$$Ce^{Кa} + De^{-Кa} = Fe^{ika} \qquad (3)$$

$$Ce^{Кa} - De^{-Кa} = \frac{ik}{К}\,Fe^{ika} \qquad (4)$$

$$C = \frac{1}{2}\left(1 + \frac{ik}{К}\right)Fe^{-Кa}e^{ika}$$

$$D = \frac{1}{2}\left(1 - \frac{ik}{К}\right)Fe^{Кa}e^{ika}$$

and $$\frac{C}{D} = \frac{\left(1+\frac{ik}{К}\right)}{\left(1-\frac{ik}{К}\right)}e^{-2Кa} = \frac{\left(1+\frac{ik}{К}\right)A+\left(1-\frac{ik}{К}\right)\mathrm{B}}{\left(1-\frac{ik}{К}\right)A+\left(1+\frac{ik}{К}\right)\mathrm{B}}$$

where in the last step we have used Eq (1) and (2). Simple manipulations give us

$$\frac{B}{A} = \frac{(k^2+К^2)\sinh ka}{(k^2-К^2)\sinh ka + 2ikК\cosh ka} \qquad (5)$$

$$\frac{F}{A} = \frac{2ikКe^{-ika}}{(k^2-К^2)\sinh ka + 2ikК\cosh ka} \qquad (6)$$

Reflection and transmission coefficients:

The above equations will lead to the following expressions for the reflection and transmission coefficients:

$$R = \frac{J_r}{J_i} = \frac{\frac{\hbar k}{m}|B|^2}{\frac{\hbar k}{m}|A|^2} = \left[1 + \frac{4\epsilon(1-\epsilon)}{sinh^2(\alpha\sqrt{1-\epsilon})}\right]^{-} \qquad (7)$$

$$T = \frac{J_t}{J_i} = \frac{\frac{\hbar k}{m}|C|^2}{\frac{\hbar k}{m}|A|^2} = \left[1 + \frac{sinh^2(\alpha\sqrt{1-\epsilon})}{4\epsilon(1-\epsilon)}\right]^{-1} \qquad (8)$$

where $\epsilon = \frac{E}{Vo}$, $\alpha = \sqrt{(2mVoa^2/\hbar^2)}$ in writing equation 7 and 8 we have used the fact that

$$Ka = \alpha\sqrt{1-\epsilon} \quad , \qquad R + T = 1$$

E>Vo:

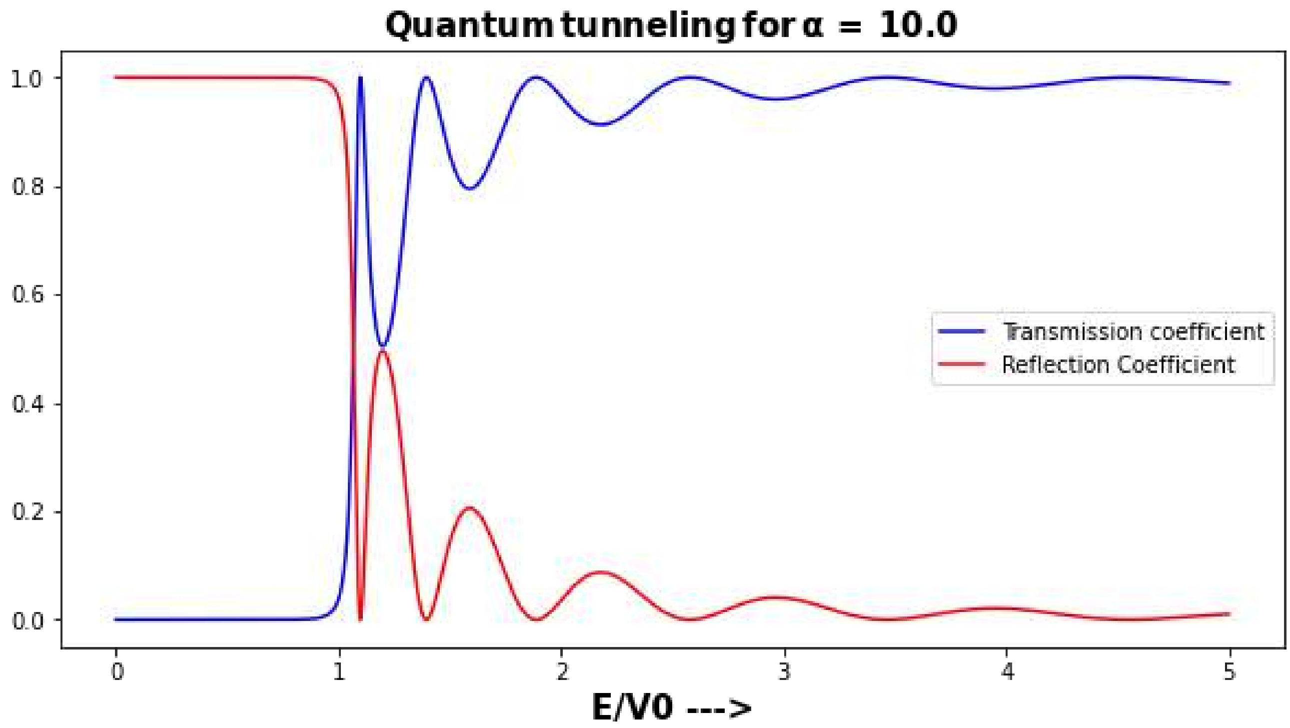
For E>Vo, the analysis is very similar, excepting that, in the region 0<x<a, instead of the solutions $e^{\pm Kx}$ we will have solutions $e^{\pm ik_1 x}$ where $k_1 = [2m(E - Vo)/\hbar^2]^{1/2}$ we have replaced K by ik1, everywhere. The final results are

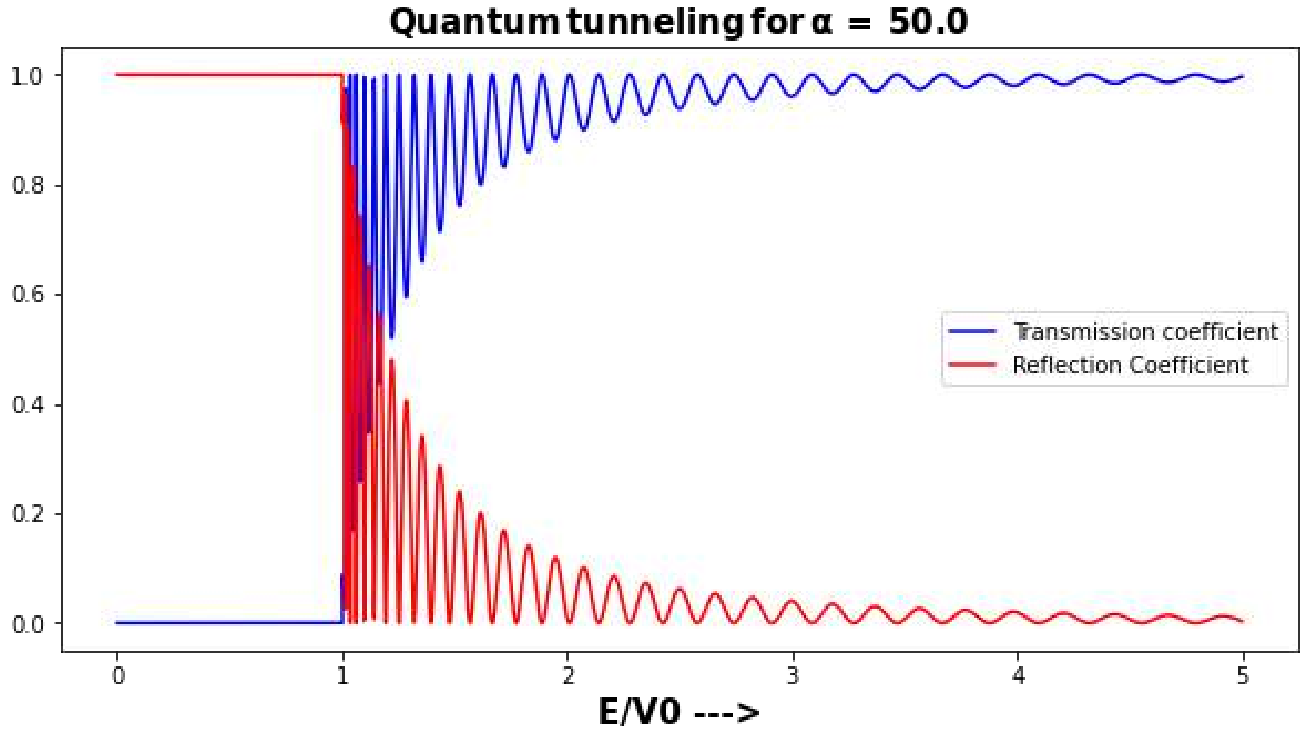
$$R = \left[1 + \frac{4\epsilon(\epsilon - 1)}{\sin^2 \alpha\sqrt{\epsilon - 1}}\right]^{-1} \qquad (9)$$

$$T = \left[1 + \frac{\sin^2 \alpha\sqrt{\epsilon - 1}}{4\epsilon(\epsilon - 1)}\right]^{-1} \qquad (10)$$

$$R + T = 1, \quad k_1 a = \alpha\sqrt{\epsilon - 1}$$

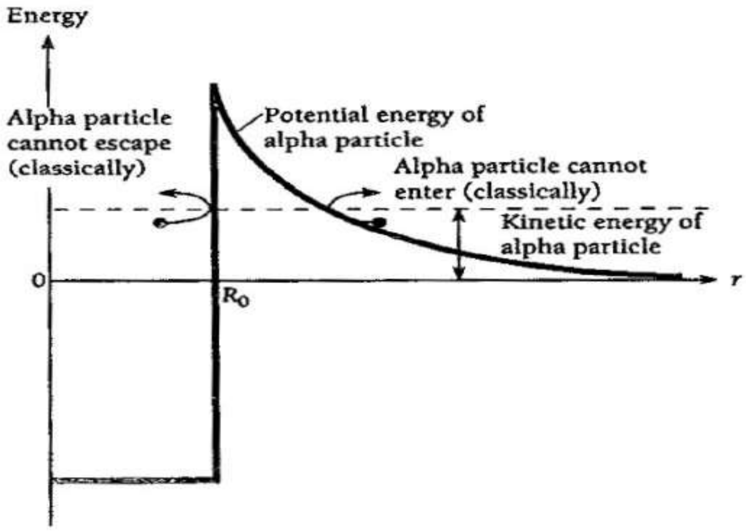
3.Numerical Plot (Quantum Tunneling):



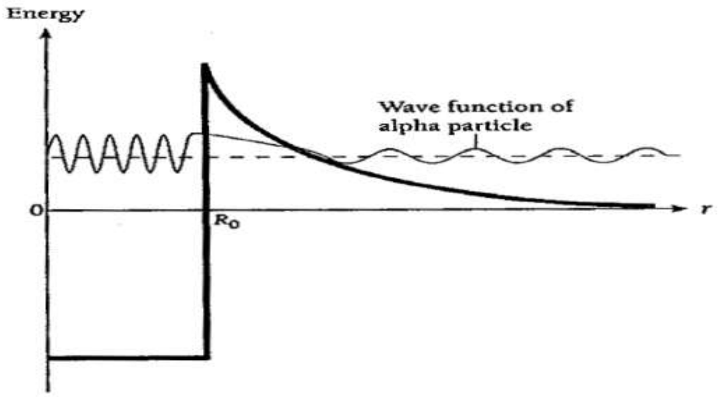


4.Application of Quantum Tunneling:

1.**Tunnel Theory of alpha decay (Gamow's theory)**: Our present-day knowledge regarding the nuclear structure tells us that the neutrons and protons forming the nucleus of an atom are very strongly attracted to one other, giving rise to a strongly bound structure known as the nucleus of the atom. The α-disintegration of the heavy nuclei shows that within such nuclei, 2 protons and 2 neutrons sometimes form a cluster the α-particle.



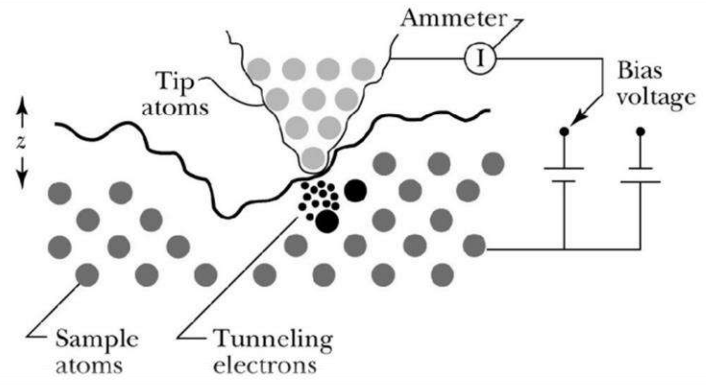
For the case of 222Rn formed during the α-disintegration of 226Ra, the value of V is 34 MeV. The α-disintegration energy Q=4.88MeV in thus case is much lower than V. This is the energy the α-particle has in the disintegrating nucleus. Classical mechanics requires that for an α-particle to escape from the nucleus or to enter it from outside its energy must at least be equal to V. If it is lower, then in some region between the two curves, the potential energy of the α-particle(V)>E (total energy). This is classically forbidden region because E=T+V, T=E-V and as V>E, T becomes negative. An α-particle in this case, can neither escape from nor enter into the nucleus from nor enter into the nucleus. from nor enter into the nucleus from nor enter into the nucleus.



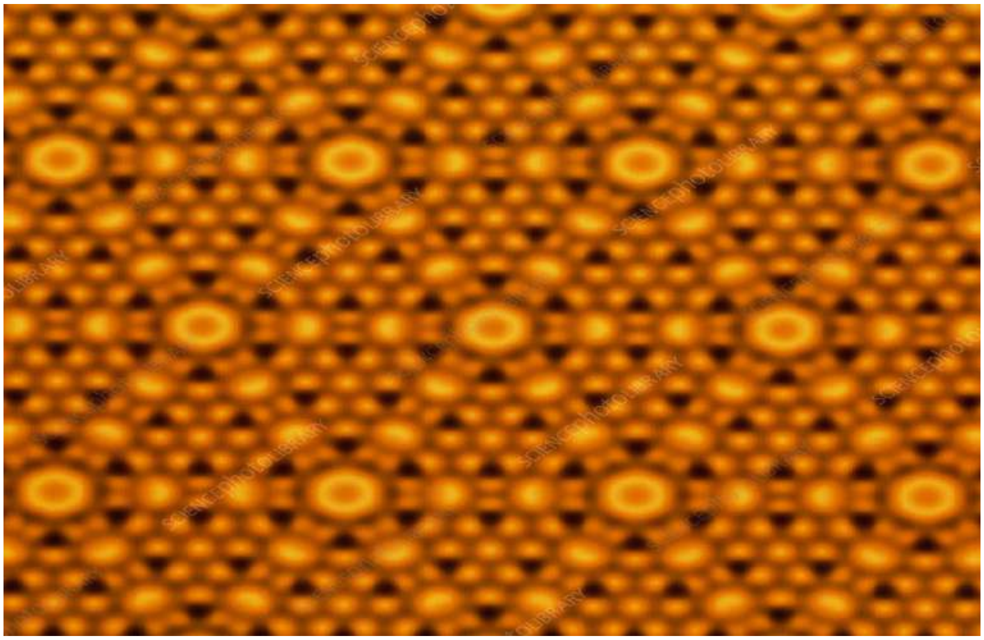
However, quantum mechanically, such classically forbidden phenomenon may occur. In quantum mechanics, the particle is represented by a wave, obeying the Schrodinger wave equation. We can write down the wave equation for the different regions by substituting the corresponding potentials acting on the α-particle in these regions. **If these equations are solved with appropriate boundary conditions, then it is found that an α-particle initially inside the nucleus has a finite probability of coming out of it.** We can picture the escape of the α-particle from the nucleus as if they leak out through tunnels in the potential barrier. Hence the effect is called **quantum mechanical tunnel effect.** The theory gives a mathematical relationship between the initial α-energy and the half-life of the nucleus.

If we calculate the transmission co-efficient $T = 16E\frac{(Vo-E)}{Vo^2}e^{-2\beta a}$ under the approximation E<<Vo, where $\beta^2 = \frac{2\mathrm{M}(Vo-E)}{\hbar^2}$. So, by this we can infer that there is a small but finite probability for the α-particle to coming out of the nucleus even though its energy(E) < potential barrier (Vo).

2. Scanning Tunneling Microscope: The ability of electrons to tunnel through a potential barrier is used in an ingenious way in the **scanning tunneling microscope** (STM) to study surfaces on an atomic scale of size. STM provides a method to image atomic structures on scales down to a few tenths of a nanometer. The STM was invented in 1981 by Gert Binning and Heinrich Rohrer. In an STM, a metal probe with a point so fine that its tip is a single atom is brought close to the surface of a conducting or semiconducting material. Normally even the most loosely bound electrons in an atom on a surface need several electron-volts of energy to escape – this is the work function. However, when a voltage of only 10mV or so is applied between the probe and the surface, electrons can tunnel across the gap between them if the gap is small enough, a nanometer or two.



According to $T = 16E\frac{(Vo-E)}{Vo^2}e^{-2\beta a}$ the electron transmission probability is proportional to e^{-a} , where a is the gap width, so even a small change in a (as little as 0.01nm, less than a twentieth the diameter of most atoms) means a detectable change in tunneling current. What is done is to move the probe across the surface in a series of closely spaced back-and-forth scans in about the same way an electrons beam traces out an image on the screen of a television picture tube. The height of the probe is continually adjusted to give a constant tunneling current, and the adjustment are recorded so that a map of surface height versus position is built up. Such a map is able to resolve individual atoms on a surface.



Silicon Atoms on the surface of a silicon crystal form a regular pattern

In this image produced by an STM

5.Conclusion:

So, in the end we can draw a conclusion that the discovery of quantum tunneling made things possible which is impossible classically. It has many other applications besides the explanation of α-decay and STM like tunnel diode, quantum computing, small microchips, micro-controllers which has a deep impact on understanding the technological aspect of quantum tunneling which undoubtedly revolutionized the new era.

6. Acknowledgements:

Throughout this project work my supervisor teacher **Dr. Atisdipankar Chakrabarti**

helps me a lot whenever I needed by clearing my doubts about this topic so I'm very thankful to him for giving me such kind of interesting topic which also helps me a lot in my future.

7. References:

i. Introduction to Quantum Mechanics by David J. Griffiths & Darrell F. Schroeter

ii. Modern Physics by Arthur Beiser

iii. Pictures (Source : Google)

USE OF HYDROGEN FUEL AS A SUSTAINABLE ENERGY

The project submitted, in partial fulfilment of the requirement for the assignments in PHSA CC-XI, PHSA CC-XII, PHSA DSE-I, PHSA DSE-II Paper (Semester V) in the Department of Physics

Submitted by

Suryashis Basak

Registration No: A01-1112-111-024-2019

Supervisor Teacher: Prof. Bhaskar Haldar

RAMAKRISHNA MISSION VIVEKANANDA CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

ACKNOWLEDGEMENT

I would like to express my special thanks of gratitude to my professor Bhaskar Haldar for his able guidance and support in completing project. I would like to extend my gratitude to him to give me this opportunity to do this project in this interesting topic and he also encourage me for this project work.

INDEX

TOPIC **PAGE**

What is hydrogen energy

Fig no. 01 , hydrogen gas stored tank

Hydrogen is one of the most abundant elements on the earth for a cleaner alternative fuel option. Hydrogen is the lightest element . it's weight is lesser than air so it rises in the atmosphere and is therefore rarely found in its pure form, H_2 . At standard temperature and pressure hydrogen is a nontoxic , tasteless and colourless highly combustible diatomic gas. Hydrogen fuel is a zero emission fuel burned with oxygen. It can be used in fuel cells or internal combustion engines. It is also used as fuel in spacecraft propulsion.

Hydrogen is the most common chemical in the universe. It can be produced as a gas or liquid or made part of other material and has many uses such as fuel for transport or heating, a way to store electricity or a raw material in the industrial processes. When it is produced using renewable energy ,hydrogen becomes a way of storing renewable energy for use at a later time for future use.

Hydrogen energy can be stored as a gas and even delivered through existing natural gas pipelines. When converted to liquid or other suitable material, hydrogen can also be transported on trucks and ships. This means hydrogen can also be exported overseas, effectively making it tradable energy commodity.

Types of Hydrogen

Green hydrogen- it is produced by electrolysis of water using renewable energy (like solar, wind) and has a lower carbon footprint .Electricity splits water into hydrogen and oxygen.

Brown hydrogen- it is produced using coal where the emissions are released to the air.

Grey hydrogen- it is produced from natural gas where the associated emissions are released to the air .

Blue hydrogen- it is produced from natural gas where the emissions are captured using carbon capture and storage.

How hydrogen are produced

There are several ways to produced hydrogen –

1. **GASIFICATION** – synthesis gas—a gas mixture of hydrogen . carbon monoxide and a small amount of carbon dioxide – is created by reacting natural gas with high temperature steam. The carbon monoxide is reacted with water to produced additional hydrogen. This method is the cheapest and most efficient ,and most common. A synthesis gas can also be created by reacting coal or biomass with high temperature steam and oxygen in a pressurized gasifier. This converts the coal or biomass into gaseous components—a process called gasification. The resulting synthesis gas contains hydrogen and carbon monoxide, which is created with steam to separate the hydrogen.

Fig no. 02, hydrogen production by gasification

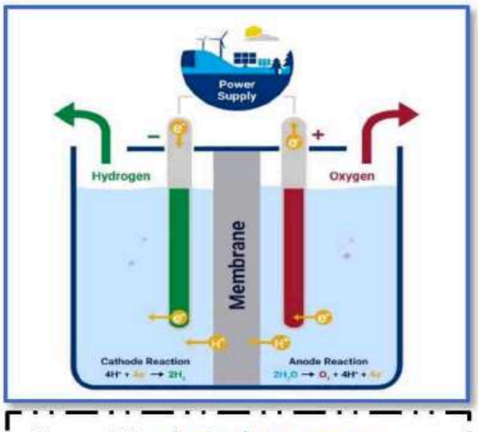


Fig no.03 , electrolysis process

2. **ELECTROLYSIS** – an electric current splits water into hydrogen and oxygen . if the electricity is produced by renewable sources, such as solar or wind, the resulting hydrogen will be considered renewable as well, and has numerous emissions benefits. Power-to-hydrogen projects are taking off, using excess renewable electricity , when available , to make hydrogen through electrolysis.

3. **RENEWABLE LIQUID REFORMING** – renewable liquid fuels, such as ethanol, are reacted with high-temperature steam to produce hydrogen near the point of end use.

4. **FERMENTATION** – biomass is converted into sugar-rich feedstocks that can be fermented to produce hydrogen.

5. **HIGH-TEMPERATURE WATER SPLITTING** – high temperatures generated by solar concentrators or nuclear reactors drive chemical reactions that split water to produce hydrogen.

6. **PHOTOBIOLOGICAL WATER SPLITTING** – microbes such as green algae, consume water in the sunlight and produce hydrogen as a byproduct.

7. **PHOTOELECTROCHEMICAL WATER SPLITTING** – photo-electrochemical system produce hydrogen from water using special semiconductors and energy from sunlight.

Salient features of hydrogen energy

1. hydrogen has highest energy content per unit mass of any chemical fuel and can be substituted hydrocarbon in a board range of application.

2. Its burning process is non-polluting.

3. Heating value of 28000 kcal/kg is 3 times greater than hydrocarbon.

4. Easy in production because it is produced from water found in abundance.

5. Hydrogen is highly flammable.

6. Hydrogen at normal pressure and temperature is a light gas with a density only (1/14) that of air and (1/9) that of natural gas under the same condition.

7. By cooling to the extremely low temperature of -253°C, the gas condensed to a liquid with a specific gravity of 0.07, roughly (1/10) that of gasoline.

8. The flame speed of the hydrogen burning in air is much greater than for natural gas, and the energy required to initiate combustion is less.

9. Mixture of hydrogen and air are combustible over an exceptionally wide range of compositions; thus the flammability limits at ordinary temperature extends from 4% to 74% by volume of hydrogen in air.

How hydrogen fuel cell works

The purpose of the hydrogen fuel cell is to produce electrical current. It is used to do work outside of the cell, like powering the electric motor , illuminating a light bulb .this current returns to the fuel cell and completing the electrical circuit. There are several kinds of fuel cells and each operates differently, but here we discuss only hydrogen fuel cell. Hydrogen atoms enter fuel cells at the anode where a chemical reaction strips them of their electrons. Now the hydrogen atoms are ionized and carry a positive electrical charge. The negatively charged electrons provide the current through the wires. If alternative current (AC) is needed , the DC output must be routed through a conversion device an inverter.

Fig no. 04, a complete fuel cell

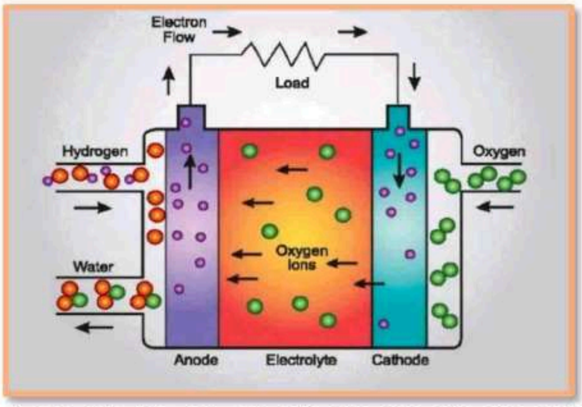


Fig no. 05, general picture of fuel cell

Oxygen enters the fuel cell at the cathode and, in some cell types (like the one illustrated above), it there combines with electrons returning from the electrical circuit and hydrogen ions that have traveled through the electrolyte from the anode.

The electrolyte plays a key role. It must permi only the appropriate ions to pass between the anode and cathode. If free electrons or other substance could pass through the electrolyte, the would disrupt the chemical reaction.

Whether they combine at anode or cathode, together hydrogen are oxygen form water, which drains from the cell. As long as a fuel cell is supplied with hydrogen and oxygen, it will generate electricity.

Since fuel cells create electricity by chemical reaction , rather than by combustion, they are not subjected to the thermodynamic laws that limit a conventional power plant. Therefore, fuel cell can also be harnessed, boosting system efficiently still further.

Different types of hydrogen fuel

- **ALKALINE FUEL CELL** – alkaline fuel cell operates on compressed hydrogen and oxygen. They generally used a solution of potassium hydroxide as an electrolyte. Alkali fuel cell were used in the APPOLO spacecraft to get electricity and water both. They required pure hydrogen fuel, but their platinum electrodes are expensive.

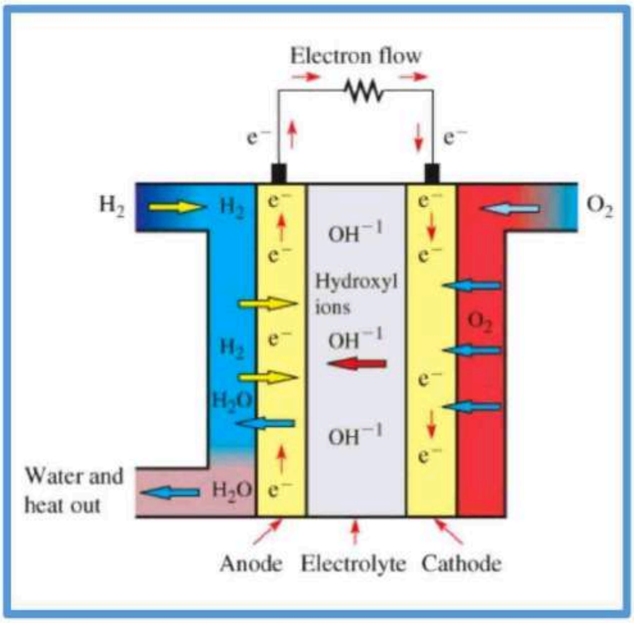


Fig no. 06, fuel cell diagram of AFC

Anode reaction:

$H2 + 2OH- \rightarrow 2H2O + 2e-$

Cathode reaction:

$\frac{1}{2}O2 + H2O + 2e- \rightarrow 2OH-$

Overall reaction:

$H2 + \frac{1}{2}O2 \rightarrow H2O$

Fig no. 07, reaction of AFC

EFFICIENCY : 70%

OPERATING TEMPERATURE : 150 – 200 (in degree C)

OUTPUT RANGE – 30 watt to 5 kilowatt

- **MOLTEN CARBONATE FUEL CELL (MCFC)** – it uses the high-temperature(like sodium and magnesium) compounds salt carbonates as their electrode. The high temperature limits damage from carbon monoxide poisoning of the cell and waste heat can be recycled to make additional electricity. Their nickel electrodes are inexpensive compared to the platinum used in the other cells. But the high temperature would be too hot for home use. Carbonate ions are used up in the reactions making it necessary to inject carbon dioxide to compensate.

Anode reaction :

H2 + CO32- → CO2 + H2O + 2e-

Cathode reaction :

½O2 + CO2 + 2e- → CO32-

Overall reaction :

H2 + ½O2 → H2O

Fig no. 08, reactions of MCFC

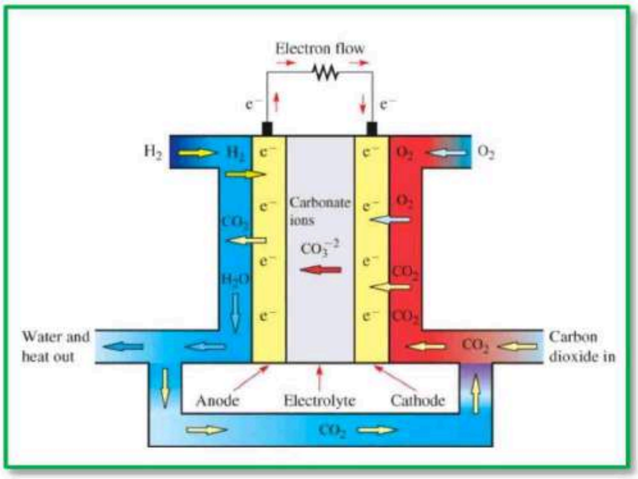


Fig no. 09, fuel cell of MCFC

EFFICIENCY: 60%-80%

OPERATING TEMPERATURE : 650 (in degree C)

OUTPUT RANGE : 2 megawatts (100 megawatts designs are ready now)

- **PHOSPHORIC ACID FUEL CELLS (PAFC) –** it uses phosphoric acid as the electrolyte. PAFC's tolerate a carbon monoxide concentration of about 1.5 percent, which broadens the choice of fuels they can use. If gasoline is used, the sulfur must be removed. Platinum electrode-catalysts are needed, and internal parts must be able to withstand the corrosive acid.

Anode reaction :

H2 → 2H+ + 2e-

Cathode reaction :

½O2 +2H+ + 2e- → H2O

Overall reaction :

H2 + ½O2 → H2O

Fig no.10, reaction of PAFC

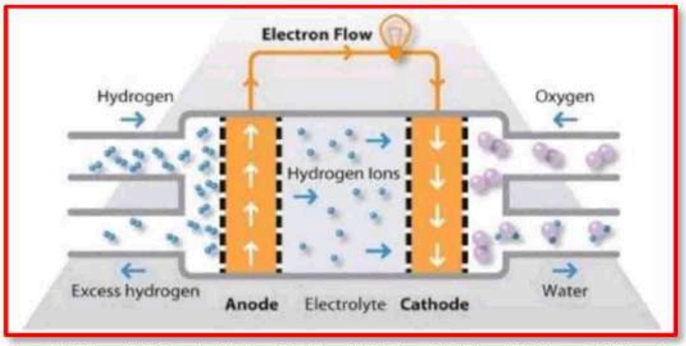


Fig no. 11, fuel cell of PAFC

EFFICIENCY: 40%-80%

OPERATING TEMPERATURE : 150-200(in degree C)

OUTPUT RANGE: 200 kilowatts

- **PROTON EXCHANGE MEMBRANE FUEL CELL(PEM):** it work with a polymer electrolyte in the form of a thin, permeable sheet. The solid , flexible electrolyte will not leak or crack, and these cells operate at a low enough temperature to make them suitable for homes and cars. But their fuels must be purified and a platinum catalyst is used on both sides of the membrane, raising costs.

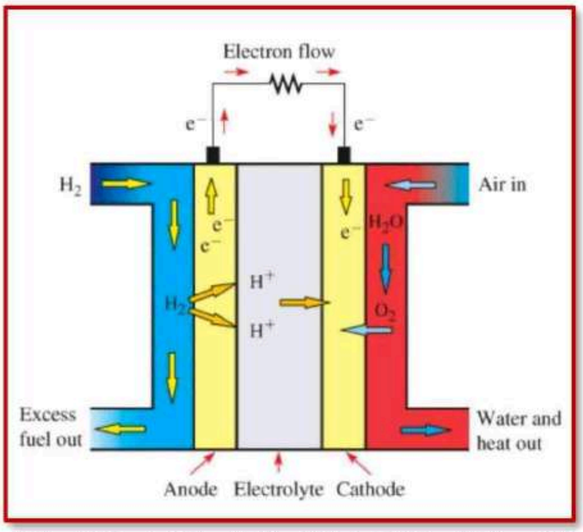
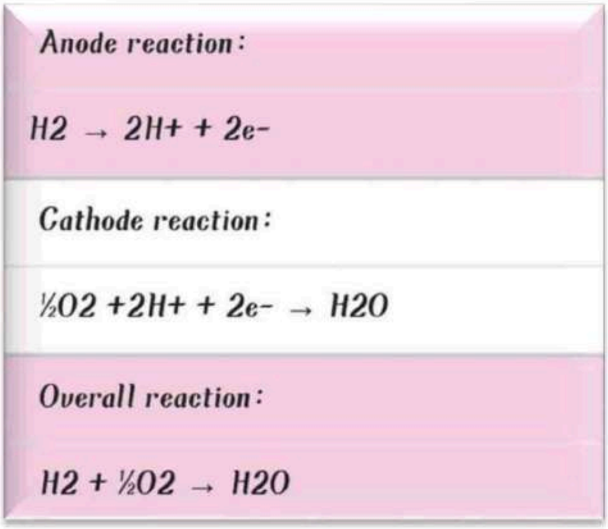


Fig no. 12, PEM fuel cell


Anode reaction :

$H_2 \rightarrow 2H^+ + 2e^-$

Cathode reaction :

$\frac{1}{2}O_2 + 2H^+ + 2e^- \rightarrow H_2O$

Overall reaction :

$H_2 + \frac{1}{2}O_2 \rightarrow H_2O$

Fig no. 13, reactions of PEM

EFFICIENCY: 40% - 50%

OPERATING TEMPERATURE: 80 (in degree C)

OUTPUT RANGE: 50 kilowatts – 250 kilowatts

- **SOLID OXIDE FUEL CELL (SOFC) :** it use a hard, ceramic compound of metal (like calcium or ziroconium) oxides(chemically, O_2) as electrolyte. High temperatures a reformer is not required to extract hydrogen from the fuel, and waste heat can be recycled to make additional electricity. However, the high temperature limits applications of SOFC units and they tend to be rather large. While solid electrolytes cannot leak, they can crack.

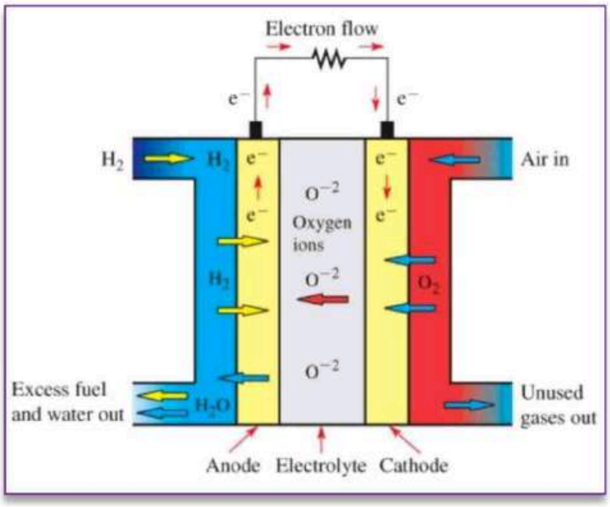


Fig no.14, Solid Oxide Fuel Cell model

Anode reactions:

H2 + O2- → H2O +2e-

CO + O2- → CO2 + 2e-

Cathode reaction:

O2 + 4e- → 22-

Overall reaction:

H2 + O2 + CO → H2O + CO2

Fig no.15, reactions of SOFC

EFFICIENCY: 60%

OPERATING TEMPERATURE: 1000(in degree C)

OUTPUT RANGE: 100 kilowatts

Steps taken by major countries for promote hydrogen energy

☆ **AUSTRALIA**- the country has had an almost negligible presence in green hydrogen market to date. But it is looking to step up its participation considerably as a way of replacing fossil fuel exports with an alternative created with the country's plentiful renewable energy sources.

For example, siemens joined a partnership to develop a 5-gigawatt combined solar and wind project aimed at powering renewable hydrogen production.

- **FRANCE –** green hydrogen was all the rage a year ago in France. In June 2018, then minister for ecological and inclusive Transition Nicolas Hulot vowed to make France a world leader in hydrogen as he unveiled a $117 million investment plan for the technology.

  Hydrogene de France (HDF) was touting a $105 million investment in a hydrogen project in France Guiana . already the country had started many hydrogen powered bus.

- **JAPAN –** the country leads the way in hydrogen fuel-cell vehicle development thanks to the efforts of auto makers such as Toyota and Honda. Policymakers are keen to stimulate green hydrogen as an alternative to liquefied natural gas, which Japan imports more of than any other country. Last month it announced a global action plan to set up 10,000 refueling stations over the next decades.

Fig no.16 , hydrogen car manufacturing factory

  In 2017, Japan formulated the basic hydrogen strategy which sets out the country's action plan till 2030, including the establishment of international supply chain.

- **SOUTH KOREA –** the government is planning to spend $ 1.8 billion in vehicle and refueling station subsidies. South Korea is operating hydrogen energy projects and hydrogen fuel cell production units under the auspices of its 'hydrogen economy development and safe management of hydrogen act,2020'.

  South Korea has also passed economic promotion and safety control of hydrogen act, which deals with three key areas – hydrogen vehicles, charging stations and fuel cells. This law is intended to bring transparency to the national hydrogen princing system.

- **US –** while the US as a whole barely merits a mention in terms of green hydrogen development, one state , California, is racing to become a world-leading market. California's interest in hydrogen is driven partly by aggressive decarbonization targets, including phasing out all the diesel or natural gas powered buses by 2040, partly by the presence of some of the industry's most high profile technology developers.

India scenario about hydrogen energy

- India has a huge edge in green hydrogen production owing to its favorable geographic conditions and presence of abundant natural elements.

- The government has given impetus in scaling up the gas pipeline infrastructure across the length and breadth of the country, and has introduced reforms for the power grid, including the introduction of smart grids. Such steps are being taken to effectively integrate renewable energy un the present energy mix.

- Capacity addition to renewable power generation , storage and transmission, production green hydrogen in India can become cost effective which will not only guarantee energy security, but also ensure self-sufficient gradually.

- India consumes about six million tones of hydrogen every year for the production of ammonia and methanol in industrial sectors, including fertilizer and refineries.

- This could increase to 28 million tones by 2050, principally due to the rising demand from the industry, but also due to the expansion of transport and power sectors.

- By 2030, the cost of green hydrogen is expected to compete with that of hydrocarbon fuels. The price will decrease further as production and sales increase. It is also projected that India's hydrogen demand will increase five fold by 2050,with 80% of it being green.

- India will become a net exporter of green hydrogen by 2030 due to its cheap renewable energy tariffs.
- The union budget for 2021-22 has announced a National Hydrogen Energy Mission (NHM) that will draw up a road map for using hydrogen as an energy source.

Advantages of hydrogen fuel cell

The technology of hydrogen fuel cell presents many advantages over other sources. These are –

Renewable and readily available energy source: Hydrogen is the basic earth element and despite the challenges associated with its extraction from water is a uniquely abundant and source of energy. It is perfect for our future zero-carbon needs for combined heat and power supply.

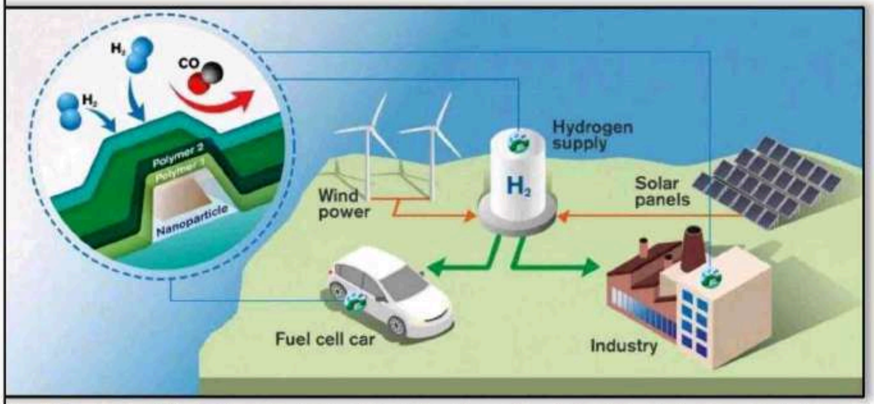


Fig no. 17, whole hydrogen energy supply chain

Practically a clean and flexible energy source: When hydrogen is burnt to produce fuel, by products (heat and water) are totally safe with no adverse environmental impact. After hydrogen is utilized, it is normally converted to drinking water. Aeronautical companies actually use hydrogen as a source of drinking water.

Non-toxic energy: Hydrogen fuel cells are non-toxic fuel source compared to other sources of fuel like nuclear energy, natural gas which are extremely hazardous or daunting to harness safely. Hydrogen cells are friendly towards the environment and does not cause any harm or destruction to human health.

More efficient than fossil fuels and other energy sources:

Hydrogen fuel cell technology provides a high-density source of energy. It is more efficient than many other energy sources, including many green energy solutions. High pressure gaseous and liquid hydrogen have around three times the gravimetric energy density (around 120MJ /Kg) of diesel and LNG and a similar volumetric energy density to natural gas. An automobile that utilizes hydrogen energy will travel more miles than one with an equal amount of gasoline. For example, a based power plant generates electricity at 33-35% efficiency compared to up to 65% for hydrogen fuel cells.

Almost zero emissions: Hydrogen fuel cells do not produce any CO2 emissions during operation, even if their production is not necessarily carbon-free. This gives them an advantage over combustion engine vehicles, which can emit small amounts of poisonous carbon monoxide.

Greatly reduces pollution: Hydrogen fuel cells do not produce noise pollution like other sources of renewable energy such as wind power. It offers a much more silent and smooth alternative to conventional energy production. It do not have the same space requirements meaning that there is less visual pollution too.

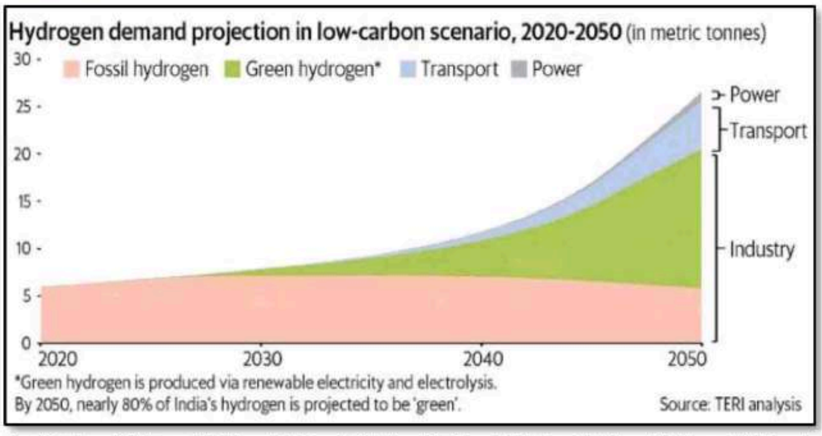


Fig no 18, data by TERI

Fast charging and long usage times: The charging time of hydrogen fuel cell power units is extremely rapid. Where electric vehicles require between 30 minutes and several hours to charge, hydrogen fuel cells can be recharged in under 5 minutes. It provides the same flexibility as conventional cars. Hydrogen fuel cells give greater efficiencies with regard to usage them. A hydrogen vehicle has the same range as those that use fossil fuels (around 300 miles).

Disadvantages of hydrogen fuel cell

There are still a few disadvantages and challenges to use hydrogen fuel cell. Some of the disadvantages of hydrogen energy include:

Extraction of hydrogen: Despite of being the most abundant element in the earth, hydrogen does not exist on its own. It needs to be extracted from water via electrolysis or separated from carbon fossil fuels. This is the real reason, it's not heavily used across the world.

Storage complications: One of the properties of hydrogen is lower density. It is a lot less dense than gasoline. This means that, it has to be compressed to a liquid state and stored at lower temperature. Hydrogen must at all times be stored and transported under high pressure.

Dependent on fossil fuels: Hydrogen energy is renewable and has a minimal environmental impact, but it's separation from oxygen requires the use of fossil fuels such as coal, oil and natural gas. So, fossil fuels are still needed to produce hydrogen fuel.

Investment is required: Hydrogen fuel cells need investment to be developed to the point where they become a genuinely viable energy source. This will also require the political will to invest the time and money into development.

Hydrogen energy is expensive: Electrolysis and steam reforming, the two main processes of hydrogen extraction are extremely expansive. Precious metal such as platinum and iridium are typically required as catalysts in fuel cells, which means that the cost of fuel cells can be high. This expense also impacts costs further down the line, such as with the price of hydrogen operated vehicles.

Not the safest source of energy: Hydrogen is highly flammable and volatile substance that frequently makes headlines for its potential dangers. Compared to gas, hydrogen lacks smell, which makes any leak detection almost impossible.

Infrastructure: As fossil fuels have been used for decades, the infrastructure for this power supply already exists. Large scale adoption of hydrogen fuel cell technology for automotive applications will require new refuelling infrastructure to support it.

Challenges faced by India

1. One of the biggest challenge faced by the industry for using hydrogen commercially is the economic sustainability of the exerting green energy.
2. For transportation fuel cells hydrogen must be the cost competitive with conventional fuels and technologies on a per mile basis.
3. Fuel cells which convert hydrogen fuel to usable energy for cars are still expensive.
4. The hydrogen station infrastructure need to refuel hydrogen fuel cell cars is still widely underdevelopment.

Research & development in India

Initiative by Government—

Ministry of new renewable energy has been supported a board based research development and demonstration programme on Hydrogen energy and fuel. Projects are supported in industrial , academic and research institution to address the challenges in production of hydrogen from renewable energy sources, its safe and efficient storage and its utilization for energy and transport applications through the combustion and fuel cells. With respect to transportation, major work has been supported to Banaras Hindu University, IIT Delhi and Mahindra & Mahindra. This has been resulted in development and demonstration of internal combustion of engines, two wheelers three wheelers and mini buses that run on hydrogen fuel. Two hydrogen refuelling stations have been established – one each at Indian Oil R&D centre , Faridabad and national institution of solar energy, Gurugram.

Fig no. 19, Faridabad hydrogen refuelling station by IOC

Hydrogen and fuel cell (HFC) programme focus to develop transformational technologies that reduce the cost of hydrogen, distribution and storage, diversify and feedstock available for economic hydrogen production, enhance the flexibility of the power grid and reduce emissions through novel uses of low-cost hydrogen.

The programmes aims to reduce the production cost of fuel cell systems to be used in transport applications, while increasing their lifetime to levels which can compete with conventional technologies, increase the electrical efficiency and the durability of the different fuel cells used for power production to levels which can compete with conventional technologies.

Recently, Indian Institute of Technology Guwahati (IIT-Guwahati) researchers are developing novel materials that can use sunlight to split water into hydrogen and oxygen. These materials are much cheaper than the 'noble metals' currently used and could lead to cost-effective solar-powered hydrogen generators.

Initiative by Indian private companies –

- Reliance (RIL) recently partnered with Danish company Stiesdal A/S through its subsidiary Reliance New Energy Solar (RNESL) to develop and manufacture hydrogen electrolyser . They collaborate to further advance the technology development of hydrogen electrolysers. Analysts have estimated that RIL will build a 2.5-gigawatt (GW) electrolyser manufacturing unit.

  The electrolyser factory will manufacture modular electrolysers for the production of green hydrogen for domestic use as well as for international sales. this will be expensive. The cost of green hydrogen is around US $ 3.6-5.8/kg.

- State-owned GAIL (India) also has ambitious plans with respect to green hydrogen. the company has finalised 2-3 sites for the unit including one at Vijaipur in Madhya Pradesh. It will take 12-14 months to set up the plant. The plant planned will have a capacity of 10 MW, the largest announced so far in the country.

  GAIL has already started mixing hydrogen in natural gas in one of the cities, on a pilot basis. The company is testing the mix percentage before it scales it up.

- NTPC plans to produce green hydrogen on a commercial scale.Currently, NTPC is running a pilot project in its Vindhyanchal unit, where the cost of hydrogen is estimated to be around US$2.8-3/kg.

  NTPC also plans to set up its first green hydrogen fuelling station in Leh, Ladakh. It will ply 5 hydrogen buses, to start with. This will put Leh as the first city in the country to implement a green hydrogen-based mobility project.

- Indian oil, the nation's largest fossil fuel retailer recently announced its plans to build a green hydrogen plant at its Mathura refinery in Uttar Pradesh. The unit is likely to have a capacity of around 160,000 barrels per day.

  It will wheel the power from its wind power project in Rajasthan to its Mathura refinery to produce absolutely green hydrogen through electrolysis. It also plans to come up with a stand-alone green hydrogen manufacturing unit in Kochi

Indian Oil has set a target of converting at least 10% of its hydrogen consumption at refineries to green hydrogen soon.

- Engineering major, Larsen & Toubro, also plans to foray into the green hydrogen space. The company has announced that it will set up a green hydrogen plant at its Hazira complex, which is slated to be completed this financial year.
  It also plans to put up a few more green hydrogen plants in its other manufacturing units.

- On 16-dec-2021, Pune-based R&D innovation firm Sentient Labs has demonstrated an indigenously developed Hydrogen Fuel Cell Bus. The start-up is incubated by KPIT Technologies which has collaborated with CSIR (Council of Scientific and Industrial Research)-NCL (National Chemical Laboratory) and CSIR-CECRI (Central Electrochemical Research Institute) to develop the hydrogen fuel cell technology.

Fig no.20, hydrogen powered bus made by Sentient Labs

Hydrogen energy in transport system

HYDROGEN FUEL CELL AUTOMOBILES

DRIVING RANGE – 300 - 400 MILES

TIME TO REFUELLING – 4-5 MIN

HYDROGEN POWERED BICYCLE

RANGE – 60 - 90 MILES

TIME TO REFUELLING – 2-3 MIN

HYDROGEN FUEL CELL TRAIN

RANGE – 900-1000 KM

TIME TO REFUELLING – 15 MINS

HYDROGEN POWERED BOAT (ENERGY OBSERVER), SAME FUEL CELL IN TOYOTA MIRAI SEDAN

8 TANKS WHICH CAN POWERED 6 DAYS AND USES SEA WATER FOR HIS OWN NEED.

HYDROGEN POWERED BOEING EC-003.

IT USES PROTON EXCHANGE MEMBRANE FUEL CELL, FLIES FOR 20 MINS. THIS PROJECT NEEDS TO DEVELOP.

HYDROGEN FUEL CELL BUS

RANGE – 300 MILES

TIME TO REFUELLING – 10 - 15 MINS

HYDROGEN POWERED 3-WHEELERS

IT WAS FIRST DEVELOPED UNDER THE PROJRCT OF UNIDO, ICHET, IITD AND M&M , SUPPORTED BY MNRE

HYDROGEN POWERED SCOOTY

IT HAS A RANGE FROM 50 - 200 KM

THIS SECTOR NEEDS DEVELOPMENT

BIBLIOGRAPHY

For making this project I have taken help from the following—

- https://electricalacademia.com/renewable-energy/fuel-cell-types-working-applications-advantages-disadvantages/
- https://www.ceb.cam.ac.uk/research/groups/rg-eme/Edu/fuelcells/types-of-fuel-cells
- https://dst.gov.in/hydrogen-research-initiative-hri
- https://www.livemint.com/industry/energy/five-indian-companies-leading-the-green-hydrogen-revolution-11636369476063.html
- https://blog.csiro.au/green-blue-brown-hydrogen-explained/
- https://www.google.com/url?sa=t&source=web&rct=j&url=https://en.m.wikipedia.org/wiki/Hydrogen_vehicle%23:~:text%3DA%2520hydrogen%2520vehicle%2520is%2520a,hydrogen%2520fuel%2520for%2520motive%2520power.%26text%3DAs%2520of%2520December%25202020%252C%2520there,reforming%252C%2520which%2520emits%2520carbon%2520monoxide.&ved=2ahUKEwitnPGJlvj0AhXZ7XMBHV8jBUAQFnoECAUQBQ&usg=AOvVaw2hgvWsuankd2uoF6ggpiAZ

Title of the Project

The project submitted, in partial fulfilment of the requirement for the assignments in CC-XI,…CC-XII, DSE-I, DSE-II……. Paper (Semester…5….) in the Department of Physics

Submitted by

(**Name of the student**)

SANKALPA GHOSH

Registration No:

A01-1152-111-025-2019

Supervisor Teacher:

KALYANBRATA CHATTERJEE

THE
BRACHISTOCHRONE
PROBLEM

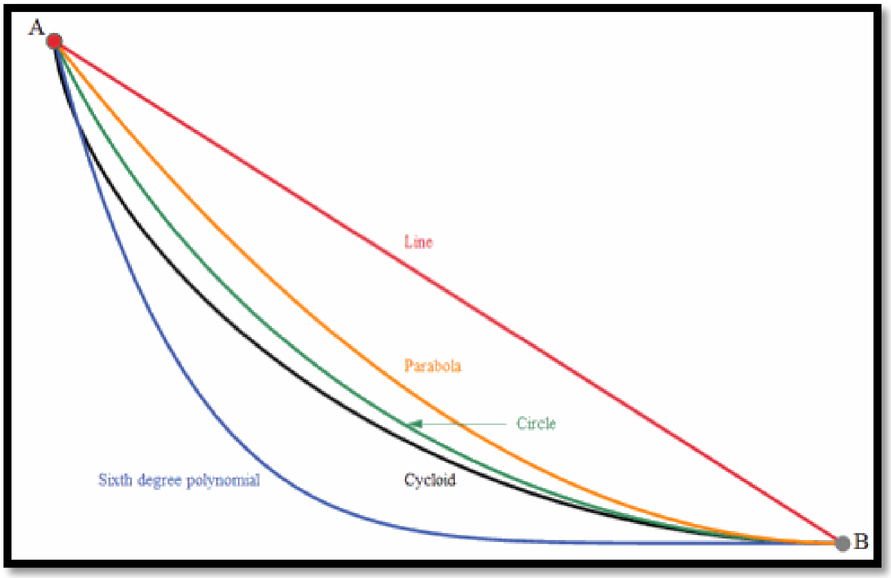
INTRODUCTION:

The brachistochrone problem was one of the earliest problems posed in the calculus of variations . The term derives from two Greek words: '*brachistos'* meaning " the shortest " and '*chronos* ' meaning " time delay."

The problem was given by Johann Bernoulli in 1696 to challenge many scientists and mathematicians wordwide including his own brother Jacob Bernoulli.

The problem as stated by Bernoulli was :

"Given two points A and B in a vertical plane, what is the curve traced out by a point acted on only by gravity, which starts at A and reaches B in the shortest time."



IN FIG: The Brachistochrone Problem: Which path from A to B is traversed in the shortest time?

HISTORICAL BACKGROUND:

Johann Bernoulli posed the problem of the brachistochrone to the readers of '*Acta Eruditorum*' in June, 1696. He said:

> " I, Johann Bernoulli, address the most brilliant mathematicians in the world. Nothing is more attractive to intelligent people than an honest, challenging problem, whose possible solution will bestow fame and remain as a lasting monument. Following the example set by Pascal, Fermat, etc., I hope to gain the gratitude of the whole scientific community by placing before the finest mathematicians of our time a problem which will test their methods and the strength of their intellect. If someone communicates to me the solution of the proposed problem, I shall publicly declare him worthy of praise. "

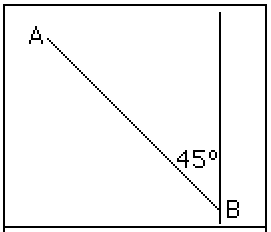
Leibniz persuaded Johann Bernoulli to allow a longer time for solutions to be produced than the six months he had originally intended so that foreign mathematicians would also have a chance to solve the problem.

Five solutions were obtained, Newton, Jacob Bernoulli, Leibniz and de L'Hôpital solving the problem in addition to Johann Bernoulli .

According to Newton's biographer Conduitt, he solved the problem in an evening after returning home from the Royal Mint.

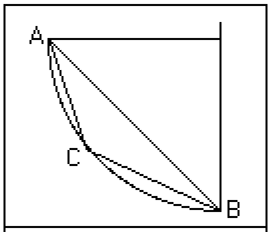
However Johann Bernoulli was not the first to consider the brachistochrone problem. Galileo in 1638 had studied the problem in his famous work '*Discourse on two new sciences*'. His version of the problem was first to find the straight line from a point A to the point on a vertical line which it would reach the quickest.

He correctly calculated that such a line from A to the vertical line would be at an angle of 45 reaching the required vertical line at B .



Galileo's best straight line

He calculated the time taken for the point to move from A to B in a straight line, then he showed that the point would reach B more quickly if it travelled along the two line segments AC followed by CB where C is a point on an arc of a circle.



Galileo's result

Although Galileo was perfectly correct in this, he then made an error when he next argued that the path of quickest descent from A to B would be an arc of a circle - an incorrect deduction.

Johann Bernoulli used the analogous one of considering the path of light refracted by transparent layers of varying density (Mach 1893, Gardner 1984, Courant and Robbins 1996). Johann Bernoulli solved this problem showing that the cycloid which allows the particle to reach the given vertical line most quickly is the one which cuts that vertical line at right angles.

Euler in ' *Methodus inveniendi lineas curvas maximi minimive proprietate gaudentes sive solutio problematis isoperimetrici latissimo sensu accepti* ' published in 1744 generalises the problems studies by the Bernoulli brothers but retains the geometrical approach developed by Johann Bernoulli to solve them. He found what has now come to be known as the Euler-Lagrange differential equation for a function of the maximising or minimising function and its derivative.

Actually, Johann Bernoulli had originally found an incorrect proof that the curve is a cycloid, and challenged his brother Jacob to find the required curve. When Jacob correctly did so, Johann tried to substitute the proof for his own (Boyer and Merzbach 1991, p. 417).

HOW TO SOLVE THE PROBLEM:

The problem of finding the curve has the following assumptions:

- There is no friction in the curve
- The bead starts at rest
- The gravitational field is constant 'g'

The time to travel from a point ' P_1' to another point ' P_2' is given by the integral

$$t_{12} = \int_{P_1}^{P_2} \frac{d\,s}{v}, \qquad (1)$$

Where 's' is the arc length and 'v' is the speed. The speed at any point is given by a simple application of conservation of energy equating kinetic energy to gravitational potential energy,

$$\frac{1}{2}\, m\, v^2 = m\, g\, y, \qquad (2)$$

Giving,

$$\boxed{v = \sqrt{2\, g\, y}\,.} \qquad (3)$$

Putting this into equation (1) together with the identity:

$$d\,s = \sqrt{d\,x^2 + d\,y^2} = \sqrt{1 + y'^2}\; d\,x \qquad (4)$$

Gives:

$$t_{12} = \int_{P_1}^{P_2} \frac{\sqrt{1+y'^2}}{\sqrt{2\,g\,y}}\,d\,x \tag{5}$$

$$= \int_{P_1}^{P_2} \sqrt{\frac{1+y'^2}{2\,g\,y}}\,d\,x. \tag{6}$$

The function to be varied is thus

$$f = \left(1+y'^2\right)^{1/2} (2\,g\,y)^{-1/2}. \tag{7}$$

To proceed, one would normally have to apply the full-blown Euler-Lagrange differential equation

$$\frac{\partial f}{\partial y} - \frac{d}{d\,x}\left(\frac{\partial f}{\partial y'}\right) = 0. \tag{8}$$

However, the function $f\,(y,\,y')$ is particularly nice since x does not appear explicitly. Therefore, $\partial f / \partial x = 0$, and we can immediately use the Beltrami identity

$$f - y'\,\frac{\partial f}{\partial y'} = C. \tag{9}$$

Computing

$$\frac{\partial f}{\partial y'} = y'\left(1+y'^2\right)^{-1/2} (2\,g\,y)^{-1/2}, \tag{10}$$

subtracting $y'\,(\partial f / \partial y')$ from f, and simplifying then gives

$$\frac{1}{\sqrt{2\,g\,y}\,\sqrt{1+y'^2}} = C. \tag{11}$$

Squaring both sides and rearranging slightly results in

$$\left[1+\left(\frac{d\,y}{d\,x}\right)^2\right] y = \frac{1}{2\,g\,C^2} = k^2, \tag{12}$$

(13)

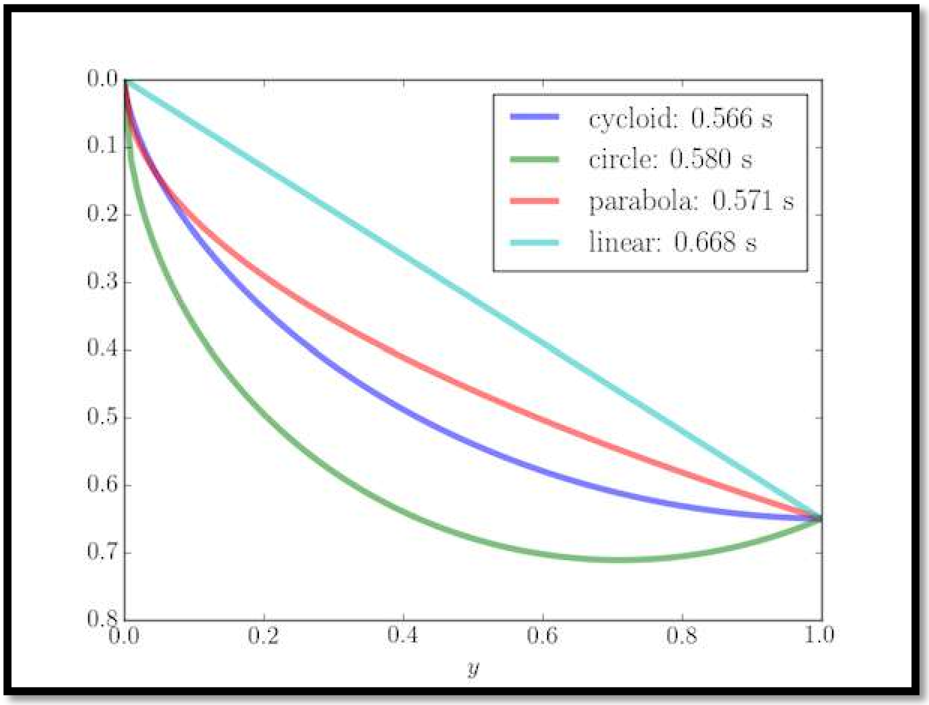
where the square of the old constant has been expressed in terms of a new positive constant. This equation is solved by the parametric equations:

$$x = \tfrac{1}{2} k^2 (\theta - \sin\theta)$$

$$y = \tfrac{1}{2} k^2 (1 - \cos\theta),$$

These are equations of a cycloid.

Paths between P1=(0,0)P1=(0,0) and P2=(1,0.65)P2=(1,0.65):



TAUTOCHRONE

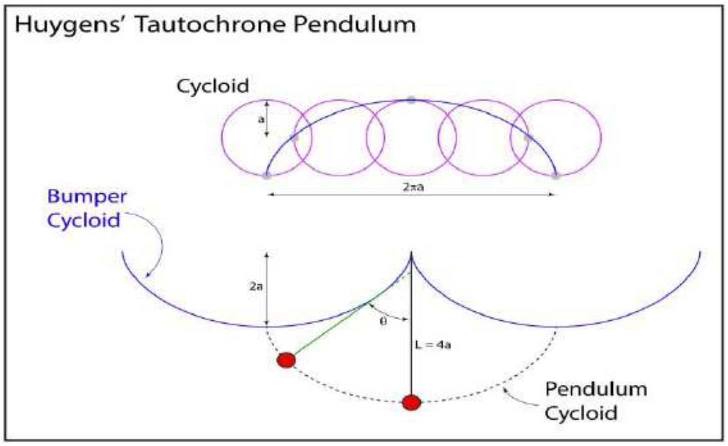
INTRODUCTION:

The term ‘ Tautochrone ’ is derived from 2 greek terms: ‘tauto’- meaning *‘same’* and ‘ chronos ’ meaning ‘ *time* ’. Tautochrone is also called ‘ isochrone ’. It is the curve for which the time taken by an object sliding without friction in uniform gravity to its lowest point is independent of its starting point on the curve..

Huygens had shown in 1659, prompted by Pascal's challenge about the cycloid.

The problem is to find the curve down which a bead placed anywhere will fall to the bottom (to its lowest point), under uniform gravity is in the same amount of time independent of its starting point. The solution is a cycloid, a fact first discovered and published by Huygens in ‘*Horologium oscillatorium* ’ (1673).

Huygens also constructed the first pendulum clock with a device to ensure that the pendulum was isochronous by forcing the pendulum to swing in an arc of a cycloid. This is accomplished by placing two evolutes of inverted cycloid arcs on each side of the pendulum's point of suspension against which the pendulum is constrained to move (Wells 1991, p. 47; Gray 1997, p. 123). Unfortunately, friction along the arcs causes a greater error than that corrected by the cycloidal path (Gardner 1984).



SOLVING THE CURVE:

To see that the cycloid satisfies the tautochrone property, let us consider the derivatives

$$x' = a(1-\cos\theta) \tag{3}$$

$$y' = a\sin\theta, \tag{4}$$

and

$$x'^2 + y'^2 = a^2\left[\left(1 - 2\cos\theta + \cos^2\theta\right) + \sin^2\theta\right] \tag{5}$$

$$= 2a^2(1-\cos\theta). \tag{6}$$

Now

$$\tfrac{1}{2} m v^2 = m g y \tag{7}$$

$$v = \frac{ds}{dt} = \sqrt{2gy} \tag{8}$$

$$dt = \frac{ds}{\sqrt{2gy}} \tag{9}$$

$$= \frac{\sqrt{dx^2 + dy^2}}{\sqrt{2gy}} \tag{10}$$

$$= \frac{a\sqrt{2(1-\cos\theta)}\; d\theta}{\sqrt{2ga(1-\cos\theta)}} \tag{11}$$

$$= \sqrt{\frac{a}{g}}\; d\theta, \tag{12}$$

So the time required to travel from the top of the cycloid to the bottom is

$$T = \int_0^{\pi} dt = \sqrt{\frac{a}{g}}\;\pi. \tag{13}$$

However, from an intermediate point θ_0,

$$v = \frac{d\,s}{d\,t} = \sqrt{2\,g\,(y - y_0)}\,, \tag{14}$$

so

$$T = \int_{\theta_0}^{\pi} \sqrt{\frac{2\,a^2\,(1 - \cos\theta)}{2\,a\,g\,(\cos\theta_0 - \cos\theta)}}\;d\,\theta \tag{15}$$

$$= \sqrt{\frac{a}{g}}\;\int_{\theta_0}^{\pi} \sqrt{\frac{1 - \cos\theta}{\cos\theta_0 - \cos\theta}}\;d\,\theta. \tag{16}$$

To integrate, rearrange this equation using the half-angle formulas

$$\sin\left(\tfrac{1}{2}\,x\right) = \sqrt{\frac{1 - \cos x}{2}} \tag{17}$$

$$\cos\left(\tfrac{1}{2}\,x\right) = \sqrt{\frac{1 + \cos x}{2}}\,, \tag{18}$$

with the latter rewritten in the form

$$\cos\theta = 2\cos^2\left(\tfrac{1}{2}\,\theta\right) - 1 \tag{19}$$

to obtain

$$T = \sqrt{\frac{a}{g}}\;\int_{\theta_0}^{\pi} \frac{\sin\left(\tfrac{1}{2}\,\theta\right)d\,\theta}{\sqrt{\cos^2\left(\tfrac{1}{2}\,\theta_0\right) - \cos^2\left(\tfrac{1}{2}\,\theta\right)}}. \tag{20}$$

Now let us assume a transform variable:

$$u = \frac{\cos\left(\tfrac{1}{2}\,\theta\right)}{\cos\left(\tfrac{1}{2}\,\theta_0\right)} \tag{21}$$

$$du = -\frac{\sin\left(\frac{1}{2}\theta\right)d\theta}{2\cos\left(\frac{1}{2}\theta_0\right)}, \qquad (22)$$

So substituting the value of the trigonometric function with ‘u’ :

$$T = -2\sqrt{\frac{a}{g}}\int_1^0 \frac{du}{\sqrt{1-u^2}} \qquad (23)$$

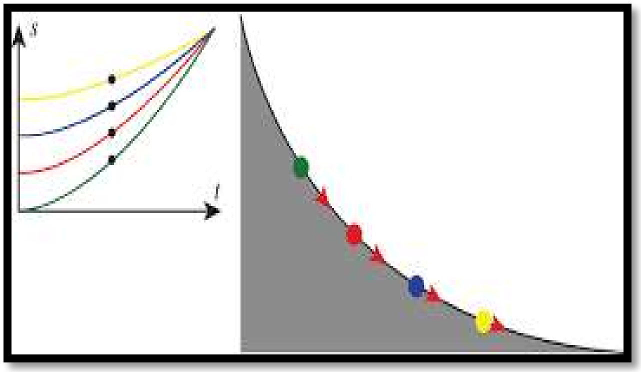
$$= 2\sqrt{\frac{a}{g}}\left[\sin^{-1} u\right]_0^1 \qquad (24)$$

$$= \pi\sqrt{\frac{a}{g}}, \qquad (25)$$

and the amount of time is the same from any point.

CONCLUSION:

The brachistochrone curve is the same shape as the tautochrone curve; both are cycloids. This curve, a cycloid, and the time is equal to π times the square root of the radius However, the portion of the cycloid used for each of the two varies. More specifically, the brachistochrone can use up to a complete rotation of the cycloid (at the limit when A and B are at the same level). In contrast, the tautochrone problem can use only up to the first half rotation, and always ends at the horizontal.



BIBLIOGRAPHY:

1. https://mathworld.wolfram.com/CalculusofVariations.html
2. https://mathshistory.st-andrews.ac.uk
3. https://www.cantorsparadise.com
4. https://wiki.math.ntnu.no
5. https://mathworld.wolfram.com/Tautochrone.html
6.

Title of the Project

A BRIEF STUDY OF RADIO TELESCOPE

The project submitted, in partial fulfilment of the requirement for the assignments in {CC-XI,CC-XII,DSE-I & DSE-II} Paper (Semester V) in the Department of Physics

Submitted by

TANOY SARKAR

Registration No: A01-1112-111-026-2019

Supervisor Teacher: Dr. ANJAN KUMAR CHANDRA

RAMAKRISHNA MISSION VIVEKANANDA
CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

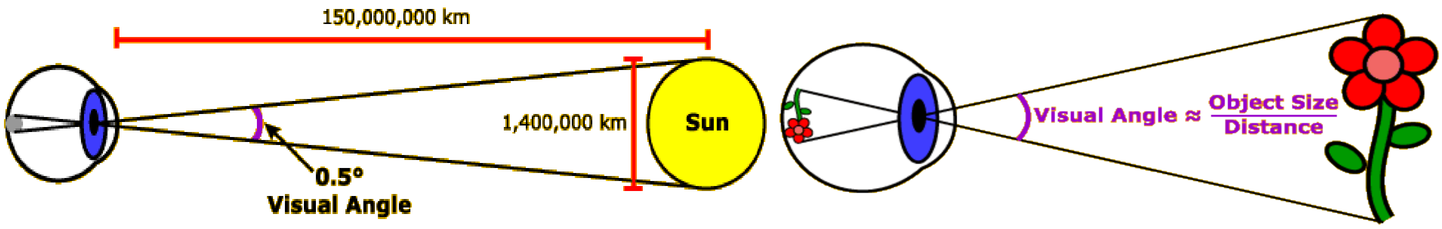
RADIO TELESCOPE

Aim of the project:

In telescope we can fulfil the basic requirement in 1700. When people try to see more-far object then they see the image of the object is unclear then they try to make a batter telescope to get a clear image of this object. When people want to see image of galaxy, pulsar etc. then it need something more advance thing and many astronomical object emits radiation Radio waves range or ultra-violate range so the optical telescope can't detect this radiation. Now Radio telescope come and it is a very special and important to discover the universe.

In this project in first part we see the historical background of the discovery of telescope we see why and how the telescope improve itself with the requirement. In the second part we will see the why this Radio telescope is very important and its work function, Y configuration, very large array and some advanced technology and how we can thought and move forward.

Historical background and some basic of telescope to understand it



In this picture the visual angle of the sun in our eye is 0.5 degree is so small so we see it small but in a telescope it bring this object in front of our eye it can't magnify it. Now assume if it is closer to our eye it make a big visual angle so we see this object big. And the next picture in formula for visual angle if we dresses the distance the visual angle will be increase.

Magnifying power of a telescope:

Magnifying power of telescope is how many times it can bring an object in front of our eye. Let say magnifying power of a telescope is 100 it means that it can bring a far object 100 times in front of our eye.

First discovery of telescope

Hans Lippershey invented first telescope in 1608.

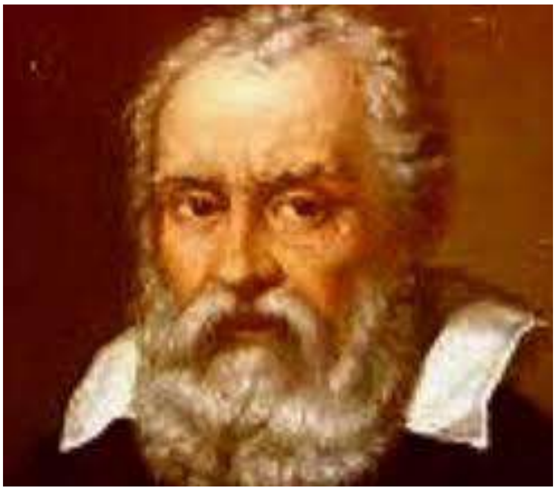
Hans Lippershey

Hans Lipperhey is known for the earliest written record of a refracting telescope, a patent he filed in 1608. His work with optical devices grew out of his work as a spectacle maker, an industry that had started in Venice and Florence in the thirteenth century, and later expanded to the Netherlands and Germany.

In 1609 Galileo Galilei made his own refracting telescope to see heavenly bodies. His telescope can magnified an object 20 times. The design of these early refracting telescope consisted of a convex objective lens and a concave eyepiece.

First invented telescope

Galileo improved on this design the following year and applied it to astronomy. In 1611, Johannes Kepler described how a far more useful telescope could be made with a convex objective lens and a convex eyepiece lens. By 1655, astronomers such as Christiaan Huygens were building powerful but unwieldy Keplerian telescopes with compound eyepieces.



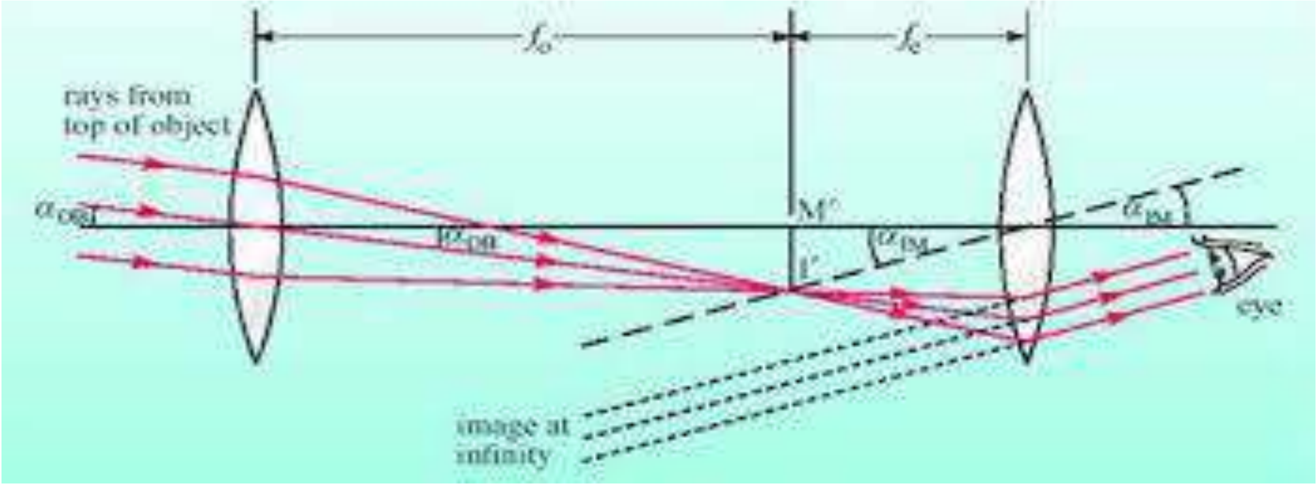
Galileo Galilei

Types of telescope

Refracting Telescope:

First invented telescope was a refracting telescope.in that time this was a most workable telescope. Many scientist were work on this to improve this.

A refracting telescope is a telescope that uses a converging lens to collect the light. A reflecting telescope is made by the combination of two lens one is objective lens(large focal length) which collect parallel ray from infinite distance object and another is eyepiece(small focal length) lens which is close to our eye.



Ray diagram of a refractive telescope

But after some time people see some disadvantage of this telescope and they try to make a different telescope

Disadvantage of refracting telescope and invention of a new telescope

Disadvantage of refracting telescope:

i) We need large objective lens to collect large number of rays to get more data but in case of that the lens should be very large and costly

ii) In case of use large lens there are some problem of spherical aberration

iii) Chromatic Aberration

If we use a mirror in place of objective lens then we can overcome this disadvantage

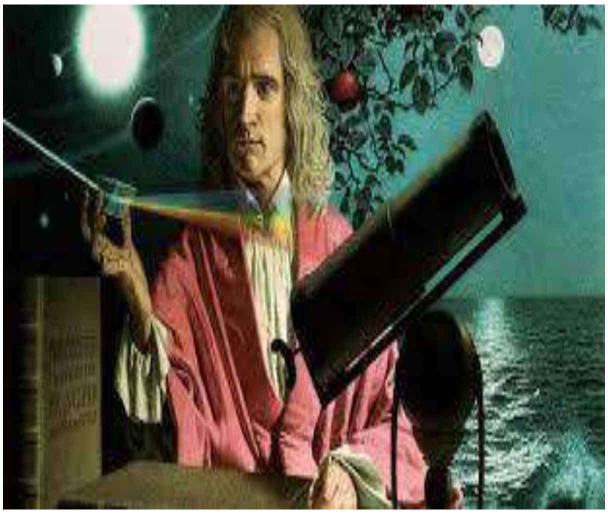
Then people placed an objective concave paraboloidal mirror and make a new telescope reflecting telescope like i) cassegrain telescope

ii) Newtonian telescope

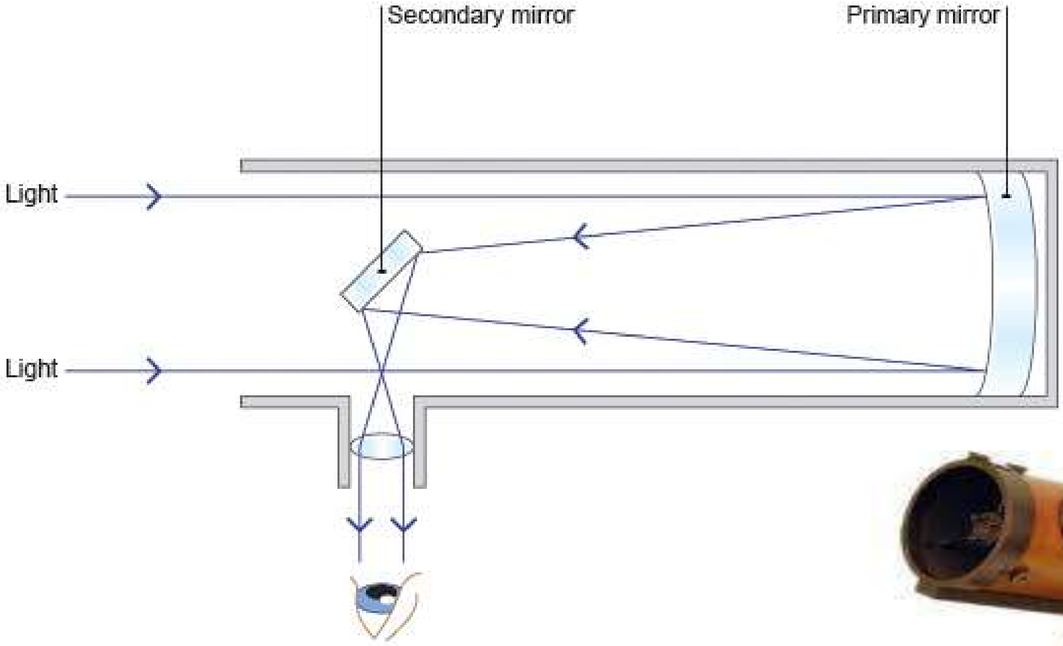
Reflecting telescope

Sir Isaac Newton was first discover of the reflecting telescope in 1668.

For reflecting telescope we place *an objective concave paraboliodal mirror in place of objective lens.* A reflecting telescope is a telescope that uses a single mirror or a combination of curved mirrors that reflect light and form an image.

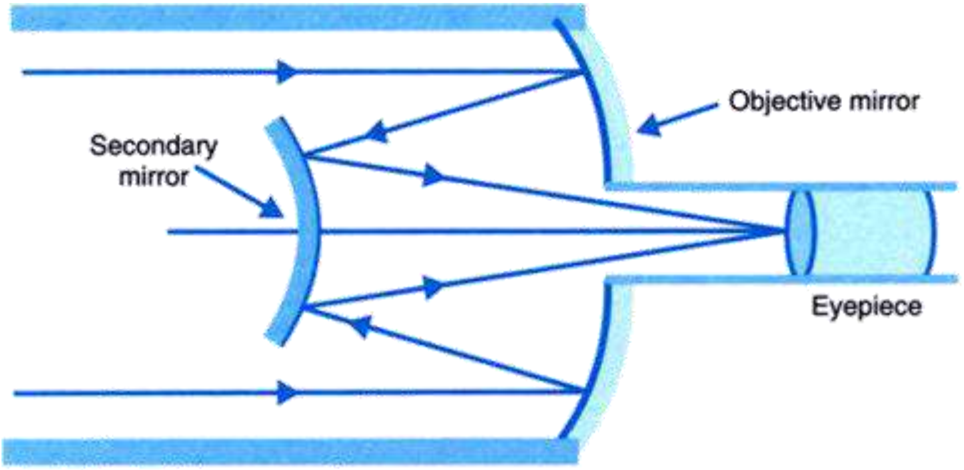


Sir Isaac Newton



Ray diagram of Newtonian telescope

Reflective telescope of Newton



Ray diagram of *cassegrain* telescope

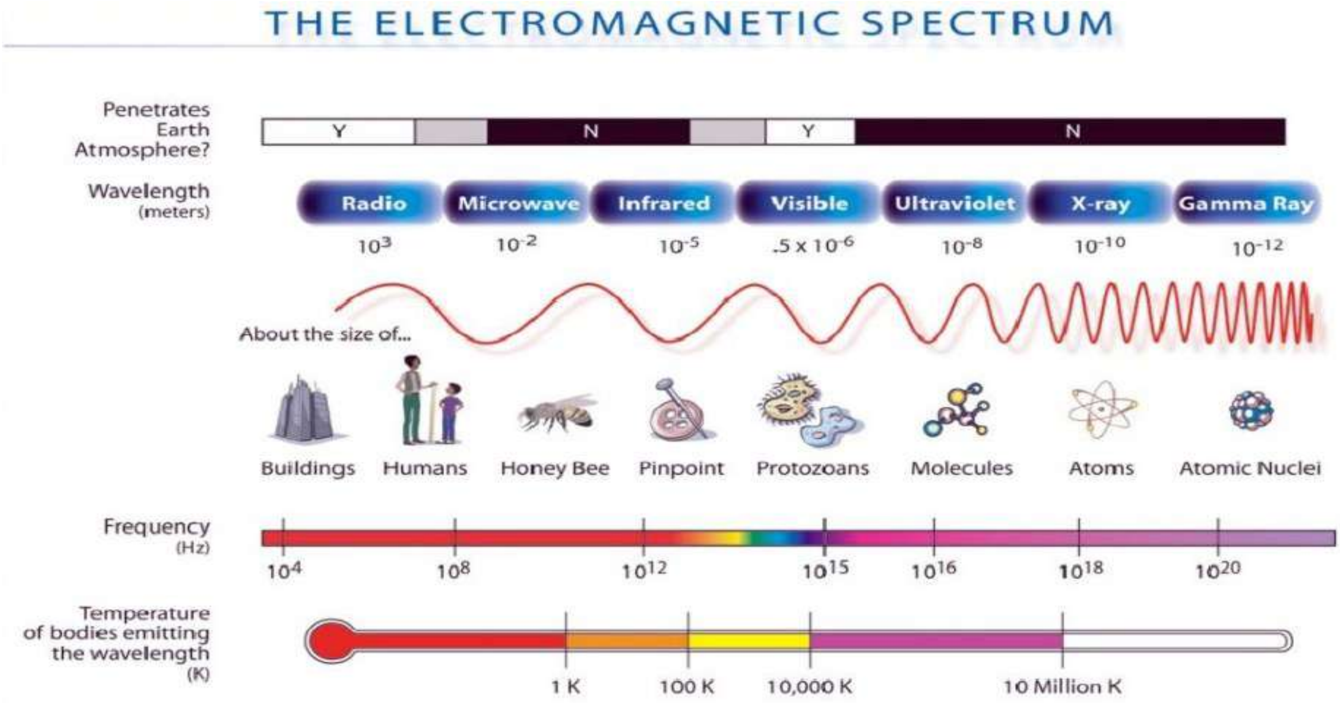
What is the Requirement of the Advance Telescope?

Till now we say only about optical telescope but when we want to get some clear picture of star nebula or some astronomical object we have to go to some advance telescope.

Many astronomical object emits radiation more strongly at lower wavelength than visible wavelength so if we want to get picture of this object we have to make telescope which can collect the wavelength and by some computation we get a clear image of this astronomical object.

Optical signal which is emits from an astronomical object can affected by earth atmosphere so we need a telescope that can collect another signal like x-ray, gamma ray, radio waves

Different objects emits different wavelength radiation. According to Wins Displacement law in increases of temperature wave length of the radiation decrease.

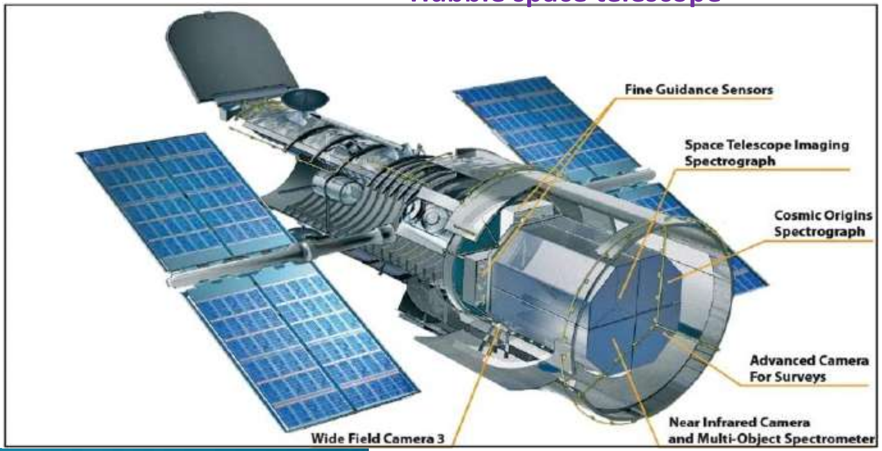


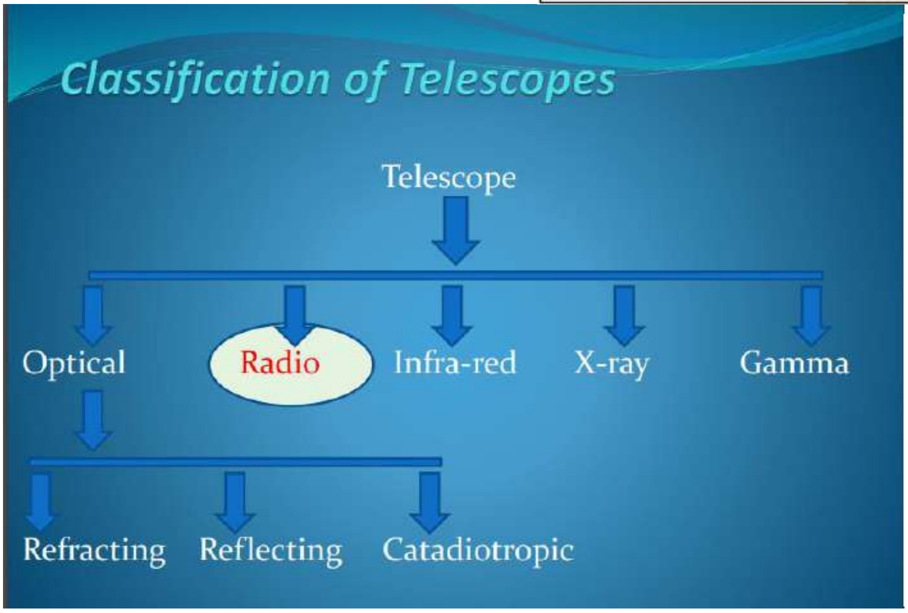
Some image of Advance Telescope

Hubble space telescope

Hubble space telescope

Celestron Advance VX 6





Classification of Telescopes

Telescope

Optical

Radio

Infra-red

X-ray

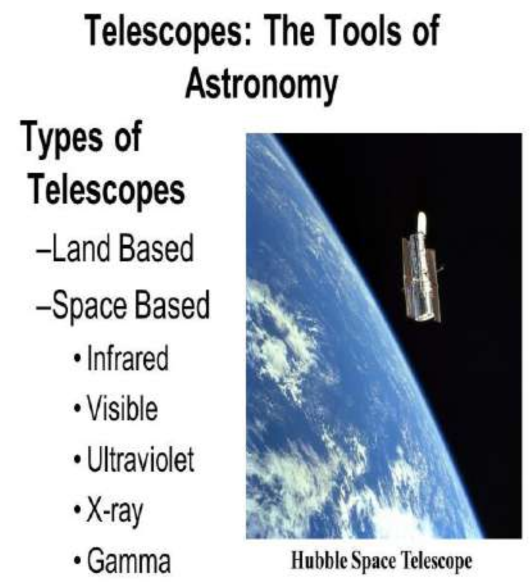
Gamma

Refracting

Reflecting

Catadiotropic

Different types of telescope



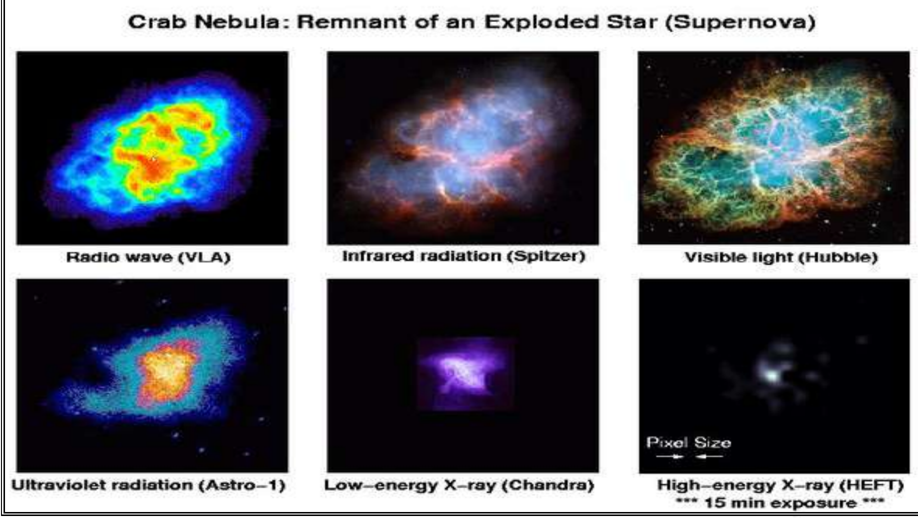
Telescopes: The Tools of Astronomy

Types of Telescopes
- –Land Based
- –Space Based
  - Infrared
  - Visible
  - Ultraviolet
  - X-ray
  - Gamma

Hubble Space Telescope

Different types of Space telescope

Some image of Advance Telescope



Crab Nebula: Remnant of an Exploded Star (Supernova)

Radio wave (VLA)

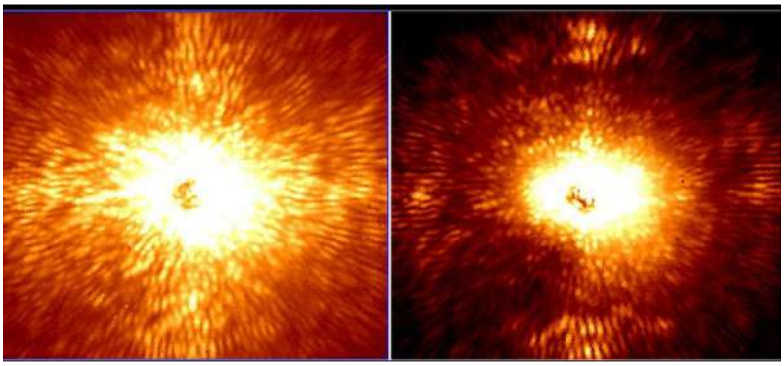
Infrared radiation (Spitzer)

Visible light (Hubble)

Ultraviolet radiation (Astro–1)

Low–energy X–ray (Chandra)

High–energy X–ray (HEFT)
*** 15 min exposure ***



Hubble's Slice of Sagittarius

Radio Telescope

Why we chose radio waves and radio telescope

☐ Many astronomical objects emit radiation more strongly at longer wavelengths (1 mm to 1o m) than at visible wavelength.

☐ Radio waves generally obey all the same laws as other electromagnetic waves including reflection, refraction and diffraction. These three properties are especially important when it comes to understanding how radio telescopes function.

☐ The angle at which a radio wave is reflected from a dish will equal the angle at which it approached the surface.

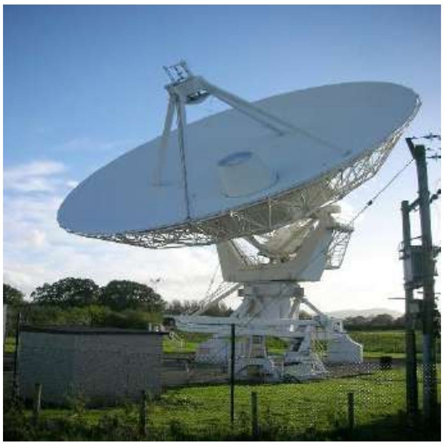
☐ Naturally occurring radio light from planets, stars, nebula, galaxies are collected, brought at a focus, amplified and analysed.

☐ When it passes through regions of varying pressure, temperature and water content, it will diffract and take different paths through the atmosphere.

☐ Propagation of radio waves in space is free, because they do not experience a strong influence of a medium, unfortunately the Earth's atmosphere is a solid barrier to much of the electromagnetic radiation reaching us from space. It absorbs most of the wavelengths smaller than the ultraviolet, most of the wavelengths between the infrared and microwaves, and most of the longest radio waves. This only leaves visible light, some ultraviolet and infrared, and the shortest radio waves to penetrate the atmosphere and bring information about the universe. The radio window ranges from about 30MHz to over 300 GHz, which corresponds to wavelengths of almost 100 m down to 1 mm.

What do radio telescope do?

- Track and collect data from satellites
- Monitor radio signals from outer space
- Detect weather, other planets stars and other astronomical objects.



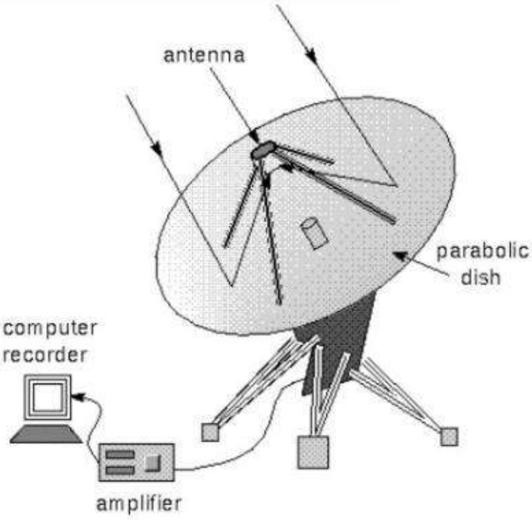
Basic Radio telescope

What is radio telescope?

- It is very sensitive radio receiver
- Measures the intensity of radio waves
- Shoots a small beam to detect the signals

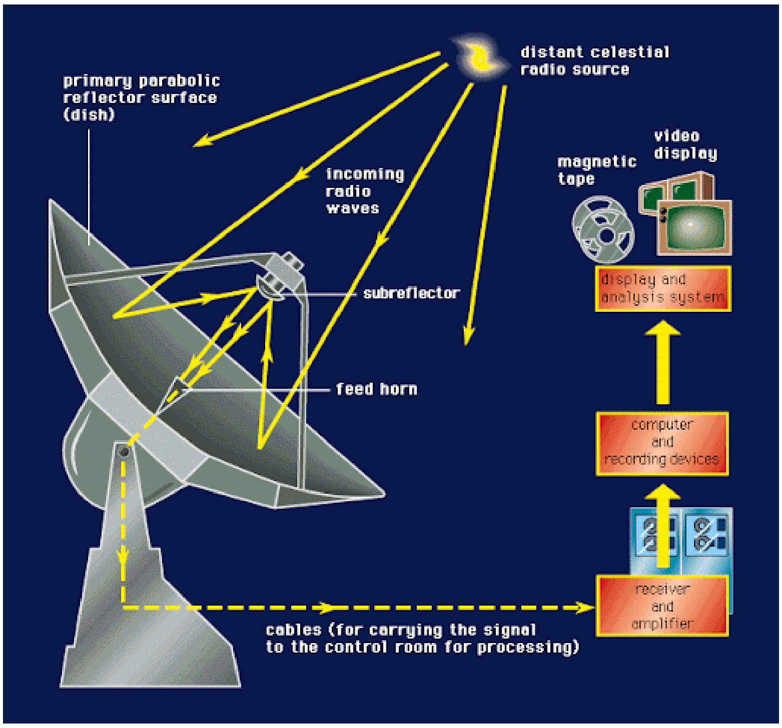
How radio telescope made

- A parabolic shaped dish
- An antenna mounted on top of the dish
- An amplifier is used gain a stronger signal
- All the information is sent to a computer



How radio Telescope work

- The dish reflected radio waves to the antenna
- The antenna sends the signal to the amplifier
- The information is recorded onto a computer
- Uses a parabolic shaped dish that reflects radio waves to an antenna.
- The signal is sent to an amplifier to magnify the signal
- The signal is processed by a computer to turn the signal into a picture



First radio Telescope

The first telescope was made by Karl Jansky on 1931

He recorded thunderstorms and categorized them in different categories.

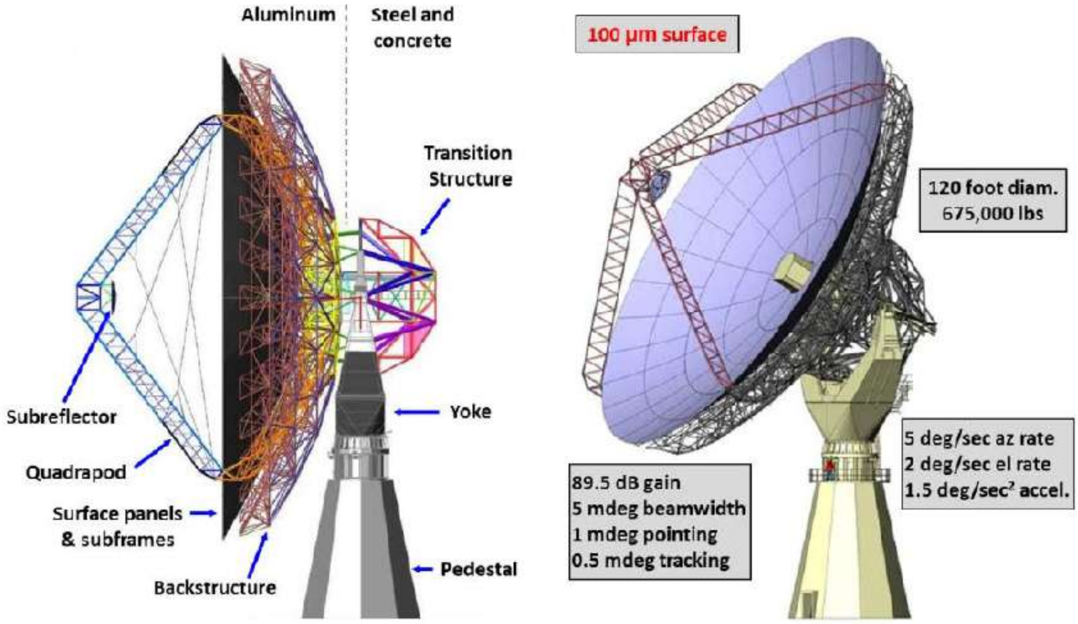
Karl Jansky

Many types of radio telescope in our world. There are different in sizes and designs

First radio telescope

Haystack radio telescope

- Controlled by remote
- Used mainly for education and space tracking
- Usually smaller telescope



a new HUSIR 36.6 m diameter Cassegrain antenna design has identical dimensions to the old antenna

Westford Radio Telescope

- Dome shaped telescope
- Made in 1961
- Used for communication technology

World Biggest radio telescope

- Located in Arecibo , Puerto Rico
- 1,001 foot dish
- It has remote antenna

Very large array

The Very Large Array is the most versatile, widely- used radio telescope in the world. It can map large-scale structure of gas and molecular clouds and pinpoint ejections of plasma from supermassive black holes.
The Karl G. Jansky Very Large Array (VLA) is a centimeter-wavelength radio astronomy observatory located in central New Mexico on the Plains of San Agustin, between the towns of Magdalena and Datil, ~50 miles (80 km) west of Socorro.

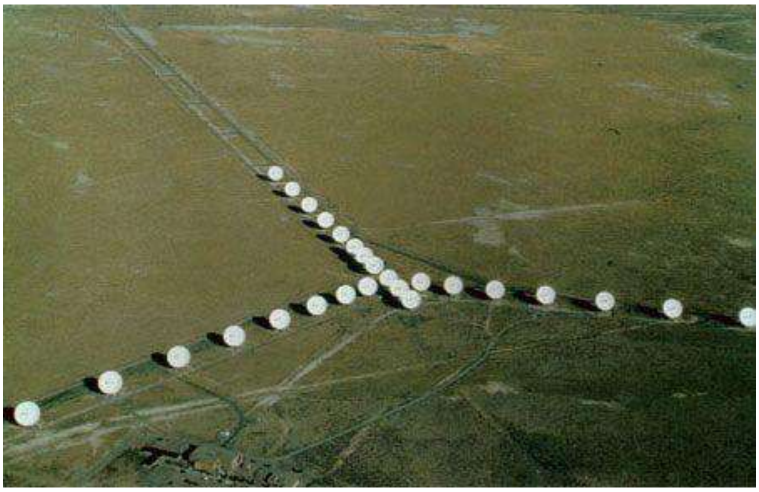
Giant Metre-wave Radio Telescope

Astronomers from all over the world regularly use this telescope to observe many different astronomical objects such as HII regions, galaxies, pulsars, supernovae, and Sun and solar winds. In August 2018, the most distant galaxy ever known, located at a distance of 12 billion light years, was discovered by GMRT.

The large size of the parabolic dishes implies that GMRT will have over three times the collecting area of the Very Large Array (VLA) in New Mexico, USA which consists of 27 antennas of 25 m diameter and is presently the world's largest aperture synthesis telescope operating at centimeter wavelengths.

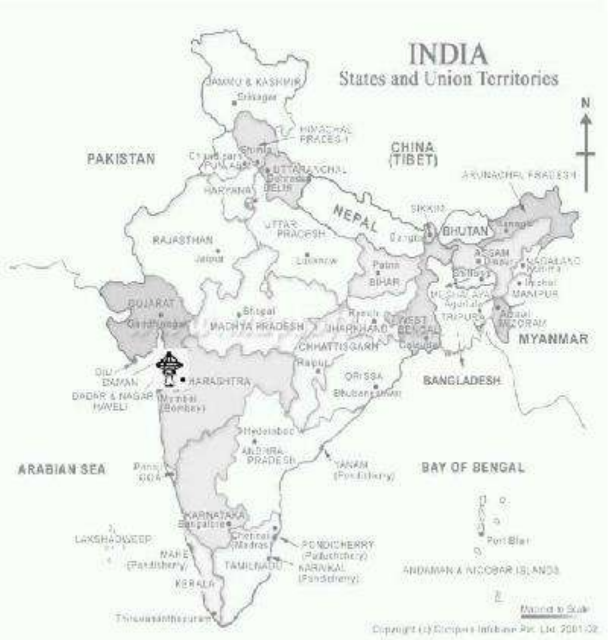
Y configuration of GMRT

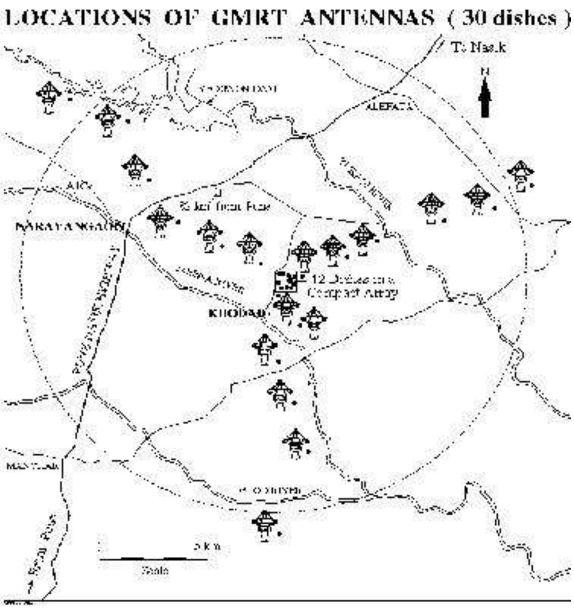
There are several reasons why the antennas that make-up the Very Large Array (VLA) are arranged in a Y-configuration. The Y-configuration produces a nearly circular resolution element ("synthesized beam" in radio astronomy parlance) on the sky for a variety of integration times and for most positions on the sky. Also, a Y-configuration is the most efficient arrangement of antennas which need to be connected to a central power and communication center. Finally, the Y-configuration makes it possible to expand and contract the array to allow for a variety of spatial resolution sizes ("synthesized beam sizes" in radio astronomy parlance) on the sky.



Location of GMRT

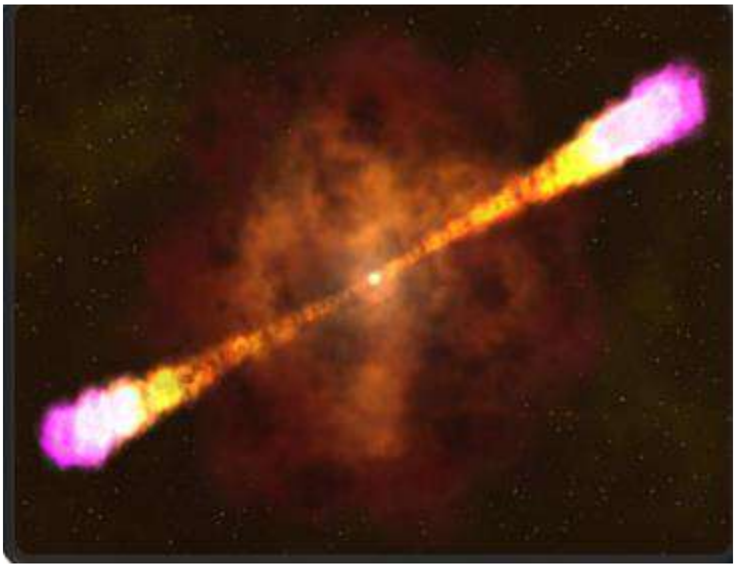
Giant metre-wave Radio telescope (GMRT) is situated in different place but they are situated in Y shaped.



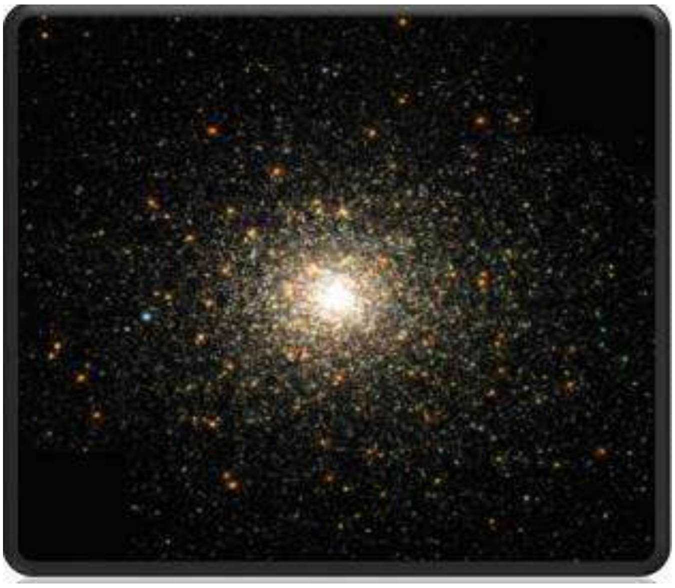


Some observation or image of GMRT

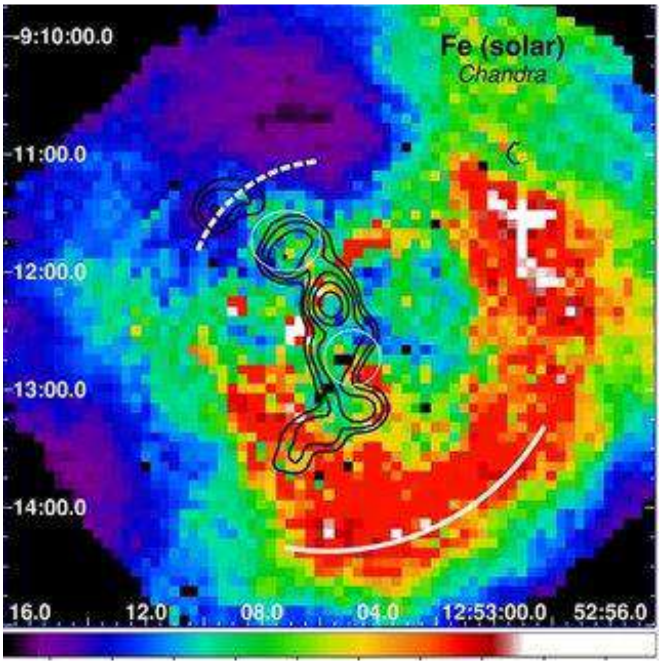
Some plusars is discover with GMRT



Gamma Ray Burster GBR030329 burst



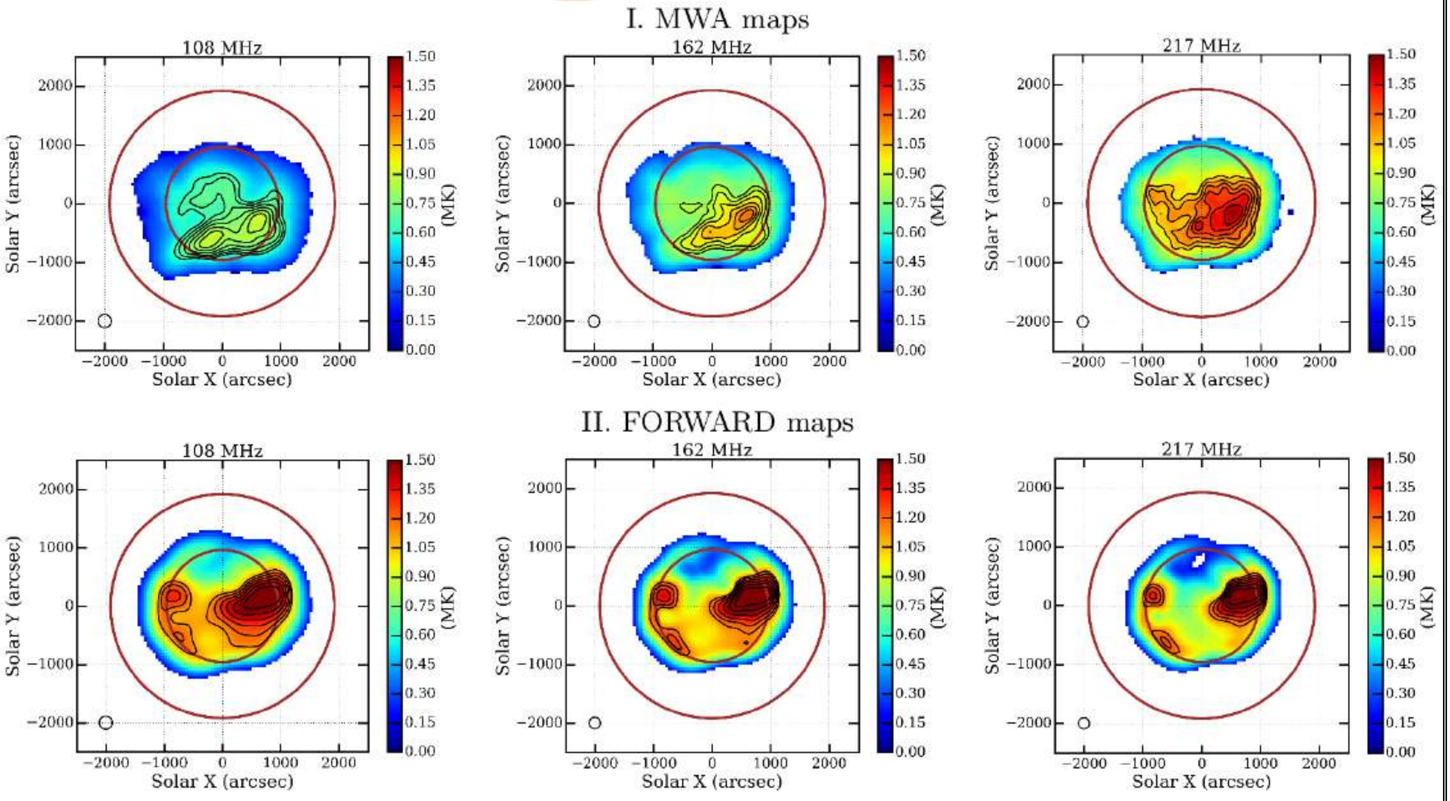
NGC 1851



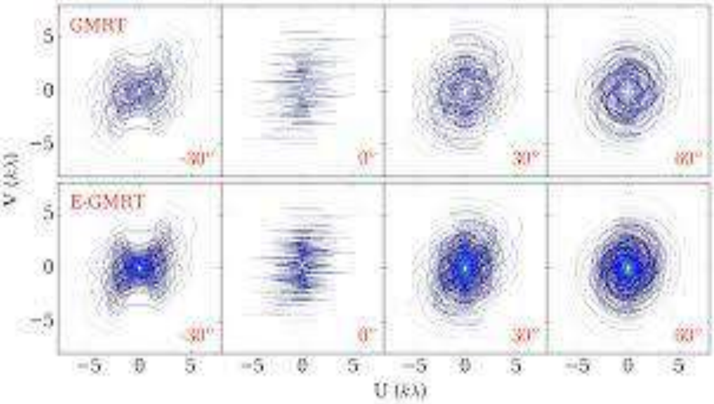
Chandra iron abundance map

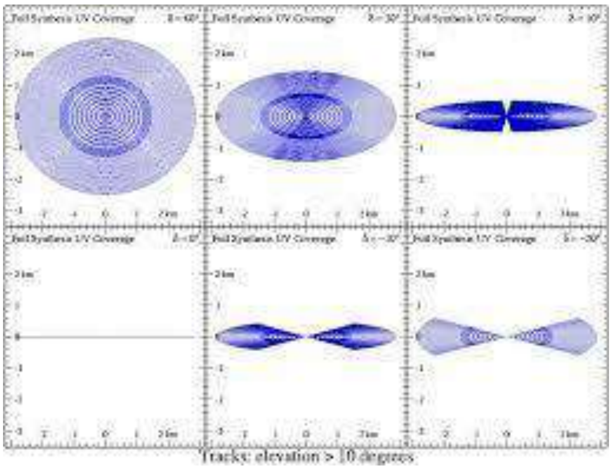
Great red spot of Jupiter

Solar Observation Image



Some metre-Wavelength observation





Conclusion:

Radio Telescope is an important and advance subject to know the universe. In help of this we can see very far object and we can also see the astronomical object who are not strongly emits visible light because of its temperature. Now in next we hope the technology of radio telescope improved more and we get a stronger telescope.

ACKNOWLEDGEMENT

I would like to express my special thanks of gratitude to my professor

Dr. Anjan Kumar Chandra for his able guidance and support in completing project. I would also like to extend my gratitude to him to give me this opportunity to do this project in this interesting topic and he also encourage me for this project work.

REFERENCE

- https://en.wikipedia.org/wiki/History_of_the_telescope
- https://www.le.ac.uk/ph/faulkes/web/telescopes/r_te_types.html
- https://en.wikipedia.org/wiki/Radio_telescope

TITLE OF THE PROJECT

A BEGINNER’S STUDY ON STELLAR EVOLUTION

The project submitted, in partial fulfilment of the requirement for the assignments in ***CC-XI, CC-XII, DSE-I, DSE-II*** *Paper (* ***Semester V*** *) in the Department of Physics.*

Submitted by

SAIKAT SAHA

Registration No: **A01-1152-111-027-2019**

Supervisor Teacher: DR. PALASH NATH

RAMAKRISHNA MISSION VIVEKANANDA
CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

STARS AND THEIR VARIOUS CATEGORIES

The very first thought arises in our mind as soon as we hear the word "**STARS**" is the vast collection of tiny dots of light everywhere lit up on a clear night sky.

Quoting **Carl Sagan** one of the renowned astronomers of the late twentieth century, "**We are made of star stuff.**"

Hence, stars are an indispensable part of our lives, but, however the study of this familiar object is not quite so familiar.

Now, beginning with this study, we should first try to categorise the stars-----

Considering our nearest star, our Sun as the reference, we can broadly categorise stars into three groups-

i) **Massive Stars or Supergiant**: These are the stars having mass more than about 10 to 100 solar masses.
ii) **Medium sized or Sun-like stars**: These are the stars having mass less than about 10 to 0.5 solar masses.
iii) **Low mass stars**: These are the stars having mass of less than about 0.5 solar masses.

Now, let us discuss the properties of these three groups of stars----

Firstly, considering massive stars---

i) Temperature: The surface temperature range spans from about 20000 K to over 50000 K.
ii) Density: Average density is of the order of $1.3x10^{-7}$ g/cm^3.
iii) Size: Average size varies from 70 to 100 solar masses.
iv) Luminosity: Average luminosity is of the order of $184000x10^{26}$ W.

Now, considering Sun-like stars---

i) Temperature: The surface temperature ranges from 6000 K to over 15000 K.
ii) Density: Average density is of the order of 10^{-7} g/cm^3.
iii) Size: Average size varies from 0.2 to about 20 solar masses.
iv) Luminosity: Average luminosity is of the order of $12000x\ 10^{26}$ W.

Last, but not the least, low mass stars---

i) Temperature: The surface temperature is less than about 5000 K.
ii) Density: Average density is of the order of more than 10^{-7} g/cm^3.
iii) Size: Average size varies 0.01 to about 0.7 solar masses.
iv) Luminosity: Average luminosity is of the order of less than 10000x10^{26} W.

Apart from these three types of stars, there are also other types of stars such as variable stars, binary stars, multiple stars, etc.

One of the most common aspects in the stellar study is the composition of stars. It is quite an astonishing fact that the types of stars discussed above and other stars present in the universe are mainly composed of hydrogen along with a little helium, the lightest gases present in the nature.

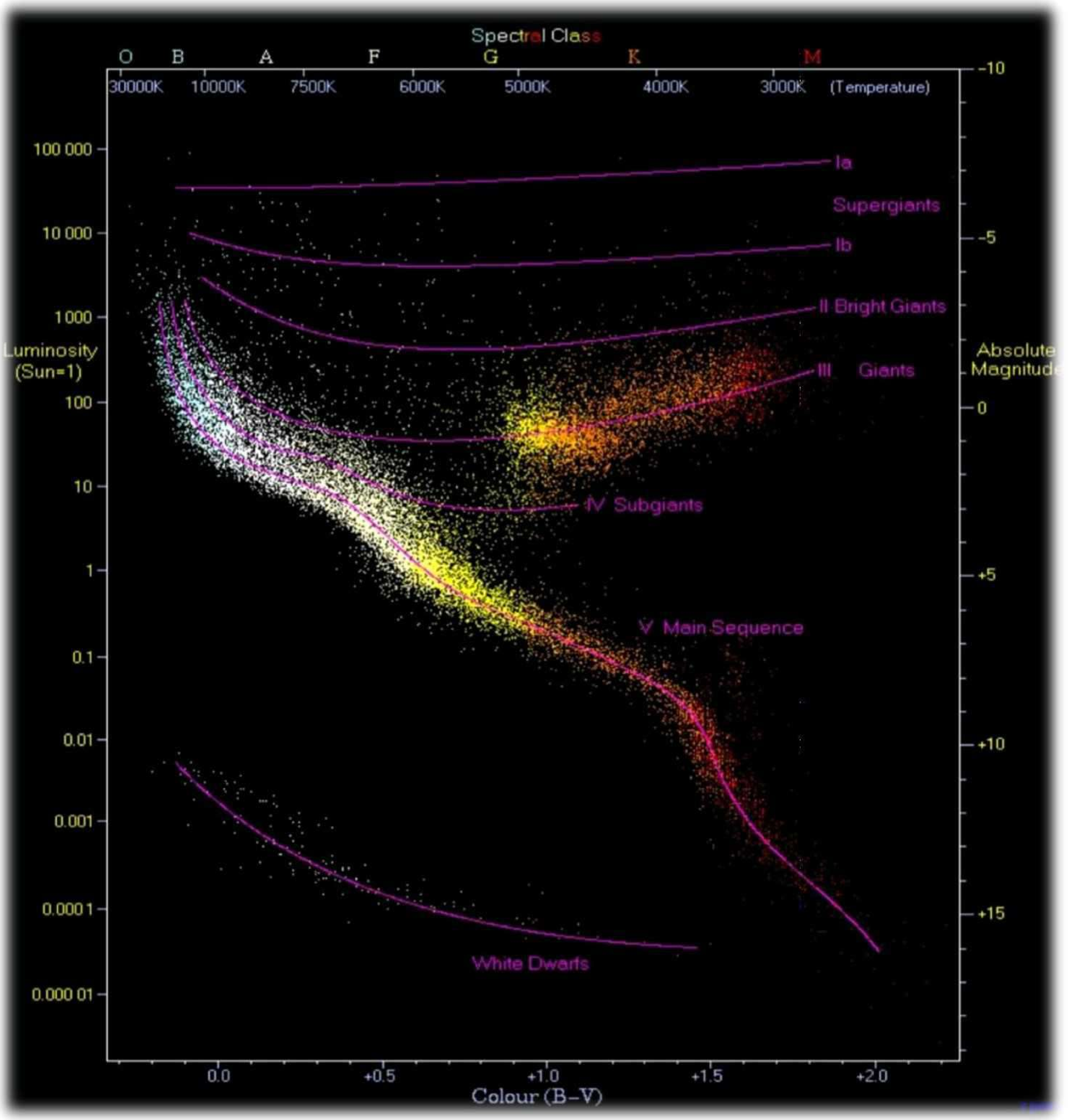
Another quite astonishing fact is that there is an order in the properties of stars!!!

The order is generally represented by the famous Hertzsprung-Russell diagram

The Hertzsprung-Russell Diagram

The Hertzsprung-Russell diagram, abbreviated as HR diagram is a scatter plot of stars showing the relationship between the star's absolute magnitudes or luminosities versus their stellar classifications or effective temperatures. It generally shows (brightness) as a function of temperature (spectral class);the ordinate "absolute magnitude" is a logarithmic measure of power. The diagram was created independently around 1910 by **Ejnar Hertzsprung** and **Henry Norris Russell**, the same person who developed the Russell-Saunders coupling.

Most of the stars lie on the "main sequence": massive stars are hot and have high power, while the small stars have lower masses, are cold and have low power.



The giant stars lie on the top-right part of the diagram, while the white dwarfs are in the bottom-left.

STAR BIRTH: !!A STAR IS BORN!!

Star formation or star birth is the process by which dense regions within molecular clouds in interstellar space, sometimes referred to as “stellar nurseries” collapse and form stars.

Interstellar clouds are generally consists of molecular hydrogen (H2) gas and dust grains.

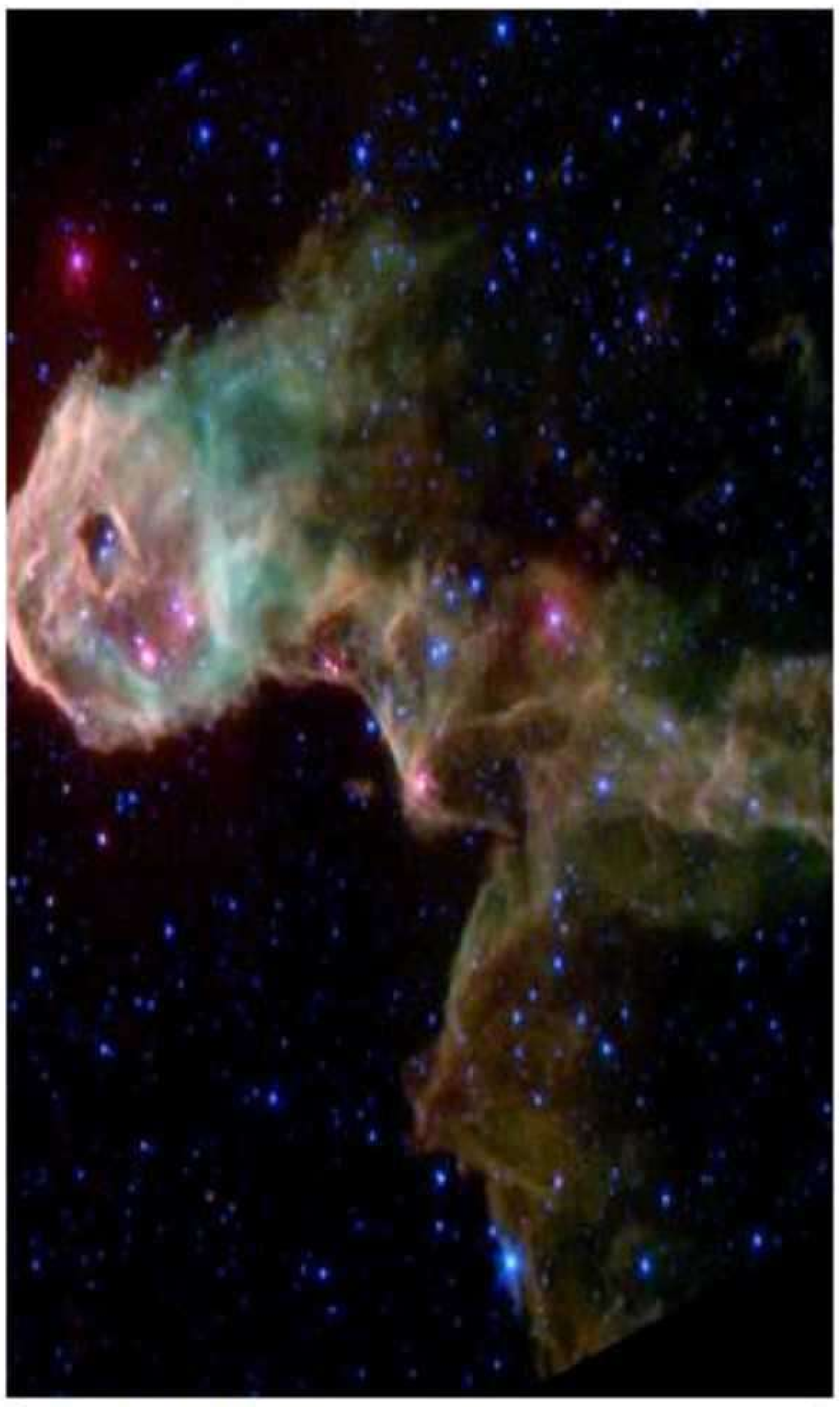
Masses are generally in the order of 10^5 to 10^6 times the Sun’s mass.

Sizes consists of ~150 light years in diameter.

These clouds are generally very cold:~10-30 K

Stars form when gravity overcomes thermal pressure.

Gravity causes dense cores in molecular clouds to collapse.



Stellar Nursery
Picture Courtesy: NASA

Collapsing cores heat up and when the core gets hot enough, fusion begins and stops the shrinking.

New star achieves long-lasting state of balance.

As a branch of astronomy, star formation includes the study of the interstellar medium(ISM) and giant molecular clouds (GMC) as precursors to the star formation process, and the study of protostars and young stellar objects as its immediate products. Most stars do not form in isolation but as part of a group of stars referred as star clusters or stellar associations.

Source of energy of stars: !!STAR IS ACTIVE!!

NUCLEAR FUSION

The energy source for all stars is nuclear fusion. Stars are made mostly of hydrogen and helium which are packed so densely in a star that in the star's centre, the pressure is great enough to initiate nuclear fusion reactions.

Proton-Proton Chain

It is the main process of fusion in the Sun and other Sun-like stars.

At high temperatures and densities, in stars like our Sun, protons overcome the electrostatic repulsion between them, and form ^{2}H(deuterium) and neutrino (ν).

Later, another proton is coupled with deuterium to form ^{3}He.

Later, the ^{3}He kernel adds another proton to form helium.

The carbon-nitrogen-oxygen cycle

In massive stars, with very hot nucleus, protons can collide with a ^{12}C (carbon) nucleus.

This begins a circular sequence of reactions in which finally four protons fuse to form a helium nucleus

A ^{12}C nucleus is recovered again at the end of the cycle, therefore it is not created nor destroyed; it acts as a nuclear catalyst.

Dynamical Equilibrium: !!STAR IS STABLE!!

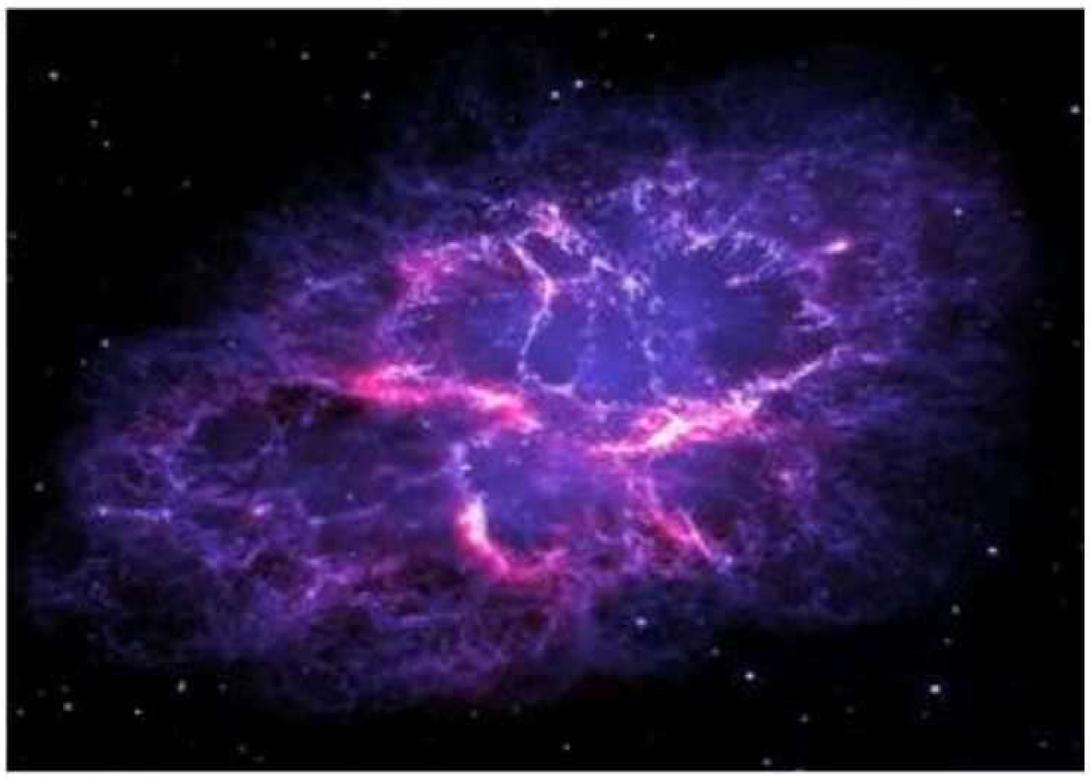
There is an equilibrium between the inward pull of gravity and the outward push of gas pressure, radiation pressure.

Without fusion, a star would collapse under its own weight.

The evolution of a star is basically a tug of war between gravity and nuclear reactions.

Stars are born from a contracting cloud of interstellar matter. As the cloud contracts, gravitational potential energy is released. Part of this energy is used to heat up the gas; in this way the cloud becomes hotter than its surroundings and starts to radiate energy away. Hence the first nuclei to react are those with the lowest charge, i.e., hydrogen, starting when the temperature reaches a few million degrees.

$$4\ {}^{1}H \rightarrow {}^{4}He + 2e^{+} + 2\nu_{e}$$



Crab Nebula
Picture Courtesy: Herschel Space Observatory

Once, hydrogen burning in the core has been established, the contraction of the stars stops. Stars in this phase of their evolution are said to be on the main sequence.

Now, after long time, hydrogen burning stops in the core, the core contracts, releasing gravitational energy and heating up. The star is now proceeding towards the Red Giant phase!!!

Decay and death of stars: !!THE FINAL DAYS OF THE STAR!!

For low mass stars like our Sun, at the point when hydrogen burning stops in core, but continues in the shell around it; the core contracts, while the outer parts of the star expand drastically and cool, until the star becomes a red giant, with a radius that may be as large as the distance between the Sun and the Earth. As in the case of the initial contraction, the contraction of the core may be halted when its temperature becomes high enough for helium to react, to produce carbon:

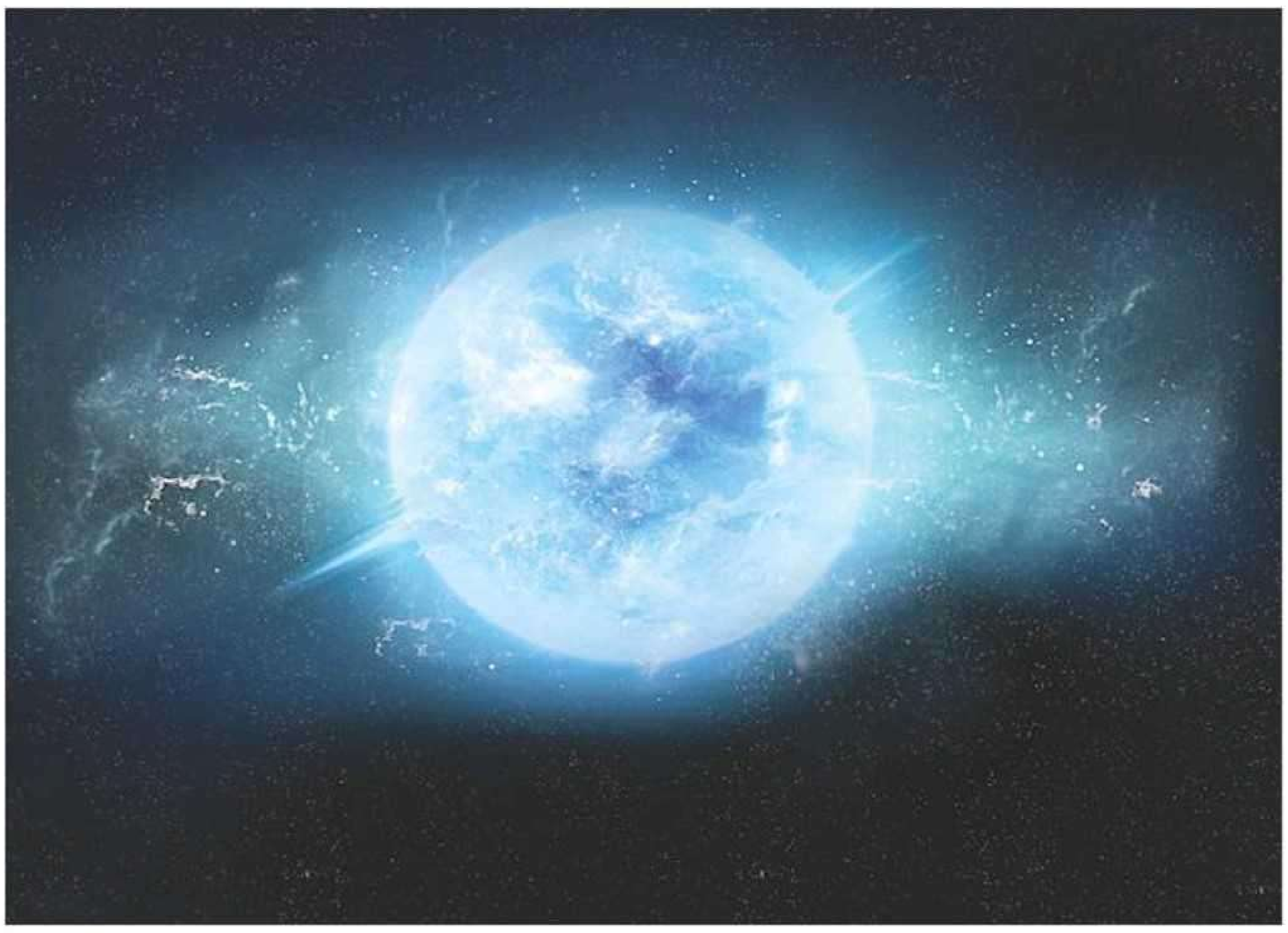
$$3\ {}^{4}He \rightarrow {}^{12}C \text{ , possibly followed by}$$

$${}^{4}He + {}^{12}C \rightarrow {}^{16}O$$

When helium is exhausted in the core, the history to some extent repeats itself: gravity again gets the upper hand, and the core, which now consists mainly of carbon and oxygen, contracts and heats up.

But if the mass of the star is less than 10 solar masses, the core never becomes hot enough for the next nuclear reactions to start. The core continues to contract and the outer layers expand until the star enters a second red giant phase. It is thought that this instability leads to the loss of large amounts of mass from the star, undoubtedly aided by the large luminosity and radius of the star. The mass that is lost ends up a planetary nebula and is later dispersed into the interstellar medium; this leaves behind the carbon- oxygen core which at that point has contracted to a radius comparable with the radius of the Earth, but is still extremely hot; such an object is observed as a white dwarf.

For more massive stars, the carbon-oxygen core heats up sufficiently to start the next type of nuclear reactions. The star continues through a sequence of nuclear burning phases of gradually heavier elements, interspersed by phases of gravitational contraction. The burning of heavier and heavier elements has to end when the material of the core has been transformed into elements of the iron group. At that point gravity has won in the core; the core continues to contract and heat up, until the temperature gets so high enough that the iron nuclei are dissociated into protons and neutrons. At that point the core can contract no further; the result is a bounce which propagates out through the outer parts of the star as a shock wave, expelling them in a supernova explosion, in which the star for a few days becomes as luminous as all the stars n a normal galaxy combined.



White Dwarf

The fate of the core depends on its mass. If the core mass is less than about 2 solar masses, a stable configuration is formed consisting almost entirely of neutrons; this has a radius of only about 10 km. If the core mass is greater, even the pressure of neutrons cannot withstand gravity, and the core collapses into a black hole, where matter is essentially crushed out of existence. Gravity smiles at last!!

CONCLUSION

After all this study, it is quite evident that stars which are quite a major part of our survival, have lives quite similar to ours. They born, constantly fight with surroundings throughout their lives and ultimately meet the ultimate fate of universe, death.

ACKNOWLEDGEMENT

In the accomplishment of this project successfully, many people have given me their blessings and the heart pledge support. Now I am utilizing this time to thank all the people who have been concerned with this project.

Primarily for being able to complete this project with success, I would like to thank our physics teacher **Dr. PALASH NATH**, whose valuable guidance has ones that helped me to complete this project with full success. Their instructions and suggestions help me very much.

Besides I would like to thank my parents and friends who have helped me with their valuable suggestions and guidance. These all help me to complete the whole project.

REFERENCES

1.Lecture Notes on Stellar Structure and Evolution

Jørgen Christensen-Dalsgaard

Institut for Fysik og Astronomi, Aarhus Universitet

2.Stellar Evolution: Birth, Life, and Death of Stars

John R. Percy

International Astronomical Union

University of Toronto, Canada

3. Image Courtesy: NASA, Caltech, Herschel Space Observatory

4.www.wikipedia.com

.ENTROPY AND PHASE TRANSITION.

The project submitted, in partial fulfilment of the requirement for the assignments in PHSA CC-11,CC-12,DSE-1,DSE-2 Paper (Semester-5.) in the Department of Physics

Submitted by

Tushar Kanti Bera

Registration No:-A01-1112-111-029-2019

Supervisor Teacher:-Sankhasubhra Nag

RAMAKRISHNA MISSION VIVEKANANDA
CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

- RAMAKRISHNA MISSION
VIVEKANANDA
CENTENARY COLLEGE.

- NAME-TUSHAR KANTI BERA.
- DEPRTMENT-PHYSICS.
- REG.NO-A01-1112-111-029-2019
- COLLEGE ROLL NO-142.
- SEM-5TH.
- EXAM ROLL-2022151026.

Project-Entropy information theory and Phase transition

SECTION-A(ENTROPY).

Introduction:-.

There are many ways to define entropy. All of them are equivalent, although it can be hard to see.

The original definition of entropy, due to Clausius, was thermodynamic. Clausius noted that entropy is a function of state, we can calculate the entropy difference between two states by connecting them however we like. If we find a reversible path to connect them, then the entropy change is determined simply by the heat absorbed.

$$\Delta S_{\text{system}} = \int_{\text{rev.}} \frac{dQ_{\text{in}}}{T} \qquad \text{(Clausius entropy)}$$

This definition of entropy (change) called Clausius entropy. It is a thermodynamic, rather than statistical-mechanics, definition.

The most important and most famous property of entropy is that it never decreases.

In statistical mechanics, we can define the entropy as-

$$S = k_B \ln \Omega \qquad \text{(Boltzmann entropy)}$$

where Ω is the number of microstates compatible with some macroscopic parameters (E; V ; N). This form is usually attributed to Boltzmann, although it was Planck who wrote it down in this form for the first time. We'll call this the Boltzmann entropy since it's on Boltzmann's gravestone.

Another way to compute entropy came from considering the number of ways N particles could be split into m groups of sizes n_i.This number is$\Omega=(N! /n_1!.n\ \ !.)$Expanding for large n_i gives.

$$S = -k_B N \sum_i f_i \ln f_i$$

where fi =(ni /N)Since ∑ni = N then ∑ fi = 1 and so fi has the interpretation of a probability: fi are the probabilities of finding a particle picked at random in the group labeled i. With the factor of KB but without the N, the entropy written in terms of probabilities is called the Gibbs entropy.

$$S = -k_B \sum_i P_i \ln P_i \qquad \text{(Gibbs entropy)}$$

Entropy of mixing:-

Although entropy is a theoretical construction it cannot be directly measured. it is nevertheless extremely useful. In fact, entropy can be used to do work. One way to do so is through the entropy of mixing. Say we have a volume V of helium with N molecules and another volume V of xenon also with N molecules (both monatomic gasses) at the same temperature. If we let the gasses mix, then each expands from V to 2V.The energy of each is constant so the entropy change of each is-

$$\Delta S = N k_B \ln \frac{2V}{V} = N k_B \ln 2$$

So we get the total entropy change of-

$$\Delta S = 2N k_B \ln 2$$

This increase Called entropy of mixing .
The entropy of mixing is a real thing, and can be used to do work. For example, say we had a vessel with xenon on one side and helium on the other, separated by a semipermeable membrane that lets helium pass through and not xenon. Say the sides start at the same temperature and pressure. As the helium inevitably diffuses through the membrane, it dilutes the helium side lowering its pressure and adds to the pressure on the xenon side. The net effect is that there is pressure on the membrane. This pressure can be used to do work. The pressure when there is a mixed solution (like salt water) in which the solute (e.g. salt) cannot penetrate a semi-permeable barrier (e.g. skin) is called osmotic pressure. For example, when you eat a lot of

salt, your veins increase salt concentration and pull more water in from your body to compensate, giving you high blood pressure.
In chemistry and biology, concentration gradients are very important. A concentration gradient means the concentration of some ion, like Na+ or K+ is not constant in space. When there is a concentration gradient, the system is not completely homogeneous, so entropy can be increased by entropy of mixing. Only when the concentrations are constant is there no way to increase the entropy more. Concentration gradients are critical for life. Neurons are when the concentration of certain chemicals or ions passes a threshold. If systems always work to eliminate concentration gradients, by maximizing entropy, how do the concentration gradients develop? The answer is work! Cells use energy to produce concentration gradients. There are proteins in the cell wall that work to pump sodium and potassium (or other ions) from areas of low concentration to areas of high concentration.

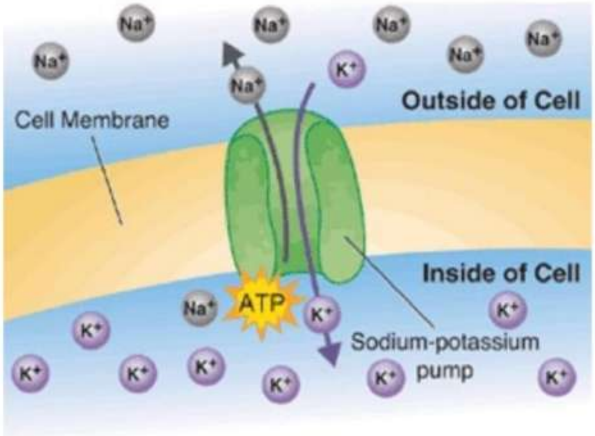


Fig-a cellular pump, present in practically every cell of every living thing. It uses energy in the form of ATP to establish concentration gradients.

Another familiar effect due to the entropy of mixing is how salt is able to melt ice. The salt draws water out of the ice (i.e. melts it) because saltwater has higher entropy.

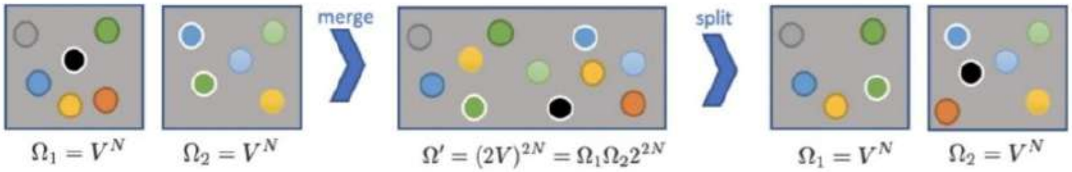
MIXING EXAMPLE DETAILS:-

Since distinguishability is an important and confusing topic, let's do an example of mixing and unmixing in full detail, with three different assumptions.

1. N colored balls + N more colored balls, all distinguishable.
2. N molecules helium + N more molecules helium, all indistinguishable.
3. N molecules helium + N molecules xenon.

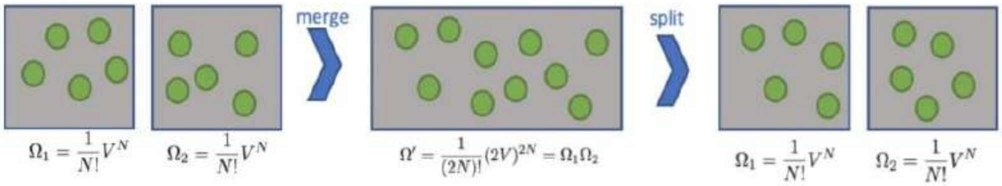
In all cases, we star with a volume V of each, then let them mix, then put a partition in to separate them.

For the first case we have something like this-



So it looks like the entropy goes up byΔ S = 2Nln2 when the two are mixed and then down by ΔS = -2N ln 2 when they are split. However, note that there are Ω(split)=2^^{2n}ways to split them. So in enumerating the final states, we should include this factor, writing Ω'' $=\Omega_1\Omega_2\Omega$split$=V^nV^n2^{2n}$ so that ΔS = 0 upon the split. If we actually look in the box and enumerate which balls are where, we lose split, but the entropy of the surroundings must go up due to the counting..

We can contrast this to the pure helium case when.

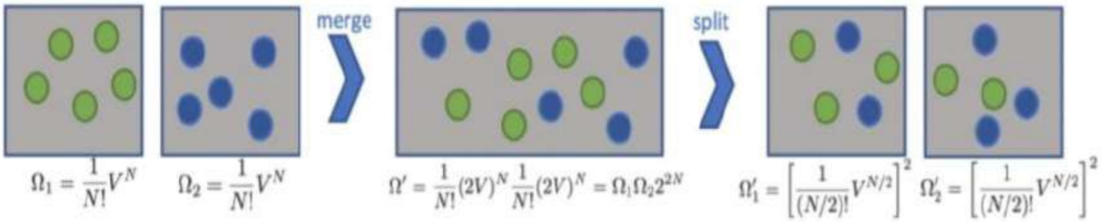


Here, we add the(1/N!) factors for identical particles. When the two sets are merged. then the number of states is

$$\Omega' = \frac{1}{(2N)!}(2V)^{2N} = \Omega_1\Omega_2\frac{N!N!}{(2N)!}2^{2N} \approx \Omega_1\Omega_2\frac{N^N N^N}{(2N)^{2N}}2^{2N} = \Omega_1\Omega_2$$

So ΔS = 0 upon merging. Similarly, $\Omega''=\Omega_1\Omega_2=\Omega'$, so ΔS = 0 upon splitting.

When we mix helium and xenon we have.



this case, after mixing, each set of N molecules occupy the volume 2V , so the entropy of mixing is ΔS = 2Nln 2, just as in the colored balls case. When we split them, since the particles are identical, there is no way to tell apart one splitting from the other. Each half has (N/2)of each species in a volume V/2. So the total number of states in this case is

$$\Omega'' = \left[\frac{1}{\left(\frac{N}{2}\right)!}V^{N/2}\right]^4 = \Omega_1\Omega_2\frac{N!N!}{\left(\frac{N}{2}!\right)^4} \approx \Omega_1\Omega_2\frac{N^N N^N}{\left(\frac{N}{2}\right)^{2N}} = \Omega_1\Omega_2 2^{2N} = \Omega'$$

And therefore ΔS = 0 for the splitting case. So we see that in the distinguishable case (colored balls) or the helium/xenon mixture case, there is a 2Nln2 entropy of mixing each of the 2N molecules now has an extra binary choice of where to be, so we get 2Nln 2. In no situation does entropy go down when we split the volume back into two halves.

SECTION-B(Phase Transitions).

Introduction:-

some phases of matter are familiar from everyday experience: solids, liquids and gasses. Solid H_2O (ice) melting into liquid H_2O (water) is an example of a phase transition. You may have heard somewhere that there are 4 phases of matter: solid, liquid, gas and plasma. A plasma is an ionized gas, like the sun. I don't know why plasmas get special treatment though perhaps it's the old ideaof the four elements. In fact, there are thousands of phases. For example, ferromagnetic is a phase,like a permanent iron magnet. When you heat such a magnet to high enough temperature, it under- goes a phase transition and stops being magnetic. Conductors, insulators, and semiconductors are also phases of matter. At very very high temperatures, nuclei break apart into a quark-gluon phase. Solids generally have lots of phases, determined by crystal structure or topological properties. For example, diamond and graphite are two phases of carbon with different lattice structure.

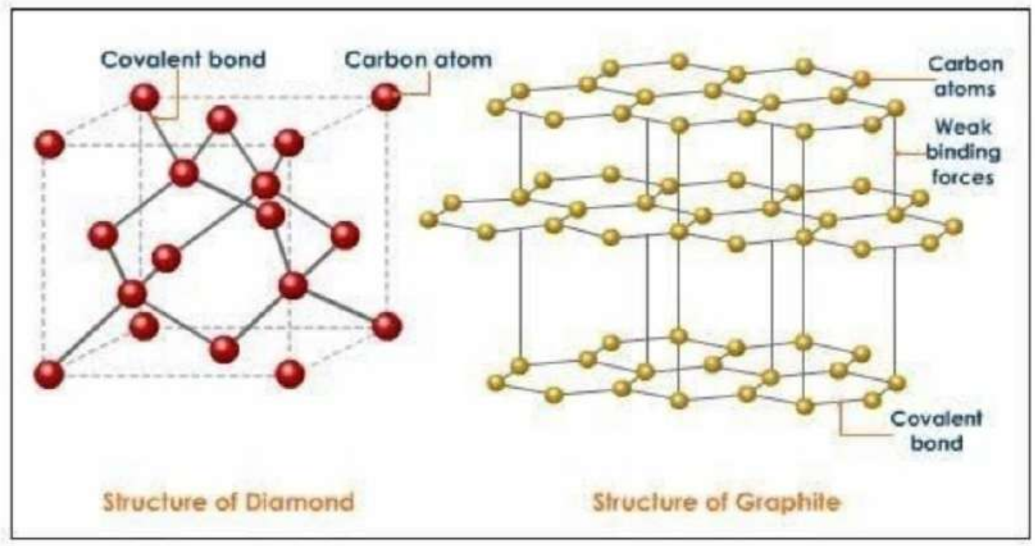


Fig-Two of the phases of solid carbon.

A more technically precise definition is A phase is a state of matter whose properties vary smoothly (i.e. it is an analytic function of P ; V ; T etc).

Latent heat-. To increase the temperature of asubstance, one needs to apply heat,and how much heat is needed can be calculated from the heat capacity because adding heat to the substance increases its entropy. The gradient of entropy with temperature is related to the heat capacity.

$$C_x = T\left(\frac{\partial S}{\partial T}\right)_x,$$

where x is the appropriate constraint (e.g.p, V , B etc).

Now consider two phases which are in thermodynamic equilibrium at a critical tem-perature Tc. Very often, it is found that to change from phase 1 to phase 2 at a constant temperature Tc, you need to supply some extra heat,known as the latent heat L, which is given by

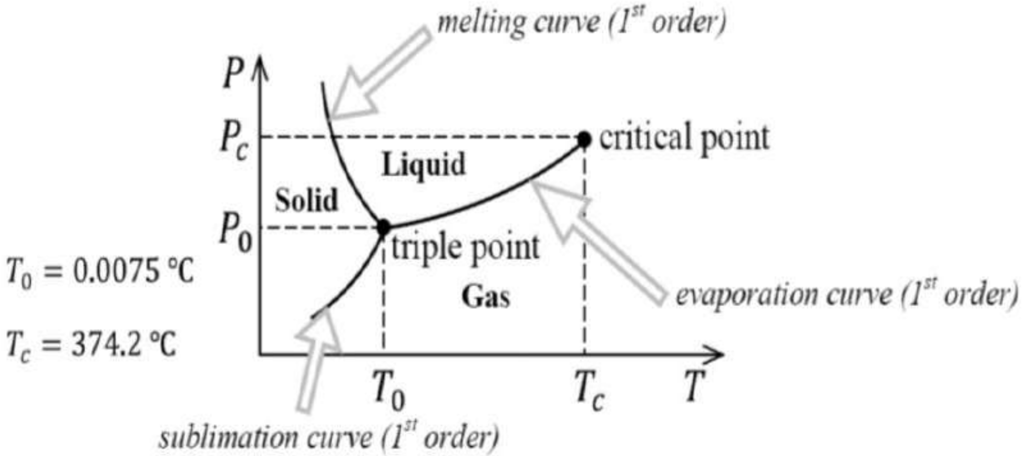
L = ∆Qrev = Tc(S2 − S1),

where S1 is the entropy of phase 1 and S2 is the entropy of phase 2. This, together with equation implies that there will be a spike in the heat capacity Cx as a function of temperature.

Phase transition of solid , liquid,gasses:-.

Gas, liquid, and solid are known as the three states of matter or material, but each of solid and liquid states may exist in one or more forms. Thus, another term is required to describe the various forms, and the term phase is used. Each distinct form is called a phase, but the concept of phase defined as a homogeneous portion of a system, extends beyond a single material, because a phase may also involve several materials. A solid has a definite shape and volume. A liquid has a definite volume but it takes the shape of a container whereas a gas fills the entire volume of a container. You already know that diamond and graphite are solids made up of the element carbon. They are two phases of carbon, but both are solids.Solids are divided into subclasses of amorphous (or glassy) solids and crystalline solids. Arrangements of atoms or molecules in crystalline solids are repeated regularly over a very long range of millions of atoms, but their arrangements in amorphous solids are somewhat random or short range of say some tens or hundreds of atoms. In general, there is only one liquid phase of a material. However, there are two forms of liquid helium, each have some unique properties. Thus, the two forms are different (liquid) phases of helium. At a definite temperature and pressure, the two phases co-exist.

First and second order phase transitions. States of matter come with their stability regions, the phase diagram. The properties of the microscopic state change by definition at the phase boundary. This change is
discontinuous+continuous =for a (first order+second order)phase transition.



The appropriate variables for the phase diagram of water are the pressure P and the temperature T.

Critical point:-The first-order phase boundary between gas and liquid becomes second order right at the critical point. The two phases have the equal densities and specific entropies (entropy per particle).

There is no critical point for the liquid-solid transition.

triple point : The point at which gas, liquid and solid coexist.

Skating on ice. The melting curve of water has a negative slope. Ice does hence melt at constant temperature $T < T_0$ when increasing the pressure P. This happens during skating on ice.

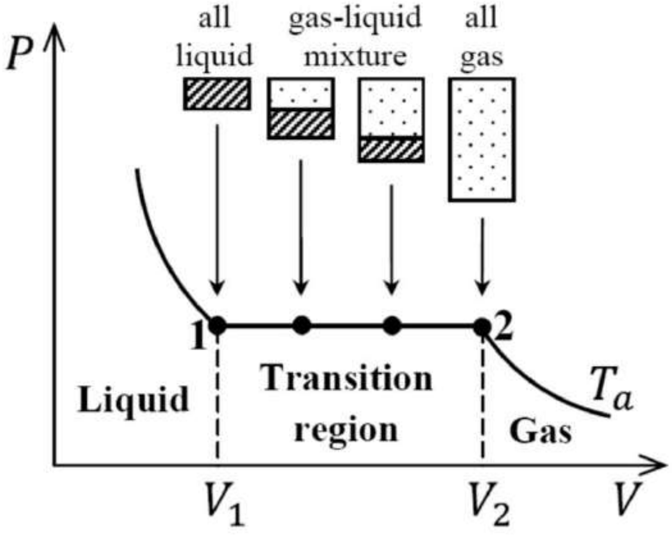


Fig-P-V diagram

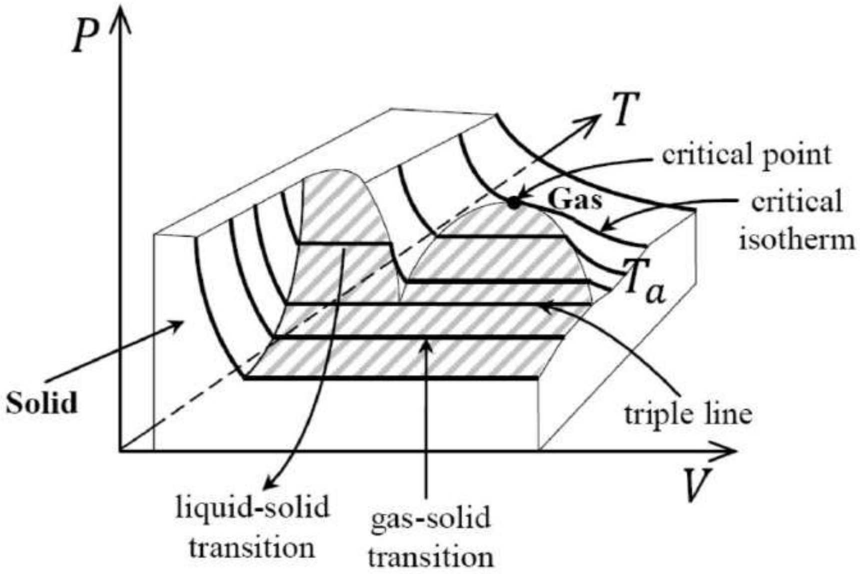


Fig- P,V,T diagram in 3d.

CONCLUSION.

Entropy is the thermodynamic property which is the measure of disorder in a system. It can be expresses by ‘S’=q/t The term is coined by Rudolf Clausius. Entropy is mainly associated with heat and temperature. Disorder can be of 3 types- Positional, Vibrational and Configurational Thermobarometric models is an excellent case study when the application of thermodynamic parameters are involved.

Phase transition is when a substance changes from a solid, liquid, or gas state to a different state. Every element and substance can transition from one phase to another at a specific combination of temperature and pressure.

. ACKNOWLEDGEMENT.

The success and the final outcome of the project required a lot of guidance and assistance of many people and I extremely privileged to have got this all along the completion of my project. All that I have done is only due to such supervision and Assistance and I would not forget to thank them.
I respect and thank our Principal, Swami Kamalasthananda and Vice Principal, Swami Vedanuragananda for providing me an opportunity to do the project work. I am extremely thankful to the Head of the Department of Physics, Prof. Asok Kumar Pal for providing such a nice support and guidance although he had a very busy schedule.

I owe my deep gratitude to our project guide Prof. Sankhasubhra Nag , who took keen interest on my project work and guided me all along, till the completion of the project work by providing all the necessary information for developing a good system. I heartily thank all my friends and classmates for their encouragement and more over for their timely support and guidance till the completion of my project work. I am thankful to and fortunate enough to get constant encouragement, support and guidance from all Teaching staffs of RKMVCC Physics Department which helped me in successfully completing my project work.

Tushar Kanti bera, physics department.

. REFERENCE.

For the completion of this project and giving it a final shape,I have Consulted the following books and website-

1)Thermal Physics (kinetic theory, Thermodynamics,and Statistics).--by S.C.Garg|R.M.Bansal| C.K. Ghosh.

2)CONCEPTS IN THERMAL PHYSICS.-by STEPHEN J. BLUNDELL AND KATHERINE M. BLUNDELL.

3)Heat and Thermodynamics.--by zeemansky and Dittman.

4)https://scholar.harvard.edu/files/schwartz/files/9-phases.
Pdf.

UNDERSTANDING THE DIFFERENCE BETWEEN IRON GROUPS IONS AND RARE EARTH IONS

The project submitted, in partial fulfilment of the requirement for the assignments in ***PHSA CC-XI, PHSA CC-XII, PHSA DSE-I, PHSA DSE-II*** *Paper (Semester 5th) in the Department of Physics*

Submitted by

Ankit Samanta

Registration No: A01-1112-111-030-2019

Supervisor Teacher: Prof. Asok Kumar Pal

RAMAKRISHNA MISSION VIVEKANANDA
CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

INDEX

PARAMAGNETISM

INTRODUCTION

Paramagnetism occurs in those atoms, ions and molecules which have permanent magnetic moments. In the absence of a magnetic field, these magnetic moments are oriented randomly and no net magnetization is produced, when a magnetic field is applied, these moments orient themselves in the direction of the field applied. The paramagnetic materials have small positive and temperature-dependent susceptibility.

The permanent magnetic moments of ions show some following contributions: -

(1). The Spin intrinsic moments of the electrons

(2). The orbital motion of the electrons.

(3). The spin magnetic moment of the nucleus.

>> Paramagnetism Occurs in: -

(1) Atoms, molecules and Lattice defects possessing an odd number of electrons, as here the total spin of the system cannot be zero.

***Examples*:-** Free Sodium atoms ,Gaseous Nitric Oxide (NO) , Organic free radicals such as Tri-phenyl methyl, C $(C_2H_5)_3$, F-centers of alkali halides.

(2) Free a atoms and ions with a partly filled inner shell, transition elements, ions isoelectronic with transition elements, rare earth and actinide elements,

Examples:- Mn^2+, Gd^3+, U^4+ Paramagnetism is exhibited by many of these ions even when incorporated into solids but not invariably.

(3) A few compounds with an even number of electrons including molecular oxygen and organic radicals,

(4) Metals

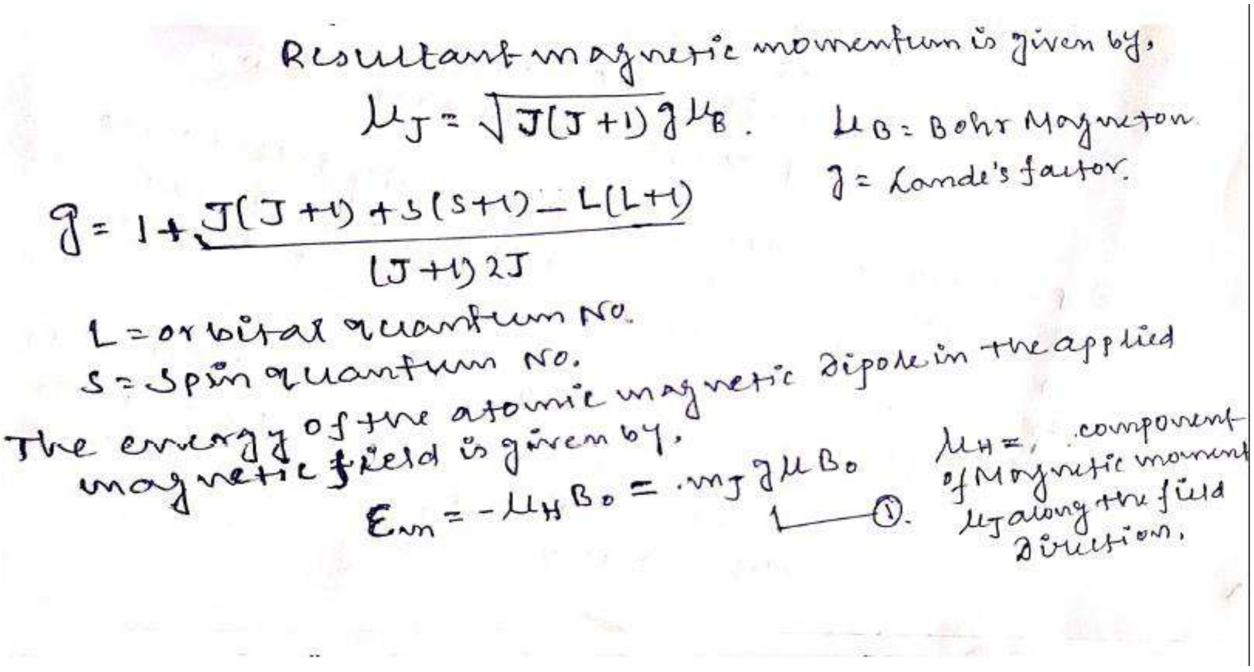
QUANTUM THEORY OF PARAMAGNETISM

- In Langevin theory of Paramagnetism it is assumed that the inclination of the atomic dipoles can vary continuously with respect to direction of the applied magnetic field. But according to Somerfield's rule for Space quantization, this assumption is not valid. According to these rules, the resultant angular Momentum Vector $\frac{Jh}{2\pi}$ of an atom can align in (2J+1) special directions such that the components of the angular momentum along the field direction can assume the discrete set of values

m_J = J , J-1, J-2,........,-J.

m_J =Magnetic quantum number

Resultant magnetic momentum is given by



Resultant magnetic momentum is given by,

$$\mu_J = \sqrt{J(J+1)}\, g \mu_B .$$

μ_B = Bohr Magneton

g = Lande's factor.

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{(J+1)2J}$$

L = orbital quantum No.

S = Spin quantum No.

The energy of the atomic magnetic dipole in the applied magnetic field is given by,

$$E_m = -\mu_H B_o = m_J g \mu B_o$$ ①.

μ_H = component of Magnetic moment μ_J along the field direction.

$$\mu_H = -m_J g \mu_B . \text{———} ②.$$

m_J can assume only finite No.s of discrete values, the magnetic energy Em can also have a discrete set of values.

to calculate the mean value of $\langle \mu_H \rangle$

$$\therefore \langle \mu_H \rangle = \frac{\sum_{-J}^{J} m_J g \mu_B \exp(m_J g \mu_B B_0/KT)}{\sum_{-J}^{J} \exp(m_J g \mu_B B_0/KT)} \text{———} ③$$

∴ Magnetization of the substance will be,

$$M_H = n\langle \mu_H \rangle = \frac{n \sum_{-J}^{J} m_J g \mu_B \exp(m_J g \mu_B B_0/KT)}{\sum_{-J}^{J} \exp(m_J g \mu_B B_0/KT)}$$

when magnetic field is not too high ($B_0 < 1T$), then at relatively high temperature,

$$\frac{m_J g \mu_B B_0}{KT} << 1 \; \rceil\!\downarrow \text{ for this condn exponential is expanded as.}$$

$$\Rightarrow \exp(m_J g \mu_B B_0/KT) = 1 + \frac{m_J g \mu_B B_0}{KT}$$

$$\therefore M_H = \frac{n g \mu_B \sum_{-J}^{J} m_J (1 + m_J g \mu_B B_0/KT)}{\sum^{J} (1 + m_J g \mu_B B_0/KT)} \text{———} ④.$$

$$= \frac{n g \mu_B \left\{ \sum_{-J}^{J} m_J + g \mu_B B_0 \sum_{-J}^{J} (m_J)^2/KT \right\}}{(2J+1) + g \mu_B B_0 \sum_{-J}^{J} m_J/KT}$$

Now, $\sum_{-J}^{J} m_J = 0$. and $\sum_{-J}^{J} m_J^2 = \frac{J(J+1)(2J+1)}{3}$

$$\therefore M_H = \frac{n g^2 \mu_B^2 B_0}{KT} \frac{J(J+1)(2J+1)}{3(2J+1)}.$$

$= \frac{n g^2 \mu_B^2}{kT} B_0 \frac{J(J+1)}{3}$ —— ⑤.

$$\therefore \boxed{\chi = \frac{M_H}{H} = \frac{\mu_0 n g^2 \mu_B^2 (J+1) J}{3kT}}$$ —— ⑥.

Now if we put,

$\mu_J^2 = J^2 (J+1)^2 g^2 \mu_B^2$.

∴ Equⁿ ⑥ assumes the same form as the classical expression

So equⁿ ⑥ shows that the quantum mechanical theory is also in agreement with curie's law.

[classical expression $\to \chi = \frac{\mu_0 n \mu^2}{3kT}$

⇒ if we say $\mu_J = P_{eff} \mu_B$

then Effective No. of Bohr magnetons is given by,

$$\boxed{P_{eff} = g\sqrt{J(J+1)}}.$$

Peff is determined by measuring χ

if we say $\alpha = \frac{J g \mu_B B_0}{kT}$

then from equⁿ ③ we can evaluate (in general case) the result,

$M_H = n g J \mu_B B_J(\alpha)$. —— ⑦

$B_J(\alpha)$ = Brillouin function.

$B_J(\alpha) = \frac{2J+1}{2J} \coth \frac{(2J+1)\alpha}{2J} - \frac{1}{2J} \coth(\frac{\alpha}{2J}$

for high fields and low temp. $\alpha \gg 1$

In the limit of $\alpha \to \infty$, we have,

$\coth \frac{(2J+1)\alpha}{2J} \to 1$ and $\coth \frac{\alpha}{2J} \to 1$

hence, $B_J(\alpha) = \frac{2J+1}{2J} - \frac{1}{2J} = 1$.

The magnetization attains the Saturation value **$\mu_B ng$** in this case, Equation **(7)** can be used to estimate the order of magnitude of paramagnetic susceptibility, since the atomic magnetic moments have values of the order of the Bohr magneton ~10^{-23}J/T and n~10^{28}/m^3

We get

$$X \sim \frac{(10^{28} \times (10-^{23})^2 \times 4\pi \times 10-7)}{(3 \times 1.38 \times 10^{-23})} \approx (\frac{3}{100})\ T$$

For T ~ 300K (Room temperature) we get X~10^{-4} for T~ 1K ; X~10^{-2}

- The permanent magnetic dipole moments of the atoms **arise from the orbital and spin motions of the electrons**. only those atoms which have partially filled electronic Sub shells have such permanent magnetic moments. There is no contribution from the completely filled shells.

  - Apart from electronic paramagnetism atoms also have nuclear Paramagnetism.

❖ DIFFERENCE IN PARAMAGNETIC BEHAVIOUR BETWEEN RARE EARTH IONS AND IRON GROUP IONS:-

Rare Earth Ions

The ions of the rare earth elements **(Table 1)** have closely similar chemical properties, and their chemical separation in tolerably pure form was accomplished only long after their discovery. Their magnetic properties are fascinating: The ions exhibit a systematic variety and intelligible complexity. The chemical properties of the trivalent ions are similar because the outermost electron shells are identically in the $5s^2 5p^6$ configuration, like neutral xenon. In lanthanum, just before the rare earth group begins, the $4f$ shell is empty; at cerium there is one $4f$ electron, and the number of $4f$ electrons increases steadily through the group until we have $4f^{13}$ at ytterbium and the filled shell $4f^{14}$ at lutecium. The radii of the trivalent ions contract fairly smoothly as we go through the group from 1.11 Å at cerium to 0.94 Å at ytterbium. This is known as the "lanthanide contraction." What distinguishes the magnetic behavior of one ion species from another is the number of $4f$ electrons compacted in the inner shell with a radius of perhaps 0.3 Å. Even in the metals the

- **Effective Magneton numbers (P_{eff}) for trivalent lanthanide group ions :-**

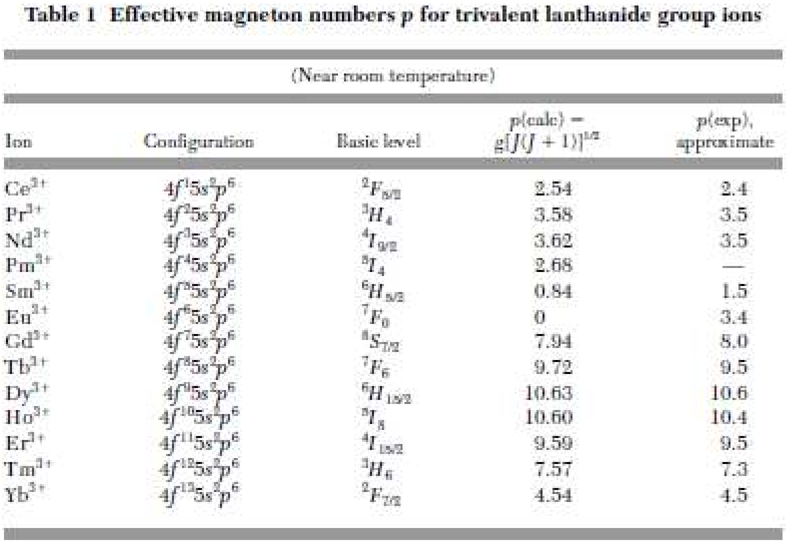


Table 1 Effective magneton numbers p for trivalent lanthanide group ions

(Near room temperature)

Ion	Configuration	Basic level	$p(\text{calc}) = g[J(J+1)]^{1/2}$	$p(\text{exp})$, approximate
Ce^{3+}	$4f^1 5s^2 p^6$	$^2F_{5/2}$	2.54	2.4
Pr^{3+}	$4f^2 5s^2 p^6$	$^3H_4$	3.58	3.5
Nd^{3+}	$4f^3 5s^2 p^6$	$^4I_{9/2}$	3.62	3.5
Pm^{3+}	$4f^4 5s^2 p^6$	$^5I_4$	2.68	—
Sm^{3+}	$4f^5 5s^2 p^6$	$^6H_{5/2}$	0.84	1.5
Eu^{3+}	$4f^6 5s^2 p^6$	$^7F_0$	0	3.4
Gd^{3+}	$4f^7 5s^2 p^6$	$^8S_{7/2}$	7.94	8.0
Tb^{3+}	$4f^8 5s^2 p^6$	$^7F_6$	9.72	9.5
Dy^{3+}	$4f^9 5s^2 p^6$	$^6H_{15/2}$	10.63	10.6
Ho^{3+}	$4f^{10} 5s^2 p^6$	$^5I_8$	10.60	10.4
Er^{3+}	$4f^{11} 5s^2 p^6$	$^4I_{15/2}$	9.59	9.5
Tm^{3+}	$4f^{12} 5s^2 p^6$	$^3H_6$	7.57	7.3
Yb^{3+}	$4f^{13} 5s^2 p^6$	$^2F_{7/2}$	4.54	4.5

4*f* core retains its integrity and its atomic properties: no other group of elements in the periodic table is as interesting.

The preceding discussion of paramagnetism applies to atoms that have a ($2J+1$)-fold degenerate ground state, the degeneracy being lifted by a magnetic field. The influence of all higher energy states of the system is neglected.

These assumptions appear to be satisfied by a number of rare-earth ions, Table 1. The calculated magneton numbers are obtained with *g*- values from the Landé result and the ground-state level assignment predicted by the Hund theory of spectral terms. The discrepancy between the experimental magneton numbers and those calculated on these assumptions is quite marked for Eu3_ and Sm3_ ions. For these ions it is necessary to consider the influence of the high states of the (*L-S)* multiplet, as the intervals between successive states of the multiplet are not large compared to k_BT at room temperature. A multiplet is the set of levels of different *J*- values arising out of a given *L* and *S*. The levels of a multiplet are split by the spin-orbit interaction.

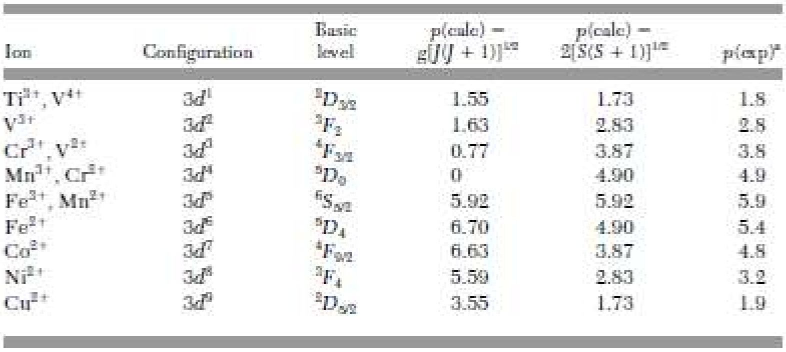
Iron Group Ions

(***Table- 2***) shows that the experimental magneton numbers for salts of the iron transition group of the periodic table are in poor agreement with (23). The values often agree quite well with magneton numbers $P = 2[S(S+1)]^{1/2}$ Calculated as if the orbital moment were not there at all.

Crystal Field Splitting

The difference in behavior of the rare earth and the iron group salts is that the 4*f* shell responsible for paramagnetism in the rare earth ions lies deep inside the ions, within the 5*s* and 5*p* shells, whereas in the iron group ions the 3*d* shell responsible for paramagnetism is the outermost shell. The 3*d* shell experiences the intense inhomogeneous electric field produced by neighboring ions. This inhomogeneous electric field is called the **crystal field**.

Table 2 Effective magneton numbers for iron group ions



Ion	Configuration	Basic level	$p(\text{calc}) = g[J(J+1)]^{1/2}$	$p(\text{calc}) = 2[S(S+1)]^{1/2}$	$p(\text{exp})^a$
Ti^{3+}, V^{4+}	$3d^1$	$^2D_{3/2}$	1.55	1.73	1.8
V^{3+}	$3d^2$	$^3F_2$	1.63	2.83	2.8
Cr^{3+}, V^{2+}	$3d^3$	$^4F_{3/2}$	0.77	3.87	3.8
Mn^{3+}, Cr^{2+}	$3d^4$	$^5D_0$	0	4.90	4.9
Fe^{3+}, Mn^{2+}	$3d^5$	$^6S_{5/2}$	5.92	5.92	5.9
Fe^{2+}	$3d^6$	$^5D_4$	6.70	4.90	5.4
Co^{2+}	$3d^7$	$^4F_{9/2}$	6.63	3.87	4.8
Ni^{2+}	$3d^8$	$^3F_4$	5.59	2.83	3.2
Cu^{2+}	$3d^9$	$^2D_{5/2}$	3.55	1.73	1.9

The interaction of the paramagnetic ions with the crystal field has two major effects:

The coupling of **L** and **S** vectors is largely broken up, so that the states are no Longer specified by their J-values; further, the $2L + 1$ sublevels belonging to a Given L which are degenerate in the free ion may now be split by the crystal Field, as in **Fig. 1**. This splitting diminishes the contribution of the orbital motion to the magnetic moment.

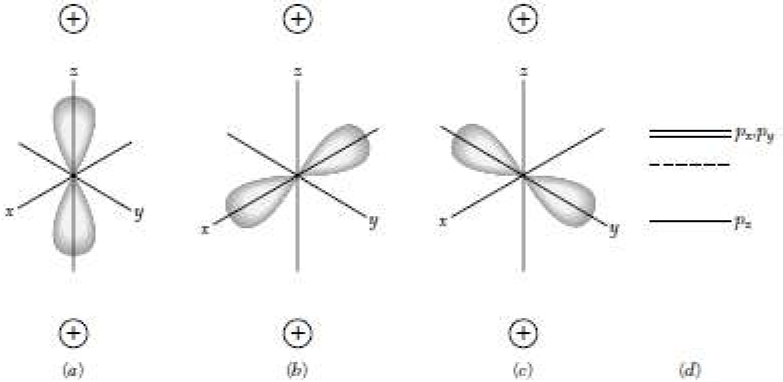


Figure 1 Consider an atom with orbital angular momentum $L = 1$ placed in the uniaxial crystallineelectric field of the two positive ions along the z axis. In the free atom the states $m_L = \pm 1, 0$ have identical energies -they are degenerate. In the crystal the atom has a lower energy when the electron cloud is close to positive ions as in **(a)** than when it is oriented midway between them, as in **(b)** and **(c)**. The wave functions that give rise to these charge densities are of the form *zf(r)*, *xf(r)* and *yf(r)* and are called the p_z, p_x, p_y orbitals, respectively. In an axially symmetric field, as shown, the p_x and p_y orbitals are degenerate. The energy levels referred to the free atom (dotted line) are shown in **(d)**. If the electric field does not have axial symmetry, all three states will have different energies.

Conclusions

Magnetic field gradients have a measurable effect on strongly paramagnetic Dy3+ and Gd3+, as well as on weakly diamagnetic Y3+ ions in solution. Paramagnetic ions move in the direction of increasing magnetic field, while diamagnetic ions move in the opposite direction. Even magnetic field gradients caused by strong NdFeB magnets are sufficient to induce sizeable concentration gradients in initially homogeneous solution of metal ions. Magnetic field gradients in a superconducting magnet of already 5 T led to concentration changes of the order of 10% in solutions containing Dy3+ ions. It was found that the concentration changes increase when the applied magnetic field gradient increases. The concentration changes however, do not change with the magnetic susceptibility of the ions, and the slightly diamagnetic Y3+ ions migrate almost as much under the same circumstances than the strongly paramagnetic Dy3+ and Gd3+ ions.

Acknowledgement

The success and final outcome of this project required a lot of guidance and assistance from many people and I am extremely privileged to have got this all along the completion of my project. All that I have done is only due to such supervision and assistance and I would not forget to thank them.

I respect and thank our Principal, **Swami Kamalasthananda** and Vice Principal, **Swami Vedanuragananda** for providing me an opportunity to do the project work. I am extremely thankful to the Head of the Department of Physics, Prof**. Asok Kumar Pal** for providing such a nice support and guidance although he had a very busy schedule.

I owe my deep gratitude to our project guide Prof. **Asok Kumar Pal** , who took keen interest on my project work and guided me all along, till the completion of the project work by providing all the necessary information for developing a good system. I heartily thank all my friends and classmates for their encouragement and more over for their timely support and guidance till the completion of my project work.
I am thankful to and fortunate enough to get constant encouragement, support and guidance from all Teaching staffs of RKMVCC Physics Department which helped me in successfully completing my project work.

Ankit Samanta, Physics Dept.

Sources

(1) Introduction to Solid State Physics by Charles Kittel

(2) Solid State physics by AJ Dekker

(3) Atomic physics by Dr. SN Ghoshal

●

Theoretical Origin and Experimental Determination of Plank’s constant

The project submitted, in partial fulfilment of the requirement for the assignments in CC-11, CC-12, DSE-1, DSE-2 Paper (Semester v.) in the Department of Physics

Submitted by

(**SIDDHARTHA MITHIYA**)

Registration No:

A01-1152-111-031-2019

Supervisor Teacher:

(**CHANDAN KR. DAS**)

RAMAKRISHNA MISSION VIVEKANANDA CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

Introduction:

Until the end of Nineteenth century classical physics was sufficient explain all natural phenomenons. Every known phenomenons were well described in terms of the existing theories which were basically a physics which is called nowadays "classical physics". The whole universe itself is made of particles which obeys (and experimentally verified) Newton's laws, one of the most powerful laws for describing physical phenomenons with magical accuracy. On the other hand, radiations of waves, electromagnetic phenomenons were well described by Maxwell's EM theory. Then from the beginning, some experiments arowsed, whose results were no more be explained through those existing theories for example for photo electric effect, black body radiation etc, also those experimental results were very surprising and some were non intuitive. One of these new phenomena was black body radiation. That time it was not any phenomena, but many theories published to explain the black body radiation from statistical approach.

Theoretical Development of Black Body Radiation:

In the year 1899 O lummere and E.Pringsheim, published the first accurate measurements of **R(λ,T)**. And they obtained a graph for R versus lambda for some fixed temperature T. Very soon many scientists tried to build up some theory which can describe experimental behaviour of R with λ.

R(λ,T) = Total emissive power of a black body that is the total power emitted per unit area of a black body.

or, Emissive power

or, Spectral emittance of the black body

λ = Wavelength of the emitted wave from a Black Body.

T = Absolute temperature of the Black Body

Now R(λ,T) be the total emissive power or total emittance of a black body, that is the total power emitted per unit area of a black body. In 1879 J. Stefan found an empirical relation between the quantity R and absolute temperature T of a black body.

$$R(T) = \sigma T^4$$

$\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4} is the constant known as **Stefan's constant**.

In 1884 Boltzmann got the above relation from thermodynamics. It is now called Stefan Boltzmann law. So that R(λ,T)dλ is the power united per unit area from the black body ay the absolute temperature T, according to relation with wavelengths between λ to λ + dλ, the total emissive power R is of course the integral of R over all wavelengths.

$$R(T) = \int_{\lambda=0}^{\infty} R(\lambda, \mathrm{T})\, \mathrm{d}\lambda$$

Advice Steven Boltzmann law $\mathbf{R = \sigma T^4}$. Since R depends only on temperature.

From the observation of spectral emittance R(λ,T), increases with increasing T. At each temperature there is a wavelength λ for which R(λ,T) has its maximum value this wavelength varies inversely with temperature.

$$\lambda_{\max} T = b$$

It is known as ***Wien's displacement law***.
b is the ***Wien's displacement constant*** (b = 2.89 x 10^{-3}).

Kirchhoff proved that the flux of radiation in the cavity is the same in all direction so that the radiation is isotopic, he also showed that the radiation is homogeneous namely the same at every point inside the cavity and that is identical in all cavities at the same temperature. Furthermore all this statements hold at each wavelength. Instead of using spectral emittance R(λ,T) it is convenient to satisfy the spectrum of black body radiation inside the cavity in terms of a quantity **ρ(λ,T).**

ρ(λ,T) = spectral distribution function
or, monochromatic energy density

It is defined so that, ρ(λ,T)dλ i.e. **the energy power unit volume of the radiation in the wavelength interval (λ, λ+dλ) at absolute temperature T**. As we expect from physical grounds ρ(λ,T) is proportional to R(λ,T) and it can be shown that the proportionality constant 4/c , where c is the velocity of light in vacuum.

$$\boldsymbol{\rho(\lambda, \mathrm{T}) = \frac{4}{c} R(\lambda, \mathrm{T})}$$

Hence measurement of the spectral emittance R(λ,T) also determine the spectral distribution function ρ(λ,T). Using general thermodynamical arguments, W. Wien showed in 1893 that the spectral distribution function has to be of the form, $\boldsymbol{\rho(\lambda, \mathrm{T}) = \lambda^{-5} f(\lambda \mathrm{T})}$

$\boldsymbol{f(\lambda \mathrm{T})}$ is a function of a single variable $\boldsymbol{\lambda \mathrm{T}}$. This cannot be determined from thermodynamics.

In order to determine this function $\boldsymbol{f(\lambda \mathrm{T})}$ and hence, $\boldsymbol{\rho(\lambda, \mathrm{T})}$ one must go beyond thermodynamics reasoning. After some attempts by Wien and lord Rayleigh and J.Jeans describe a spectral distribution function, $\boldsymbol{\rho(\lambda, \mathrm{T})}$ from the laws of classical physics in the following way,

First, from electromagnetic theory, follows that the thermal radiation within the gravity must exist in the form of standing electromagnetic waves. P number of search wheels or in other words the number of modes of oscillation in the electromagnetic field in the cavity per unit volume with wavelengths within the interval **(λ, λ+dλ)** can be shown to be $(8\pi/\lambda^4)$ so that $n(\lambda) = 8\pi/\lambda^4$ is a number of modes per unit volume and per unit wavelength range. The number is independent of the size of the shape of a sufficient large cavity. ε denotes the average energy in the node with wavelength λ, The spectral distribution.

$$\rho(\lambda, \mathrm{T}) = \frac{8\pi}{\lambda^4}\varepsilon$$

From this equation Rayleigh and Jeans add into the equation, that $\rho(\lambda, \mathrm{T}) = (8\pi/\lambda 4)\mathrm{kT}$ known as **Reyleigh Jeans spectral distribution**.

In this limit of long wavelengths, Rayleigh Jeans result approaches the experimental values of $\rho(\lambda, \mathrm{T})$ does not exit the observed maximum it diverges as $\lambda \rightarrow 0$. This behavior at short wavelengths is known as "**ultraviolet catastrophe**". Is it going to be total energy per unit volume, ρ_{tot} is seen to be infinite. Which is clearly incorrect.

$$\rho_{tot} = \int_{\lambda=0}^{\infty} \rho(\lambda, \mathrm{T})\, d\lambda$$

Plank's Quantum theory:

No solution of these difficulties can be found using classical physics. However in December 1900 **Max Planck** presented a new form of black body radiation spectral distribution based on the revolutionary hypothesis. He postulated that "The energy of a oscillator of a given frequency **ν** cannot take arbitrary values between 0 and ∞, but can only take on the discrete values $(n\varepsilon_0)$ where n is a positive integer or 0, ε_0 is a finite amount of energy or "*quantum*" of energy which only depends on frequency **ν**".
h he's a fundamental physical constant called plank's constant. Difference spectral distribution law for $\rho(\lambda, \mathrm{T})$ is given by,

$$\rho(\lambda, \mathrm{T}) = \frac{8\pi\mathrm{hc}}{\lambda^5}\frac{1}{\exp(\mathrm{hc}/\lambda\mathrm{kT}) - 1}$$

We see that,

$$f(\lambda\mathrm{T}) = \frac{8\pi\mathrm{hc}}{\exp(\mathrm{hc}/\lambda\mathrm{kT}) - 1}$$

Now, **for long wavelength**, $\rho(\lambda, \mathrm{T}) \rightarrow \frac{8\pi\mathrm{hc}}{\lambda^4}$ in agreement with Rayleigh-Jeans formula. For short wavelength, the presence of $\exp(\mathrm{hc}/\lambda\mathrm{kT})$ in the denominator of the plank's radiation law, ensures that $\rho \rightarrow 0$ as $\lambda \rightarrow 0$.

Now, $h=6.626 \times 10^{-34}$ J.s

$$\text{Dimension of } h \rightarrow [h] = [\text{energy}][\text{time}]$$

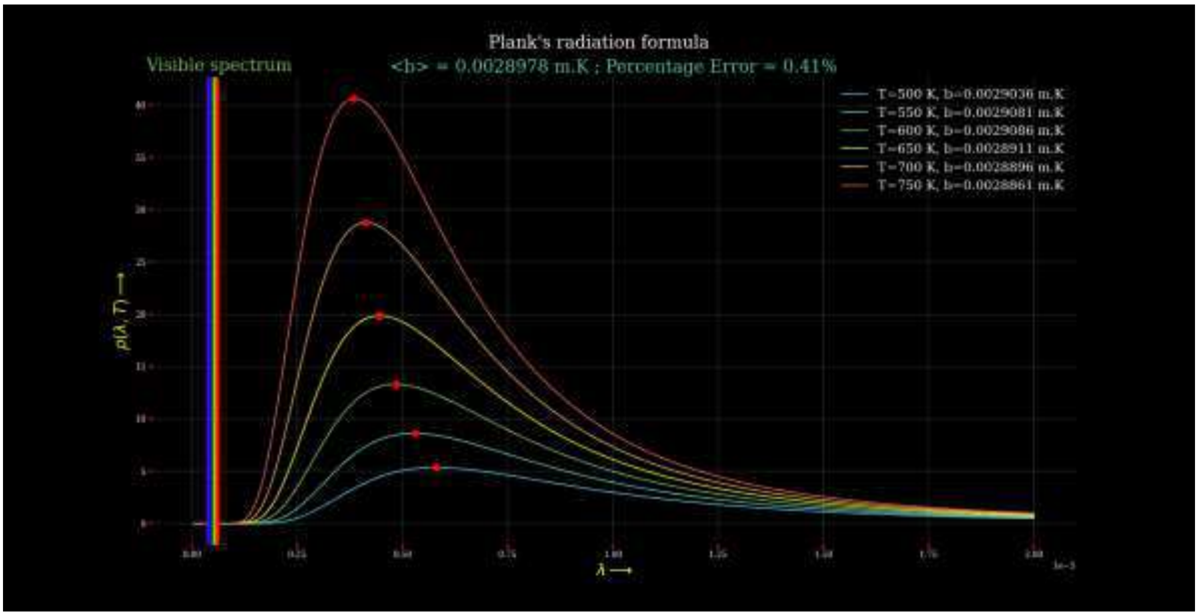
$$= [\text{length}][\text{momentum}]$$

$$= [L][MLT^{-1}]$$

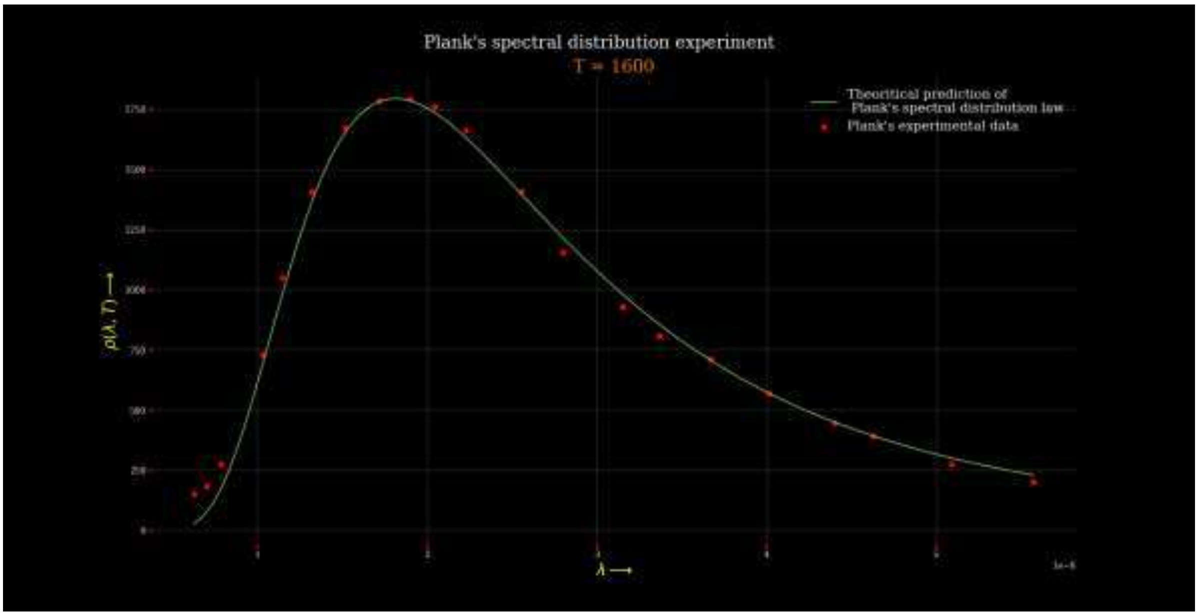
$$= [ML^{2}\,T^{-1}]$$

Thus dimension is those of a physical quantity called ‘action’, and consequently , plank’s constant h, is also known as “Fundamental quantum of action”.

Thus, the “true origin” of plank’s constant was Black Body Radiation, where it comes quite naturally.



Graph-1



Graph-2

Graph-1: Plank’s Spectral Distribution for several temperatures

Graph-2: Comparison of Plank’s spectral distribution with his original experimental data.

Experimental Determination:

After the theoretical origin of plank’s constant in plank’s quantum theory for Black Body radiation, it was very important to determine it’s value through experiment.

Few experimental method to determine it’s value are,

- Using LED
- From Photoelectric Effect
- Millikan’s method
- Etc.

Here, I’m going to discuss the **<u>1st one</u>** of them.

Experimental Determination of (h) using LED:

Light Emitting Diodes (LEDs) are semiconductor devices characteristically defined by their ability to emit electromagnetic radiation in the visible spectrum when a potential is applied to the semiconductor materials. LEDs are composed of p-type (electron acceptor) and n-type (electron donor) materials that form a physical connection referred to as the p-n junction. Excited electrons transitioning from the conducting band down to the valence band (hole-electron recombination) releases a quanta of energy equal to that of the quanta required to create the electron hole pair as a photon of discrete energy (E = hν). An energy band diagram shows the relationship of these hole-electron energy transitions within the semiconductor material (Fig. 1).

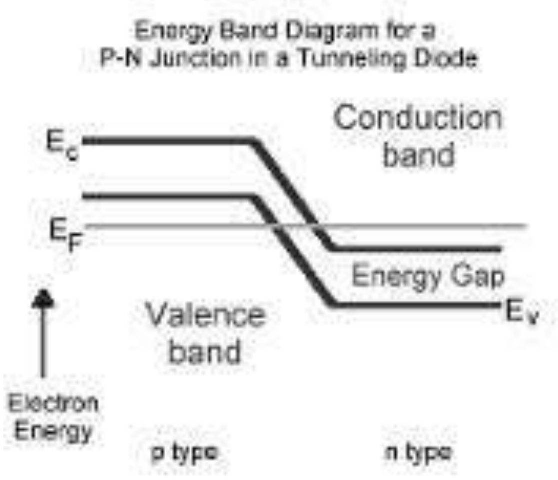


Figure 1: Typical Energy Band Diagram for a P-N Junction Diode. The electron transition from Ec to Ef releases a photon with an E equal to the difference between Ec and Ev. Ef represents threshold or forward voltage.

The relationship between the wavelength of the emitted photon, the applied potential and discrete quanta of energy E is:

$$E = h\nu = \frac{hc}{\nu} = eV_0$$

Where e = $1.602*10^{-19}$C is the magnitude of the electron charge and V_0 is the magnitude of the cut in voltage of the LED.

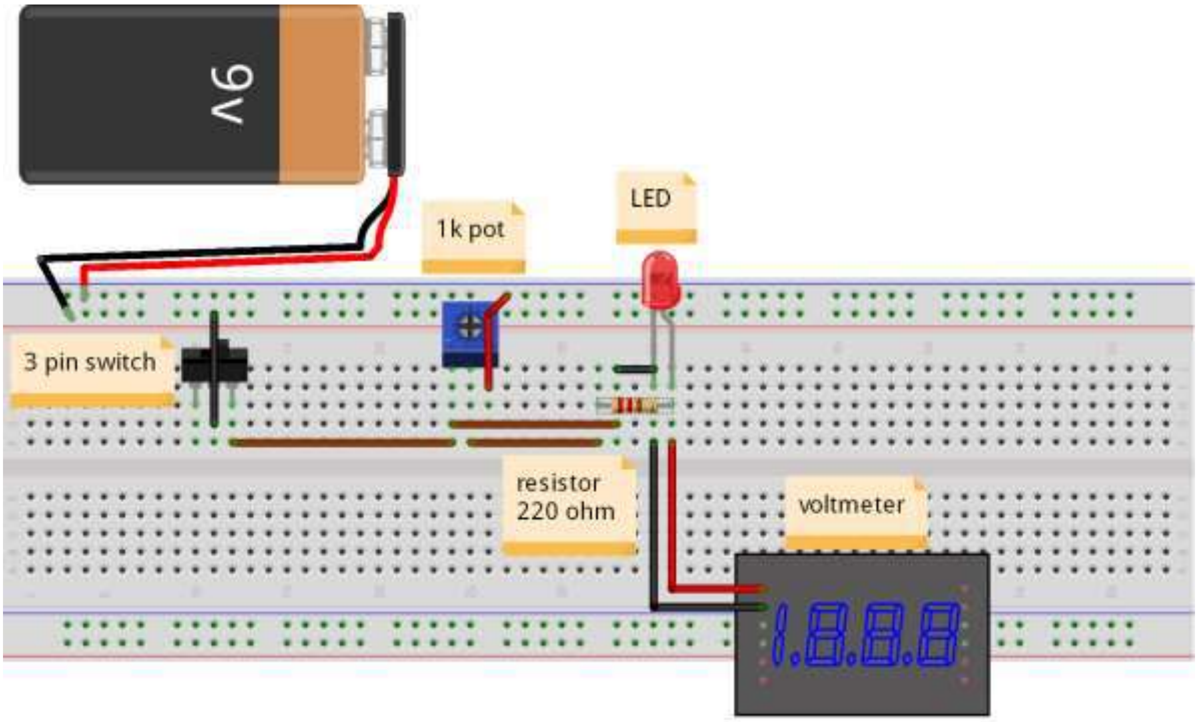
The experimental determination of Planck's constant is then easily obtained by determining the wavelength of the emitted radiation from an LED and applied voltage over the LED concurrently, solving for h to yield:

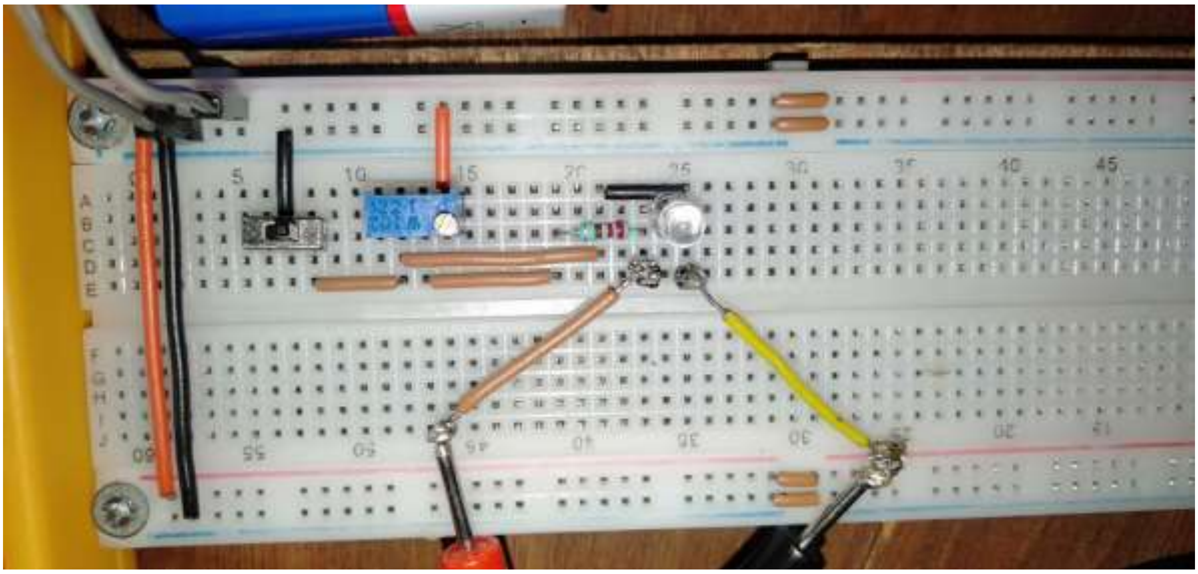
$$V_0 = \left(\frac{h}{e}\right)\nu$$

and substituting the empirical values.

a) Circuit used:

I am attaching some pictures of the circuit.





b) Experimental method:

Data was collected for three different LEDs using a simple circuit to measure

1) **Cut in voltage (V_0).**

2) **Wavelength (λ) of the emission spectra of those LEDs.**

- ### Measuring Cut in Voltage (V0):

  Cut in voltage is the voltage across the LED in when current just starts flowing. So in a darkened room, I try to observe when the LED just turns on. At that point the voltage across the LED is very near to the cut in voltage.

- ### Wavelength (λ) of the emission spectra:

  Collecting spectral data using the chart supplied by the manufacturer. The specifications of the mean wavelength (λ) of the emission spectra of those LEDs are in the data table.

c) Collected Data:

TABLE-1 (Collected Data)

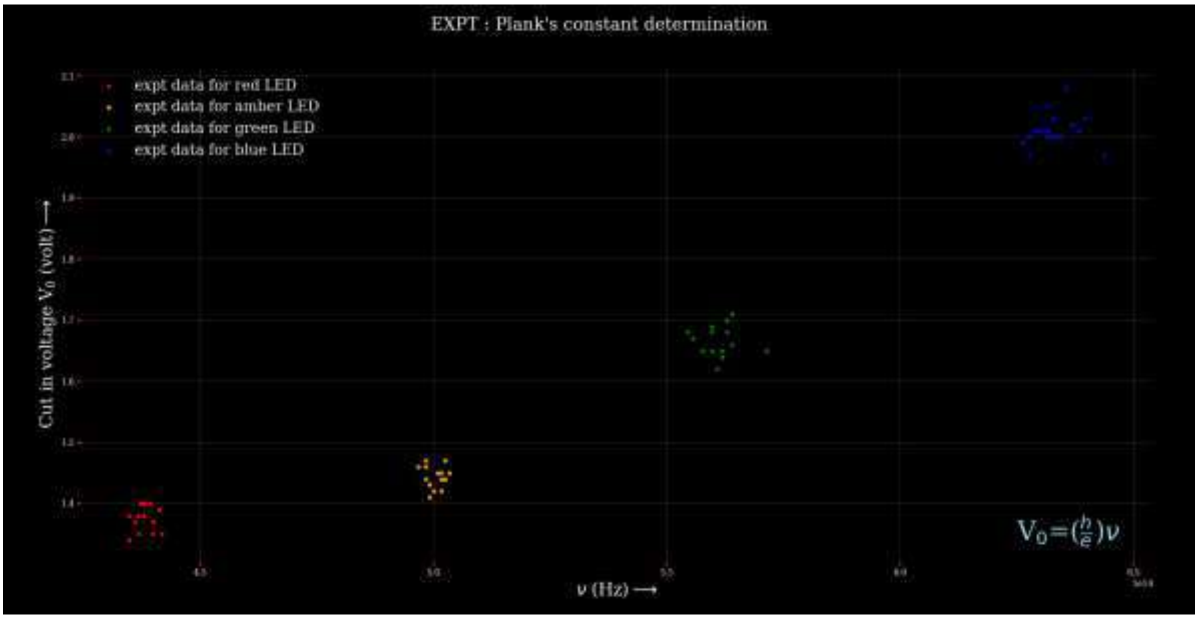
	Red $\lambda = 610\ nm$	Amber $\lambda = 590\ nm$	Green $\lambda = 500\ nm$	Blue $\lambda = 450\ nm$
Forward Cut in Voltage V_0(volt)	1.4	1.44	1.65	2.0
	1.4	1.45	1.7	2.03
	1.38	1.44	1.68	2.01
	1.37	1.46	1.65	1.99
	1.35	1.43	1.66	2.0
	1.37	1.46	1.71	1.97
	1.38	1.47	1.67	2.01
	1.34	1.44	1.69	2.03
	1.38	1.43	1.65	2.08
	1.37	1.45	1.68	2.01
	1.38	1.47	1.69	2.00

	1.40	1.42	1.64	2.05
	1.35	1.45	1.65	2.02
	1.35	1.42	1.68	2.01
	1.39	1.44	1.65	1.97
	1.37	1.41	1.62	2.00

d) Analyzing Data:

Now, assuming the emission spectra follows the Gaussian distribution, I incorporate a standard deviation of 3% of their mean wavelength.

Now, plotting the data I get the following graph.



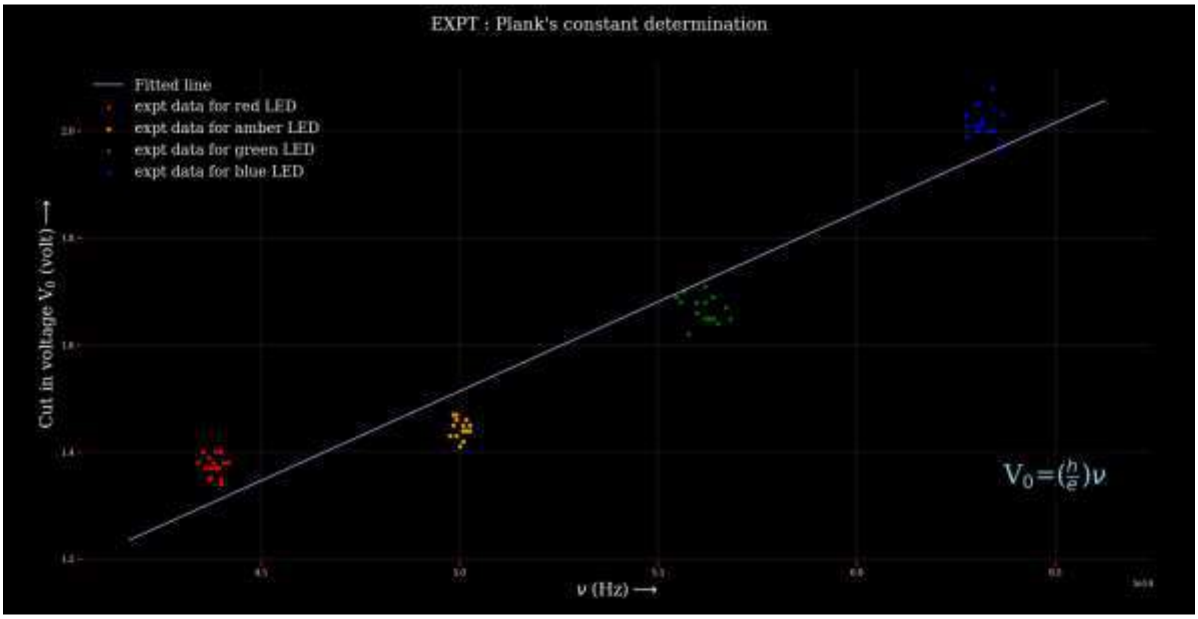
Now,

$$V_0 = \left(\frac{h}{e}\right)\nu$$

Here, in the V_0 vs ν graph the slope is (h/e).

Now, by doing the least square fitting, I can get the slope. Then by multiplying by e (=1.6 * 10^{-19} C), theoretically it will be the plank's constant.

By least square fitting, the graph I get,

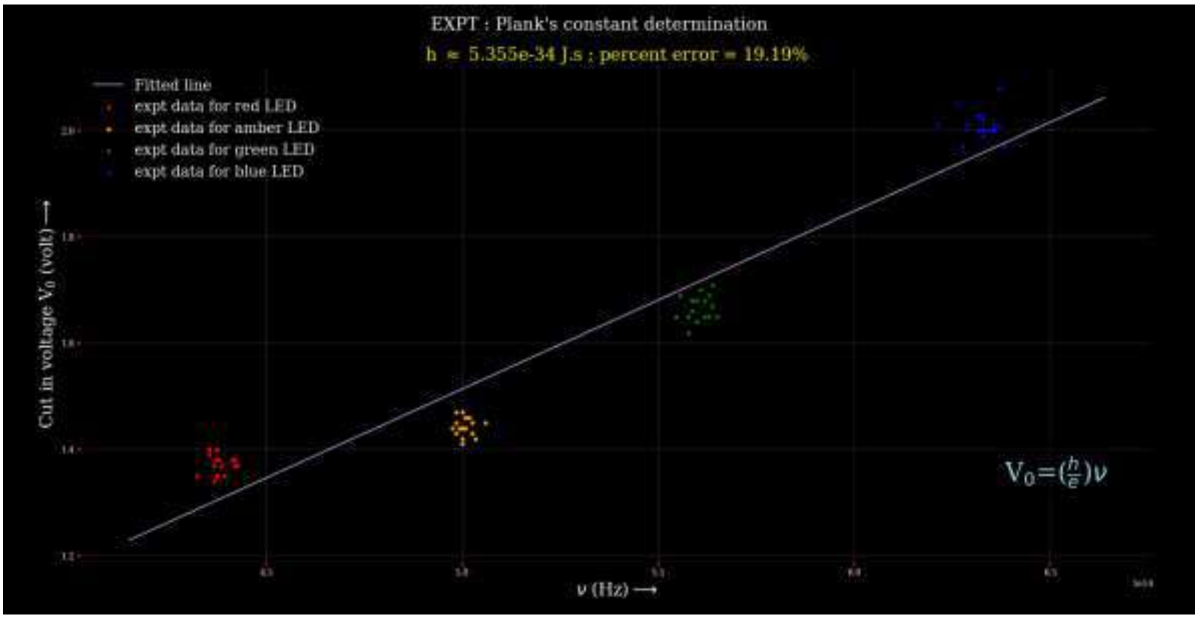


Determined slope, m = $3.346*10^{-15}$

Now, m * e = $5.354*10^{-34} \approx h$

So, I got the correct order (10^{-34}) of plank’s constant.

Percent error = 19.19%



e) Sources or error:

- Careful measurement of the wavelength can improve the result.
- Use of some sort of voltage amplifier can help detecting the cut in voltage.

f) Conclusion:

Overall it is found that the experimentally determined value for Planck's constant h = $5.354*10^{-34}$ J.s is within acceptable limits as compared to the

accepted value h = 6:626x10^{-34}J.s with a difference of 19.19%. Regarding random error and measurement uncertainty, total differentials proved to be insignificant with regard to the final precision of the experimental value of h as evident by the magnitude of in the final experimental value.
Further investigation and refinement of experimental execution and techniques would most likely decrease random error.

~: END :~

PROJECTILE MOTION IN AIR RESISTANCE

The project submitted, in partial fulfilment of the requirement for the assignments in ***PHSA CC-XI, PHSA CC-XII, PHSA DSE-I, PHSA DSE-II*** *Paper (Semester 5TH) in the Department of Physics*

Submitted
ARNAB GARAI

Registration No: A01-1112-111-034-2019

Supervisor Teacher: Prof. Atis Dipankar Chakrabarti

RAMAKRISHNA MISSION VIVEKANANDA CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

Acknowledgement

Primarily I would like to express my gratitude to Ramakrishna Mission Vivekananda Centenary College, Rahara for including the project as part of our academic curriculum. This has provided me an opportunity to learn something beyond the text book. I wish to express my deep gratitude and sincere thanks to revered Principal Maharaj Swami Kamalasthananda, Ramakrishna Mission Vivekananda Centenary College, Rahara, for his encouragement and the facilities provided for this project work. I extend my sincere thanks to Dr. Atis Dipankar Chakrabarti, Professor, Ramakrishna Mission Vivekananda Centenary College, Rahara and my mentor who guided me to successful completion of the project. I take this opportunity to express my deep sense of gratitude for his valuable support and constructive comments.

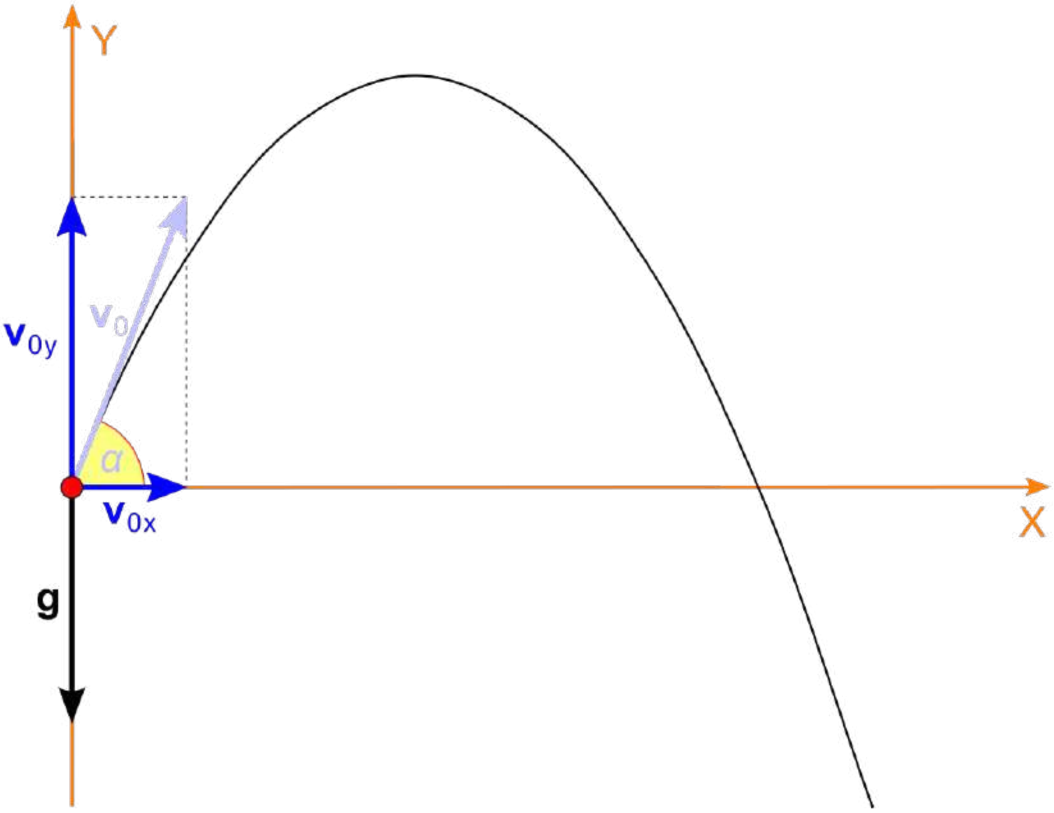
PROJECTILE MOTION

PROJECTILE MOTION :-

Projectile motion is a form of motion experienced by an object or particle (a projectile) that is projected near the Earth's surface and moves along a curved path under the action of gravity only (in particular, the effects of air resistance are passive and assumed to be negligible). This curved path was shown by Galileo to be a parabola, but may also be a line in the special case when it is thrown directly upwards. The study of such motions is called ballistics, and such a trajectory is a ballistic trajectory. The only force of mathematical significance that is actively exerted on the object is gravity, which acts downward, thus imparting to the object a downward acceleration towards the Earth’s center of mass. Because of the object's inertia, no external force is needed to maintain the horizontal velocity component of the object's motion. Taking other forces into account, such as aerodynamic drag or internal propulsion (such as in a rocket), requires additional analysis. A ballistic missile is a missile only guided during the relatively brief initial powered phase of flight, and whose remaining course is governed by the laws of classical mechanics.

In projectile motion, the horizontal motion and the vertical motion are independent of each other; that is, neither motion affects the other. This is the principle of *compound motion* established by Galileo in 1638, and used by him to prove the parabolic form of projectile motion.[

A ballistic trajectory is a parabola with homogeneous acceleration, such as in a space ship with constant acceleration in absence of other forces. On Earth the acceleration changes magnitude with altitude and direction with latitude/longitude. This causes an elliptic trajectory, which is very close to a parabola on a small scale.



Since there is only acceleration in the vertical direction, the velocity in the horizontal direction is constant, being equal to V_0cos . The vertical motion of the projectile is the motion of a particle during its free fall. Here the acceleration is constant, being equal to g.[note 1] The components of the acceleration are:

$$a_x = 0$$
$$a_y = -g$$

Let the projectile be launched with an initial velocity $v(0) = v_0$, which can be express as the sum of horizontical and vertical components as follows:-The

$$\mathbf{v}_0 = v_{0x}\hat{\mathbf{x}} + v_{0y}\hat{\mathbf{y}}$$

horizontal component of the velocity of the object remains unchanged throughout the motion. The vertical component of the velocity changes linearly, because the acceleration due to gravity is constant. The accelerations in the x and y directions can be integrated to solve for the components of velocity at any time *t*, as follows:

$$v_x = v_0 \cos(\theta)$$

$$v_y = v_0 \sin(\theta) - gt$$

Displacement

At any time, the projectile's horizontal and vertical displacement are:

$$x = v_0 t \cos(\theta)$$

$$y = v_0 t \sin(\theta) - \frac{1}{2} g t^2$$

Time of flight or total time of the whole journey:-

The total time t for which the projectile remains in the air is called the time of flight.

$$y = v_0 t \sin(\theta) - \frac{1}{2} g t^2$$

After the flight, the projectile returns to the horizontal axis (x-axis), so y=0

Maximum height of projectile.

$$0 = v_0 t \sin(\theta) - \frac{1}{2} g t^2$$

$$v_0 t \sin(\theta) = \frac{1}{2} g t^2$$

$$v_0 \sin(\theta) = \frac{1}{2} g t$$

Maximum height of projectile:-

The greatest height that the object will reach is known as the peak of the object's motion. The increase in height will last until $v_y = 0$that is,

$$0 = v_0 \sin(\theta) - g t_h$$

time to reach the maximum height(h):

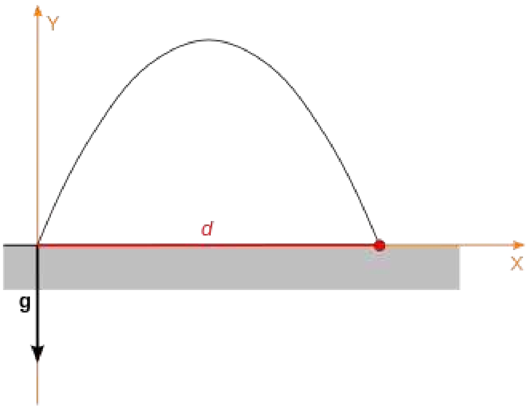
$$t_h = \frac{v_0 \sin\theta}{g}$$

for vertical displacement of the maximum height of projectile:-

$$h = v_0 t_h \sin\theta - \frac{1}{2} g t_h^2$$

$$h = \frac{v_0^2 \sin^2\theta}{2g}$$

Maximum distance of projectile:-



The range and the maximum height of the projectile does not depend upon its mass. Hence range and maximum height are equal for all bodies that are thrown with the same velocity and direction.. The horizontal range d of the projectile is the horizontal distance it has traveled when it returns to its initial height (y=0)

$$0 = v_0 t_d \sin\theta - \frac{1}{2} g t_d^2$$

time to reach ground :-

$$t_d = \frac{2 v_0 \sin\theta}{g}$$

from the horizontal displacement the maximum distance of projectile :

$$d = v_0 t_d \cos\theta$$

so;

$$d = \frac{v_0^2 \sin(2\theta)}{g}$$

PROJECTILE MOTION WITH AIR RESISTENCE:-

Finally ,let us consider much less trivial problem of a projectile subject also to a resistive force, such as atomospheric drug. We assume that the retarding force is propotional to the velocity, so that the equation of motion becomes

$$m\frac{d^2r}{dt^2}=-\lambda\frac{dr}{dt}+mg$$

the actual dependence of atomospheric drag on velocity is definitely non linear, but this equation nevertheless gives a reasonable qualitative picture of the motion .

Defining $\gamma=\frac{\lambda}{m}$, we may write as,

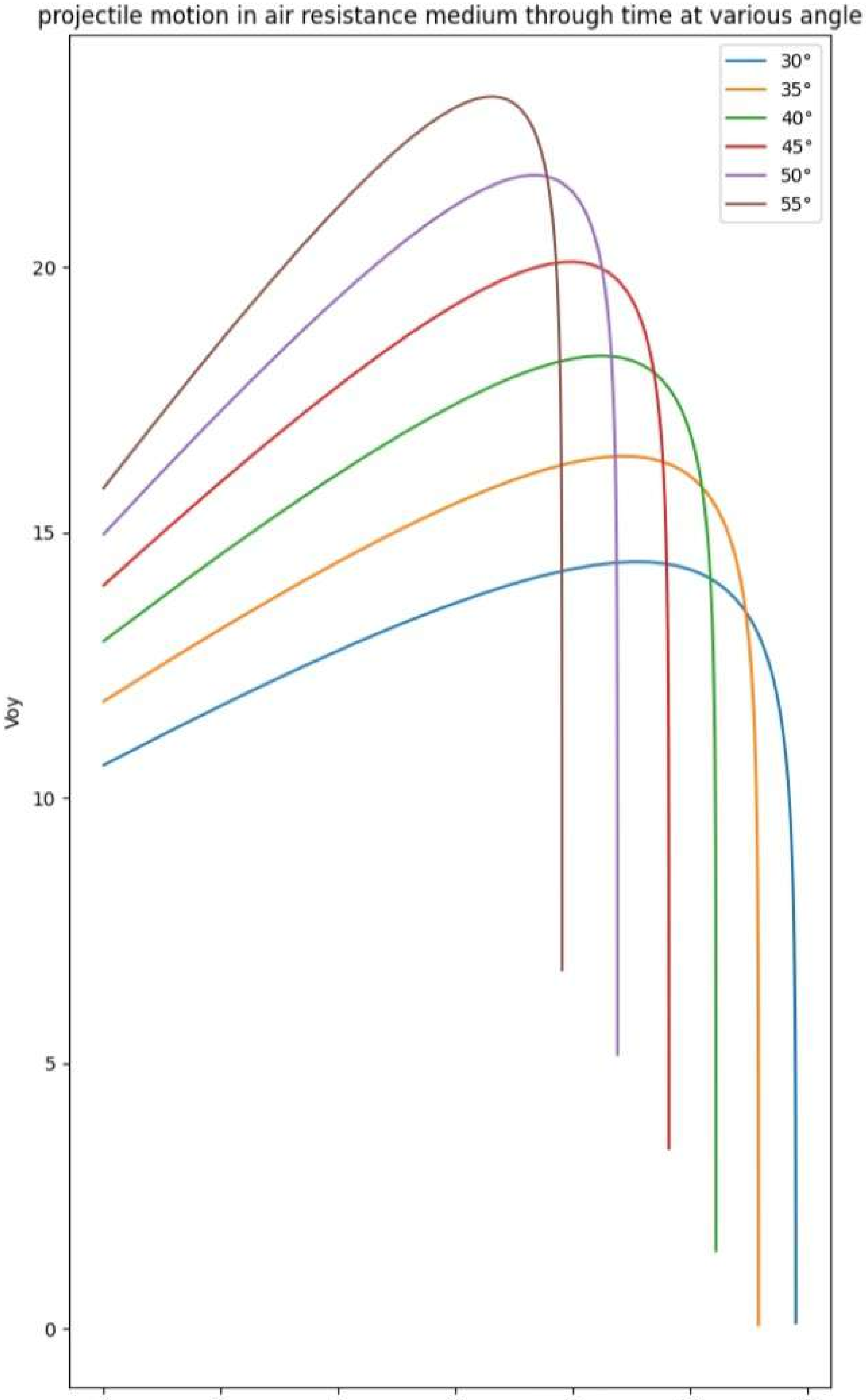
$$\frac{d^2x}{dt^2}=-\gamma\frac{dx}{dt},$$
$$\frac{d^2z}{dt^2}=-\gamma\frac{dz}{dt}-g$$

These equation may be integrated by the use of integrating factors . If the initial velocity is v= (u, 0, w), the required solution is

$$x=\frac{u}{\gamma}(1-e^{-\gamma t}),\quad \text{and}\quad z=(\frac{w}{\gamma}+\frac{g}{\gamma^2})(1-e^{-\gamma t})-\frac{g\,t}{\gamma}$$

Eliminating t between these two equations yields the equation of the trajectory,

$$z=(\gamma w+g)\frac{x}{\gamma u}+\frac{g}{\gamma^2}\ln(1-\gamma\frac{x}{u})$$


projectile motion in air resistance medium through time at various angle
30°
35°
40°
45°
50°
55°
Voy
20
15
10
5
0
0
2
4
6
8
10
12
Vox

A distinctive feature of this equation is that, even if the launch point is far above the ground , there is a maximum value of x. The projectile can never reach beyond $x=u/\gamma$, and as it approaches that value, $z\rightarrow-\infty$. Thus the trajectory ends in a near- vertical drop, when all the forward momentum has

been spent.

References

For the completion of this project and giving it a final shape , I have consulted the following books and websites

sites:-

https://en.wikipedia.org/wiki/Projectile_motion

https://scipython.com/book2/chapter-8-scipy/examples/a-projectile-with-air-resistance/

http://dynref.engr.illinois.edu/afp.html

BOOKS:-

1. Introduction to CLASSICAL MECHANICS by David Morin

2 Classical Mechanics by Herbert Goldstein
3. Classical Mechanics by John R. Taylor

:WHICH WAY THE YO-YO MOVES :

AN EXPERIMENTAL PROJECT OF ROTETIONAL DYNAMICS

The project submitted ,in partial fulfilment of the requirement for the assignments in PHSA CC-XI,PHSA CC-XII,PHSA DSE-I,PHSA DSE-II Paper(semester 5th)in the department of physics

Submitted By

Sandipan Ghosh

Registration No: A01-1152-111-035-2019

Supervisor Teacher : Prof. Rajen Kundu

RAMAKRISHNA MISSION VIVEKANANDA
CENTENARY COLLEGE

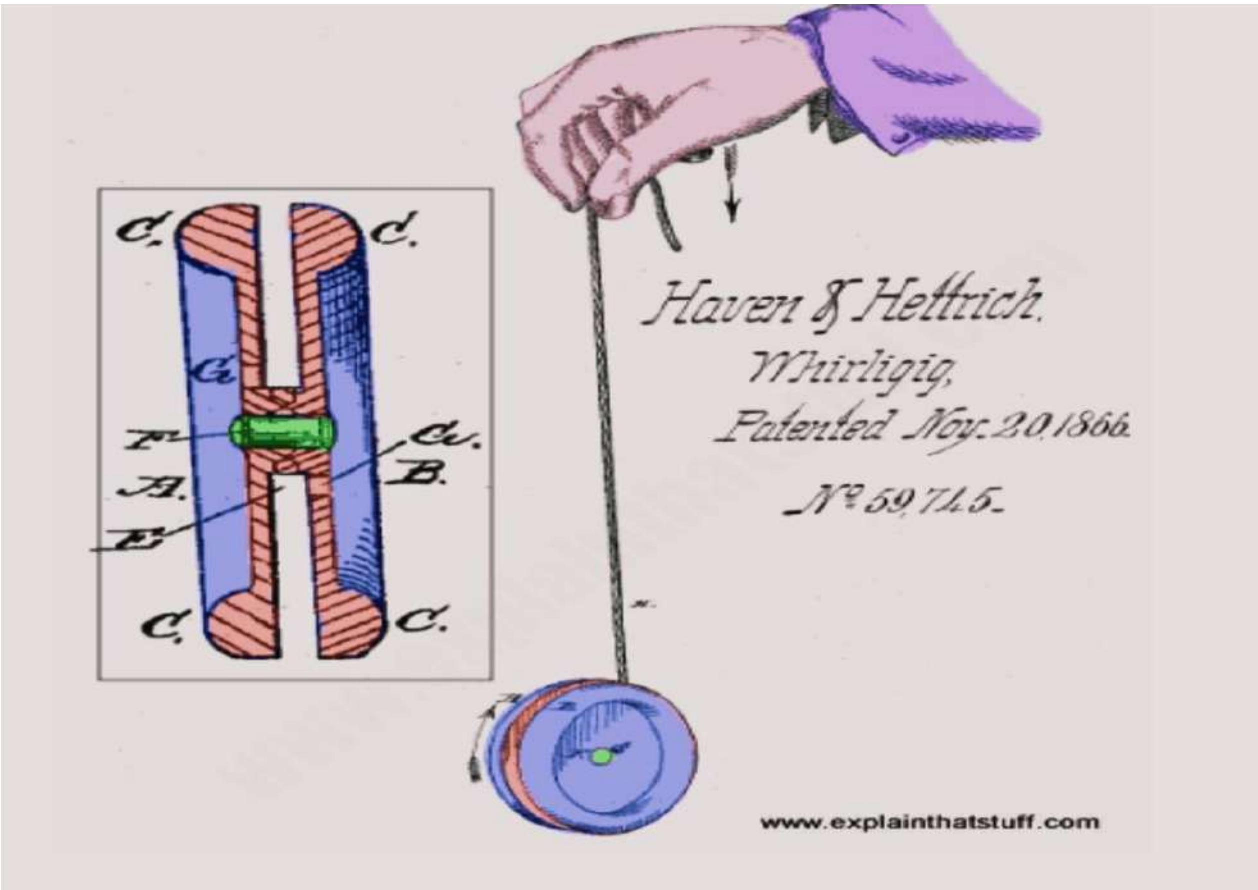
P.O. RAHARA , KOLKATA-700118

:WHICH WAY THE YO-YO MOVES : AN EXPERIMENTAL PROJECT OF ROTETIONAL DYNAMIC

- Abstract: **In this project , we try to demonstrate some fact , connected with rotational dynamics using simple experiments with a common toy known as yo-yo and by explaining it's motion .**

INTRODUCTION

Most people are familiar with the yo-yo as a fun and cool toy. It's fun learing to make it work properly and cool to learn different ticks to perform with it. It is thought that the yo-yo originated in china more then 2500 years ago! The first historical reference to the yo-yo was in 500 BC . The next historical reference to the yo-yo as a toy came in 1765 on box made in india . Within 25 years , the yo-yo made its way from India to the upper classes in Scotland and France , and then into England . The first mention of the yo-yo in America was in 1866 in a patent for an improved design .However, the yo-yo finally caught on the United states in the 1920s . It was then that a Filipion immigrant , Pedro Flores , brought over the "Filipion" yo-yo ,an even further-developed design . Instead of the yo-yo string being tied directly to the axle between the two discs ,the string is tied in a loop around the axle



The yo-yo might seem like magic, but it is just physics at work .There are three parts to the basic yo-yo : the two disks and the axle that joins them . When the it has two kind of potential energy . Potential energy is the energy stored in an object .For example , if hold a ball string out in front of you , ready to drop it, it now has potential energy . The first kind of potential energy in the yo-yo is the potential to fall to the ground . Since the string is wound the axle , the second kind of potential energy is the potential to spin. When the yo-yo thrown , the potential energy is converted into kinetic energy . Kinetic energy is the energy of motion . In example of the ball held in front of ,once we let go of the ball and it is moving through the air , it has kinetic energy . The potential energy in the ball changes to kinetic energy as soon as we let go of it . As the yo-yo falls, the string unwinds and the yo-yo spins . The yo-yo is building angular momentum .

SECTION : 1

Theoretical analysis of the yo-yo dynamics

We consider a yo-yo , with mass m, hanging from a string and moving only in the vertical direction . The string is considered as weightless and inextensible. The applied forces are the tension of the string T and the weight mg as indicated in Fig.1 both acting in the vertical direction . Assuming the y-axis oriented downwards , the second Newton law can be written as

$$ma_y = mg - T \qquad \text{............} \qquad (1)$$

Where a_y is the vertical acceleration of the center of mass .

Similarly , let us consider the rotational version of the Newton's second law in the center of mass . We assume that the tension is applied upwards , at the right of the center of mass , at a perpendicular distance which coincides with the radius of the inner cylinder , r, as shown in Fig .1. Assuming the positive torques in the counterclowiese direction , the rotational law can be expressed as

$$I \alpha = T r \qquad \text{............} \qquad (2)$$

Where α is the angular acceleration of the yo-yo .

The third equation needed to solve the problem comes from the non- slipping condition of the yo-yo , relative to the string , and relates the vertical acceleration and the angular acceleration . In the configuration shown in the figure this constraint reads as

$$a_y = \alpha r \qquad \text{..............} \qquad (3)$$

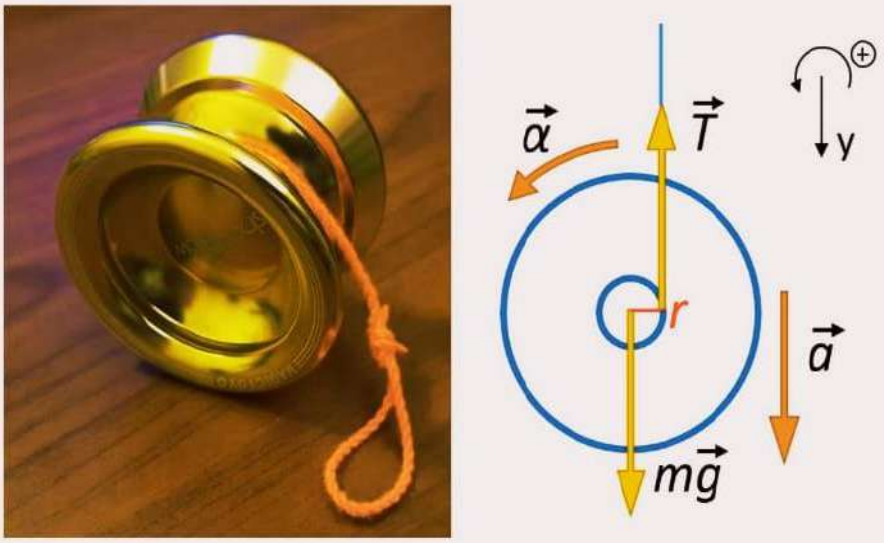


Figure 1. In the left panel the traditional toy consisting of two disks joined by an axle. A string, with a passing knot in the free end is wound around the axle as can be seen in the picture. When the yoyo is either climbing or falling two forces are exerted on it as shown in right panel: the weight, $m\vec{g}$ and the tension of the string $\vec{T}$. In the diagram it is also indicated, the inner radius, r, the acceleration of the center of mass, a, and the angular acceleration α.

The angular acceleration can be obtain solving Eqs . 1-3 . Considering the expression relating the radius of gyration and the inertia , *I* = mr_g^2

And substituting , we obtain the relation

$$\boldsymbol{\alpha = g\frac{r}{r^2 + r_g{}^2}} \text{............} \qquad \textbf{(4)}$$

Equation 4 links the (constant) angular acceleration to the physical characteristics of the yo-yo and the gravitational acceleration when thw yo-yo is either climbing or falling on the left of the string as shown in Fig. 1

SECTION : 2

How do yo-yos work

When we release a yo-yo ,gravity acts on its center of mass to pull the yo-yo downwards. Because the string of the yo-yo is wrapped around the yo-yo's axle , and because one end of the string is attached to our finger , the yo-yo is forced to rotate as it drops. If the yo-yo could not rotate, it would not drop .

Just as any object falling in a gravitational field, the rate of drop increases with time and so, necessarily, does the rotation rate of the yo-yo. The rate of drop and the rotation rat are greatest when the bottom is reached and the string is completely unwound . the spinning yo-yo contains angular momentum derived from the gravitation potential energy through which the yo-yo has dropped .

Usually, the string is tied loosely around the axle so that the yo-yo can continue to spin at the bottom . Because the full length of the string has been paid out, the yo-yo can drop no further and consequently , the rotation rate can not increase further. If left in this condition, the friction between the axle and the string will eventually dissipate the energy of rotation or equivalently the angular momentum of the yo-yo and the yo-yo will come to rest .

However , a momentary tug on the string causes the friction between the string causes the friction between the string and the axle briefly to increase so that the axle no longer slips within the string . When the axle thus stops slipping, the angular momentum of the spinning yo-yo is sufficient to cause the string to wind around the axle. This , of necessity , causes the yo-yo to begin to 'climb' back up the string. After the first one or two rotations , the string can no longer slip, so the process of climbing up the string continues beyond the momentary application of tug.

As the yo-yo continues to climb back up the string, the angular momentum of the yo-yo is converted back into gravitational potential corresponding to the increasing height of the center of mass of the yo-yo. For this reason , the yo-yo's angular momentum and hence its rotation rate steadily decreases as the yo-yo rises .This is of course the reverse of process when the yo-yo was dropped .

If not for frictional losses, the yo-yo would climb all the way back up the string to your hand just as its rotational rate decreases to zero . But due to friction , the yo-yo does not in fact quite get back up to your hand before it stops rotating.

Thereafter, the process repeats , with the yo-yo returning short of its previous height on each cycle . Eventually the yo-yo comes to rest at the bottom .

Of course , as everyone knows, it is possible to keep the yo-yo going indefinitely by giving it a slight upward pull on each cycle. This pull can be combined with the tug required to initiate the climb back up the string . the pull serves to give the center of mass of the yo-yo a little extra kinetic energy to compensate for frictional losses , so that the yo-yo can be kept going indefinitely .

Yo-yos can also be thrown horizontally or launched in other directions. The principal of operation is then just the same except that the kinetic energy of the center of mass , which is converted into spin as the string unwinds , result from being thrown , rather than from falling through a gravitational potential.

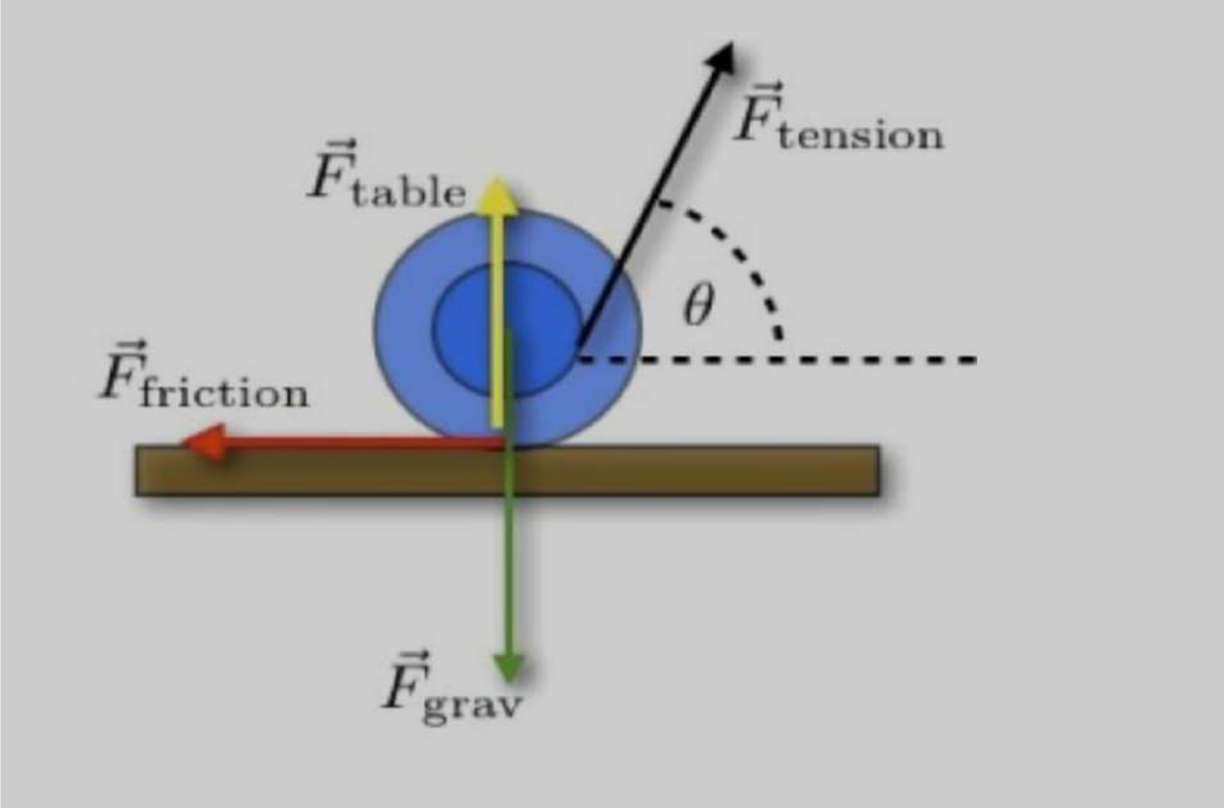
SECTION: 3

YOYO : PULLING AND ROLLING

PULLING : **I am going to pull the yo-yo at different angles and two different surfaces.**

WHAT IS GOING ON HERE?

Let me look at the first case where I pull the yo-yo and it slides without rolling . Hear is a diagram .



Normally a free body diagram treats an object as though it were a point mass . we can't do that in this case because we have to consider rotation also . when I draw a diagram as a point , this is the key thing I am looking at: ,

$$\vec{F}_{\text{net}} = m\vec{a}$$

Which I could brake into 2 or 3 component equation such as :

$$\vec{F}_{\text{net-x}} = m\vec{a}_{\text{x}}$$

$$\vec{F}_{\text{net-y}} = m\vec{a}_{\text{y}}$$

Since this object can rotate ,I must also consider that with

$$\vec{\tau} = I\vec{\alpha}$$

- **Tau is the torque about some axis .We can think of torque as the rotational equivalent of force .**
- ***I* is the momentum of inertia of that object about the same axis as the torque . The moment of inertia can be a complicated things , but in this case it can be thought of as the object's resistance to change in rotational motion . The moment of inertia depends on both the mass of the object and how this mass is distributed about the axis of rotation .**
- **Alpha is the rotational (angular) acceleration.**

I have three equation – the x equation , the y equation and the rotational equation . I need to note a couple of extra things . First I will call the radius of the inner part of the yo-yo and the outer radius R .Also the mass is , and the coefficient of static and kinetic friction will be mu$_s$ and mu$_k$. This gives the following :

$$\mathbf{F}_{\text{net-x}} = \mathbf{F}_{\text{T}}\, cos\theta - \mathbf{F}_{\text{f}} = 0$$

$$\mathbf{F}_{\text{net-y}} = \mathbf{F}_{\text{T}} sin\theta + \mathbf{F}_{\text{N}} - \mathbf{mg} = 0$$

$$\tau = F_{\text{T}} r - F_{\text{f}} R = 0$$

- *A couple of notes :*
  - **I picked the case of the sliding and not rolling yo-yo because ; the accelerating and angular acceleration are zero. The friction is kinetic friction . This means that I can determine its value. For static friction. I can only calculate the maximum friction .**
  - **The acceleration in the y-direction is zero since the yo-yo stays on the table .**
  - **I can use the model for friction to get an expression for F_f**
  - **Also , I have shorter notation for the force from the table(F_N) , tension (F_T) and the gravitational force (mg).**
  - **There are 4 force . However, I only show two torques . the torque from the force the table exerts is zero about the axis since this forces points right through the axis . The torque due to the gravitational force is also zero . This is because gravity pulls on all parts of the yo-yo.**

Here is the model for kinetic friction .Note the this an expression for the magnitude of the friction force – it is not a vector equation .

$$F_f = \mu_k F_N$$

With this , I can replace F_f and I get :

$$F_T cos\theta - \mu_k = 0$$

$$F_T sin\theta + F_N - mg = 0$$

$$F_T r = \mu_k F_N R$$

Now , I will get an expression for F_T from the last equation :

$$F_T = \frac{\mu_k F_N R}{r}$$

And now I can substitute this in the other two equation . I get :

$$F_N\left(\frac{\mu_k R}{r}cos\theta - \mu_k\right) = 0$$

$$F_N\left(\frac{\mu_k R}{r}sin\theta + 1\right) = 0$$

From the top expression , if F_N is not zero , then :

$$\left(\frac{\mu_k R}{r}cos\theta - \mu_k\right) = 0$$

$$\frac{R}{r}cos\theta = 1$$

$$\theta = cos^{-1}\left(\frac{r}{R}\right)$$

So, this says that the angle needed to pull the yo-yo it doesn't slip only depends on the ratio of the inner and outer radius . Note that r would be smaller than R . So that the ration would be less than 1. This is good because the cosine function must produce a number less than one .

- **What happens if I increase the angle of the string to pull the yo-yo?**
- **The fractional force will be less . This is because I pull at a greater angle with the string, then the normal force will be smaller .This smaller normal force means the fractional force will smaller and thus a smaller torque from the friction . Both of these together make the torque large in the direction that makes it roll to the left .**
- **What happens if I decrease the angle of the string to pull the yo-yo ?**
- **If the angle of the string is too small , the frictional force will be greater(basically because of the opposite of above).**

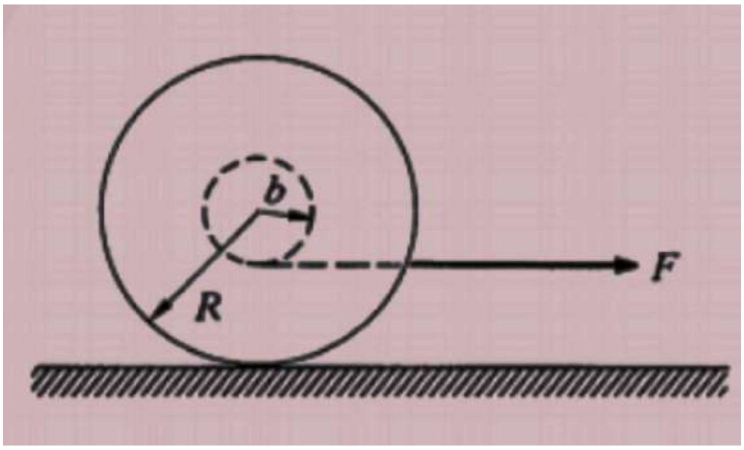
- **ROLLING :** A Yo-Yo of mass M has as axis of radius b and a spoool of radius R . its moment of inertia can be taken to be $MR^2/2$. The yo-yo is placed upright on a table and the string is pulled with a horizontal force F as shown . The coefficient of friction between the yo-yo and the table is μ.

  What is the maximum value of F for which the Yo-Yo will roll without slipping?

- Imagine there was no friction to begin with ; since the force is acting on the rope wound around the circular axle , will the yo-yo only slid to the right or will it slide to the right + roll backwards due to the pulling of the rope that is wound around the axle ? It seems like it should be the second if I try to imagine it . Now , regardless of that , the center of mass velocity will be directed to the right I think ; was only assuming this based on physical grounds that the yo-yo will slide to the right but if the pulling of the rope around the axle causes it to roll backwards then idk if it will slide to the right or not anymore so if someone could clarify this that would be great . Going with that assumption , the friction force would be directed to the left . The equation of motion are :

$$ma = F - f$$

$$= F - \mu mg$$



This is another point I would like someone to clarify for me because I assumed that since we want it to roll without slipping the rotation must be directed clockwise so that the tangential velocity at the contact point(left) is directed opposite to the assumed CM velocity direction(right) but I'm not sure if I'm supposed to assume this a priori or if its somehow supposed to come out of the equations at the end so if someone could clarify this as well ,

$$-\frac{1}{2}mR^2\alpha = -\frac{1}{2}mRa = Fb - \mu mgR$$

And putting all this together gives :

$$F = \frac{3\mu mg}{(1+\frac{2b}{R})}$$

CONCLUTION

YO -YO motion can be an effective way to help us learn about simple energy conversions . The learning cycle provides a useful way to structure hands-on-inquiry yo-yo activities. We should now appreciate that the scientific principals that make a simple toy like a yo-yo possible are also very important in many aspects of our daily life.

Acknowledgement

Primarily I would like to profound gratitude to Ramakrishna Mission Vivekananda Centenary College., Rahara for including the project as part of our academic curriculum. This has provided me an opportunity to learn something beyond the text book. I wish to express my deep gratitude and sincere thanks to revered principal Maharaj Swami Kamalasthananda , Ramakrishna Mission Vivekananda Centenary College ,Rahara , for his encouragement and the facilities provided for this project work .

I extend my sincere thanks to Dr. Rajen Kundu , Professor , Ramakrishna Mission Vivekananda Centenary College , Rahara and my mentor who guided me to successful completion of the project . I take this opportunity to express my deep sense of gratitude for his valuable support and constructive comment .

I can’t forget to offer my sincere thanks to my family and friends whose immense motivation and support had helped me to complete the project in limited time frame.

SANDIPAN GHOSH

PHYSICS HONOURS (5TH SEMESTER)

COLLEGE ROLL NO : 149

BIBLIOGRAPHY

For the completion of this project and giving it a final shape, I have consulted the following books and websites.

Sites :

- **https://www.wired.com/2010/01/yo-yo-rolling-sliding-pulling/**
- **https://www.wikipedia.org/**
- **https://www.sciencebuddies.org/**
- **https://blog.pitsco.com/blog/the-physics-that-make-yo-yos-fun**

Books :

- **Problems in General physics I. E. Irodov**
- **La physique du yo-yo , *pour la science* No 265 February 1999**

On the way the nature transits from Classical behaviour to Quantum behaviour and Max Planck

*The project submitted, in partial fulfilment of the requirement for the assignments in **PHSA CC 11, PHSA CC 12, PHSA DSE 1, PHSA DSE 2** Paper (Semester **V**) in the Department of Physics*

Submitted by

SUDEEPTA ROY

Registration No: A01-1122-111-036-2019 of 2019-2020

Supervisor Teacher: BHASKAR HALDAR

RAMAKRISHNA MISSION VIVEKANANDA
CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

On the way the nature transits from Classical behaviour to Quantum behaviour and Max Planck

by

Sudeepta Roy

December 15, 2021

Department of Physics

'All science is either physics or stamp collecting.'

ERNEST RUTHERFORD

1871-1937

Acknowledgments

I am grateful to Dr. Rajen Kundu and Professor Bhaskar Halder for the discussions on the problem. They really helped me for thinking and doing mathematics that how Planck thought about disreet energy rather than continuous spectrum.I want to thank my friend Debojyoti Ballav for choosing this topic and my close friend Chandrima Sarkar, both of them helped me for searching important informations in the whole work.

"Anyone who is not shocked by quantum theory has not understood it"

-NIELS BOHR

1885-1962

Table of Contents

List of Figures

Abstract

In this work I address several issues occurred at early stages of nineteenth century where our deterministic classical theory didn't explain the practical experiments. After Rutherford's comment on atomic structures there were some serious problems rising up. To describe internal structures of atoms physicists asked a question that, how matter interacted with radiation, radiation means electromagnetic radiation. I make a detail analysis performed by Rayleigh and Jeans and then the statistical calculation made by Max Planck on 1900 in non-relativistic situations. Here I also include the origin of Planck's constant by performing an action integral.

Using this ideas Albert Einstein able to explain the particle Nature of Light or "Photoelectric Effect" and after that, Arthur Compton able to show after his name "Compton Scattering" and then Niels Bohr made the "Hotchpotch Model of Atom."

Chapter1

Preliminary idea of Light and Atoms

Sir Issac Newton invented physics to describe nature almost three hundred years ago. Newton certainly built upon the work of others, but it was the publication of his three laws of motion and theory of gravity, that sets off on the road that has led to space flight, lasers, atomic energy, genetic engineering, an understanding of chemistry, and all the rest of today science. For two hundred years, Newtonian physics (”Classical” physics”) reigned supreme; in the twentieth century revolutionary new insights took physics far beyond Newton, but without those two centuries of scientific growth those new insights might never have been achieved.

But Newton's ideas and laws are only applicable for solid objects like planets and massive objects which can be seen by everyday. His three laws of motion predicts the future of a system. The first law of Newton is a statement of **Newtonian Relativity**. These three laws of motion together with Newton's laws of gravity explained the orbits of planets around the sun and the moon around the earth. But these laws had puzzling philosophical implications. Actually Newton was the first person who taught us how to predict the future of the behavior of a particle if it were possible to know the initial position and velocity of that particle. So that we can easily know the fate of universe.[1]

But this mechanics are not applicable for ”Non uniqueness in Newtonian Mechanics”. Because those systems are not fully integrable, and for all times the initial conditions are not maintained so that this is ”**Non-uniqueness in classical mechanics**.”

Using the particle nature Newton got success so he tried to explain the nature of light in terms of particles. which are travelling in straight lines,"**Newton's Corpuscular Theory**." After that Christiaan Huygens was contemporary of Newton, said "**light is a wave, propagating through invisible ether medium**." Young used this idea and design his double slit experiment. At 1870s, James Clerk Maxwell establish existence of waves involving changing electric and magnetic fields.

Now physicist want to discover, matter is made up by which ? Answer of this question is "**Atoms**". After that Rutherford's gold foil experiment with α-particles made the picture of atoms so, electron clouds are seen now. Now the main problem is ,"What is the trajectory of electron ?" If they follows a particular trajectory then due to **accelerating electrons** continuous electromagnetic radiation produced. But still actual picture was different, because no continuous spectrum found. Then a group of physicist arise a question, **"How matter interacts with electromagnetic radiation ?"**, It's really important to get a picture of atoms.[1]

Chapter2

Atomic model and the Black body clue

Let's see what did Rutherford get the picture about atoms and whats is the next:

- There must be a small region of atom which he called nucleus. It contains all positive charge of atom and an amount od exactly equal and opposite charge to the amount of negative charge in the cloud of electrons that surrounded the nucleus, so that nucleus and electrons together make up an electrically neutral atom.
- **But if that is the actual picture then why don't negative charges fall into nucleus ?**

To answer the question, asked at the end of Chapter 1, the first version of quantum theory was born.At the end of twentieth century, the scientific view of natural world require a dualistic philosophy. **Matter might be described in terms of atoms, but electromagnetic radiation, (i.e.light) had to be thought in terms of waves.**[1] The answer of the question mentioned at second bullet is a **nuclear repulsive force** but here I will not interested on this topic.

Our main focus is to observe at a hot object, which is radiating continuous electromagnetic radiation due to movement of tiny electric charges. At that time our best classical theories - statistical mechanics and electromagnetism - predicted a form of radiation which is completely different from actually coming from hot objects. Physicist made an imaginary idealized example, **a perfect absorber and emitter of radiation.** So independently

KIRCHHOFF and BALFOUR STEWART, had shown that, ***the nature of radiation in thermal equilibrium in an enclosure whose walls are kept at a fixed temperature, is independent of properties of any material bodies, including the walls, which are in completely equilibrium with the radiation.*** When radiation falls on the surface of a body then some of it gets reflected and some is absorbed, as for example, dark bodies absorb most of the radiation falling on them and white bodies reflect most of it. The *absorption coefficient* (a_λ) of a material surface at a given wavelength is defined as the fraction of the radiant energy, incident on surface, which is absorbed at that wavelength. If a body is in thermal equilibrium with its surroundings, and therefore is at constant temperature, it must absorb and emit the same amount of radiant energy at unit time, otherwise its temperature would rise or fall.

A ***black body*** is defined as a body which absorbs all the radiant energy falling on it hence appear as black under reflection when illuminated from outside. When an object is heated then it radiates electromagnetic energy as a result of thermal agitation of electrons in its surface. **The intensity of this emitted radiation depends on its frequency and on the temperature; the light it emits over entire spectrum.** In 1859 Kirchhoff proved using general thermodynamical arguments, for any wavelength the ratio of the *emissive power* or spectral emittence (e_λ) (energy emitted per unit area at a given wavelength) to the *absorption coefficient* is same for all bodies at the **same temperature.** As the maximum absorption coefficient corresponds to black body, then according to Kirchhoff's law it is also the most efficient emitter of electromagnetic energy. (Physicists made those types of idealization for the problem to eliminate material dependence of a body.) i.e. for black body,[2]

$$\frac{a_\lambda}{e_\lambda} = 1 \tag{2.1}$$

A black body can be closely approximated by taking a hollow cavity whose internal walls are

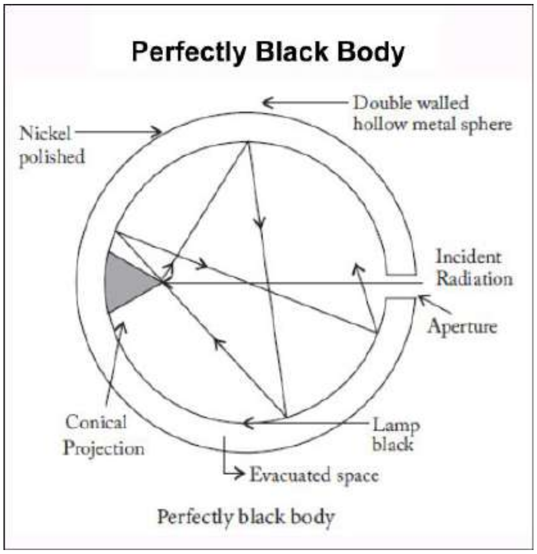


Figure 2.1: A good approximation to a black body. A cavity kept at constant temperature and having blackened interior walls is connected to the outside by a small hole. To an outside observer, this small hole appears like a black body surface because any radiation incident on the hole will completely absorbed after multiple reflections on the interior surface of the cavity.

perfectly reflect electromagnetic radiation (e.g. metallic walls), which has a very small hole on its surface. Radiation that enters through the hole will be trapped inside the cavity and gets completely absorbed after successive reflections on their inner surfaces of the cavity.The **small hole** absorbs radiation like a black body. On the other hand when this cavity is heated to a temperature $\boldsymbol{T}$, the radiation that leaves the hole is black body radiation and the hole behaves as a perfect emitter, as temperature increases the hole will eventually glows. ***So the radiation leaving the hole of a heated hollow cavity is known as black body radiation.***[3]

Chapter3

Need of Black body and Wien's prediction

Black body is an idealization which is seriously needed for material independence, otherwise our experiment will material dependent, and a black body is a prefect emitter and absorber. Our main aim is to make our life and calculation easier so that we take black body. Now we are going to heat our taken black body to proceed our experiment.[4][5]

I am going to introduce several properties of the universal function of temperature $\boldsymbol{T}$ and frequency which describes this equilibrium spectral distribution had been established predicting many decades. In order to formulate these, it is convenient to introduce the function as $\rho(\nu, T)$, where $\rho(\nu, T)d\nu$ is the energy per unit volume in thermal radiation, at absolute temperature $\boldsymbol{T}$ which lies in the frequency interval from ν to $\nu + d\nu$. STEFAN[1] had

---

[1] Before him BOLTZMANN had already found that expression by applying a Carnot cycle, using radiation particles as working fluid. In a piston cylinder system the pressure exerted on the piston,

$$p = e/3 \tag{3.1}$$

where e is the energy density, then by using conservation of energy we can write,

$$Tds = d(eV) + pdV = edV + V\frac{de}{dT} + \frac{e}{3}V \tag{3.2}$$

$$Tds = V\left(\frac{\partial e}{\partial T}\right)dT + \frac{4}{3}edV \tag{3.3}$$

By taking cross derivatives of entropy w.r,t T and V are,

$$\frac{1}{T}\frac{de}{dT} = \frac{4}{3}\left(T\frac{de}{dT} - e\right)\frac{1}{T^2} \tag{3.4}$$

$$\frac{de}{dT} = 4\frac{e}{T} \rightarrow e = \sigma T^4 \tag{3.5}$$

found experimentally in 1879 that the total energy density, integrated over all frequencies, is proportional to the fourth power of the temperature[6], thus $\rho(\nu, T)$ satisfies the equation,

$$\int_0^\infty \rho(\nu, T) d\nu = \sigma T^4 \tag{3.6}$$

where σ is a constant. This experimental result of STEFAN was derived theoretically in 1884 by BOLTZMANN [7] who applied the second law of thermodynamics in radiation and treating it as a gas whose pressure was radiation pressure of MAXWELL'S electromagnetic theory. In 1893 WIEN drew another conclusion from the second law of thermodynamics which imposed a significant limitation of the energy distribution function $\rho(\nu, T)$. This displacement law of WIEN'S [8] requires that $\rho(\nu, T)$ have the form,

$$\rho(\nu, T) = \nu^3 f\left(\frac{\nu}{T}\right) \tag{3.7}$$

The plot of this function is listed in list of figure section, where $f(\nu/T)$ depend only the ratio of frequency to temperature[2]. There was also an important result known to Planck when he started his work in the problem, was the *distribution law proposed by* WIEN 1896 which gave the explicit form for the function $\rho(\nu, T)$, or the function $f(\nu, T)$:

$$\rho(\nu, T) = \alpha \nu^3 exp\left(\frac{-\beta\nu}{T}\right) \tag{3.8}$$

where α and β are constants. Here WIEN had given a theoretical argument for the correctness of the given equation. This expression the high frequency data remarkably well but fails badly at low frequency, see fig ().

The energy density shows a pronounced maximum at a frequency, which increases with temperature , that is *the peak of the radiation spectrum occurs at a frequency that is propor-*

---

[2] It is worth noting that the WIEN displacement law implies the STEFAN-BOLTZMANN law, since $\int_0^\infty \rho(\nu, T) d\nu = \int_0^\infty \nu^3 f(\nu/T) d\nu = T^4 \int_0^\infty x^3 f(x) dx$, and the last integral, so long as it exists, is just a pure number

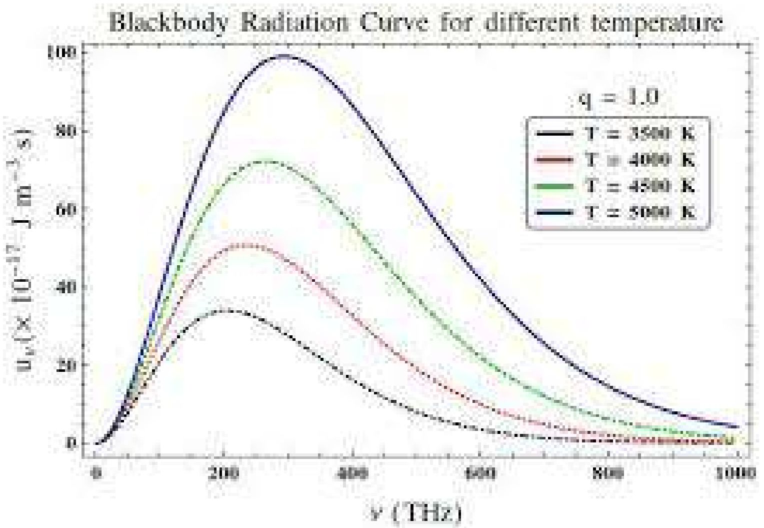


Figure 3.1: A plot of the energy of the radiation of a black body versus frequency for various temperatures

tional to the temperature.

The first accurate measurements of $\rho(\nu, T)$ were made by O. LUMMER and E. PRINGSHEIM in 1899.[9] The observed spectral radiation density is already shown. We see that for fixed ν , $\rho(\nu, T)$ increases with increasing T. At each temperature there is a maximum frequency for which $\rho(\nu, T)$ has its maximum value.

Using thermodynamical arguments and second law of thermodynamics KIRCHHOFF proved that flux of radiation inside cavity is isotropic and the radiation is homogeneous. This condition holds for each frequency.

So from WIEN'S prediction and arguments it is seen that it is applicable for only when the black body emits very large range of frequency in radiation. The experimental situation was not clear PASCHEN'S latest measurements supported WIEN'S law but the work of LUMMER and PRINGSHEIM extended to lower frequencies. Then a new distribution law had been proposed by THIESEN to fit the experimental data and at a same time to be consistent with Stefan-Boltzmann law and displacement law.

By October 1900, the experimental picture had changed considerably, . The very careful work of RUBEN'S and KURLBAUM [9] with long waves over wide range of temperatures had shown beyond any doubt that WIEN'S distribution was inadequate. These measurements indicated clearly that for very long wavelengths the distribution function $\rho(\nu, T)$ approached a very different form , which is becoming proportional to absolute temperature T.

Chapter4

Rayleigh and Jeans calculation and the problem

Rayleigh had published a short note [10] in June 1900 issue of *Philosophical Magazine* under the title *"Remarks upon the Law of Complete Radiation.* In two pages he had shown that ***if the equipartition theorem of statistical mechanics could be applied to the modes of aetherial vibration, then the distribution law for black body radiation is uniquely determined to have a form which is completely different from WIEN'S distribution*** . RAYLEIGH suggested a modification of WIEN'S distribution which appeared to him more probable a priori.

RAYLEIGH'S method for arriving at the radiation distribution law was different from any other approach. His argument concerned itself directly with the radiation and didn't need to refer to a material system with which it was in equilibrium. (Actually RAYLEIGH focused on understanding the nature of the electromagnetic radiation inside cavity.) The number of standing waves allowed modes of electromagnetic vibration of enclosure, whose frequencies lie in the interval from ν to $\nu + d\nu$, is proportional to $\nu^2 d\nu$. Then according to the equipartition theorem, the average energy of everyone of these modes, regardless of its frequency, would be proportional to T at thermal equilibrium, , with a universal proportionality constant. It follows at once that the distribution function $\rho(\nu, T)$ must have the form,

$$\rho(\nu, T) \propto \nu^2 T \tag{4.1}$$

so this is in accord with the displacement law.

RAYLEIGH didn't trouble to point out explicitly in this note, it must have been quite obvious to him that a distribution law of this form could not possibly hold for all frequencies, the integral of $\rho(\nu, T)$ over all frequency would diverge. After having obtained the result expressed in Eq. (prev. equn.), RAYLEIGH added if we introduce the exponential factor, the complete expression will be,

$$\rho(\nu, T) \propto \nu^2 \; T \; exp\left(\frac{-\beta\nu}{T}\right) \tag{4.2}$$

This equation represents the facts observation as well as (the WIEN'S distribution).[9].

4.1 The picture inside the radiating hollow cavity

The simplest model to regard the radiating system as compared of a collection of **charged linear harmonic oscillators, which according to the electromagnetic theory of light, radiate electromagnetic waves because of their accelerated motion**[11]. Consider a cavity full of such radiation, then the atomic oscillators in the walls enclosing the cavity will continually exchange energy with the radiation in the cavity. Severally an equilibrium condition will be established when the energy density of the electromagnetic radiation will assume an equilibrium value determined by the temperature T of the cavity walls.

When the temperature of the walls is increased, the amplitudes of existing modes of vibration of the oscillators are increased. Also new modes are excited for which the frequencies are higher.

To get the equilibrium radiation density in the cavity for a given frequency $\nu = c/\lambda$, we have to find the number $N(\nu)$ of oscillators per unit volume having the frequency ν and to multiply this by the average energy of the oscillators present on the walls of the cavity $\langle E \rangle$. $N(\nu)$ can be calculated by determining the number of modes of stationary vibrations, which can be excited in a three dimensional box of specified dimension and appropriate boundary conditions and can be calculated as follows.

Consider a cavity full of electromagnetic radiation with frequencies ranging from 0 to ∞. The cavity is in thermal equilibrium with its walls which are at temperature T. Now we assume the cavity to be cubical with the sides of length a each. The radiation is incident normally on the walls. Due to reflections from the walls standing waves are generated in the cavity with nodes at the walls.

The three dimensional equation for propagation of electromagnetic waves can be written as,

$$\nabla^2\phi = \left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}\right) = \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} \tag{4.3}$$

where ϕ is the variable electromagnetic field. Now consider a sinusoidal wave with time dependence of the form,$\phi \sim exp(i\omega t)$ where $\omega = 2\pi\nu$ is the concular frequency, now putting into Eq.(4.3) we get $\partial^2\phi/\partial t^2 = -\omega^2\phi$. Hence from Eq.(4.3) we get,

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} + \frac{\omega^2\phi}{c^2} = 0 \tag{4.4}$$

where ϕ represents the space part of the field variable. Our preferred boundary conditions to solve the above equation,

$$\phi = 0; x = y = z = 0$$

and

$$\phi = 0; x = y = z = a$$

Using separation of variables, we can write,

$$\phi = \phi(x, y, z) = \phi_x(x)\phi_y(y)\phi_z(z) = \phi_x(x)\phi_{yz}(y, z) \tag{4.5}$$

substitute this in place of ϕ we get,

$$\frac{1}{\phi_x}\frac{d^2\phi_x}{dx^2}+\frac{\omega^2}{c^2}=-\frac{1}{\phi_{yz}}\left(\frac{d^2\phi_{yz}}{dy^2}+\frac{d^2\phi_{yz}}{dz^2}\right)=\frac{\omega_{23}^2}{c^2}=constant \tag{4.6}$$

We can see the L.H.S. member in the above equation is a function of x only and the R.H.S. is a function of (y,z), each must be equal to be a constant, which I write as ω_{23}^2/c^2 , by rearranging,

$$\frac{d^2\phi_x}{dx^2}+\frac{\omega_1^2\phi_x}{c^2}=0 \tag{4.7}$$

where $\omega_1^2=\omega^2-\omega_{23}^2$. So the solution of the Eq.(4.7) of the form,

$$\phi_x=A_1sin\frac{\omega_1 x}{c}+B_1cos\frac{\omega_1 x}{c} \tag{4.8}$$

The boundary condition $\phi_x=0$ at $x=0$ requires us to put $B_1=0$ (must be) which gives,

$$\phi_x=A_1sin\frac{\omega_1 x}{c} \tag{4.9}$$

also $\phi_x=0$ for $x=a$, then we get,

$$\frac{\omega_1 a}{c}=n_1\pi\Rightarrow\frac{\omega_1}{c}=\frac{n_1\pi}{a} \tag{4.10}$$

where n_1 is an integer then we get,

$$\phi_x=A_1sin\frac{n_1\pi x}{a} \tag{4.11}$$

Using appropriate boundary condition and separation of variables and solving for ϕ_y and ϕ_z we get

$$\phi_y=A_2sin\frac{n_2\pi y}{a} \tag{4.12}$$

and

$$\phi_z = A_3 sin\frac{n_3\pi z}{a} \tag{4.13}$$

where n_2 and n_3 are integers to satisfy boundary conditions $\phi = 0$ at $y = a$ and $z = a$, then finally I get

$$\phi = \phi_0 sin\frac{n_1\pi x}{a} sin\frac{n_2\pi y}{a} sin\frac{n_3\pi z}{a} \tag{4.14}$$

from the Eq. (4.10) we get n_1 , n_2 and n_3 are following,

$$n_1 = \frac{\omega_1 a}{\pi c}, n_2 = \frac{\omega_2 a}{\pi c}, n_3 = \frac{\omega_3 a}{\pi c} \tag{4.15}$$

here $\omega^2 = \omega_1^2 + \omega_2^2 + \omega_3^2$ so,

$$n_1^2 + n_2^2 + n_3^2 = \frac{\omega^2 a^2}{\pi^2 c^2} = \frac{4a^2\nu^2}{c^2} \tag{4.16}$$

A set of values of n_1, n_2 and n_3 satisfying the above Eq.(4.16) represents a particular mode of vibration. To calculate the number of modes of vibration in the frequency interval ν to $\nu + d\nu$ we represent the n_1, n_2,n_3 values in a three dimensional diagram with n_1 along x axis, n_2 along y axis and n_3 along z axis. Each combination of n_1, n_2, n_3 values is then represented by a point in this diagram whose coordinates are (n_1,n_2,n_3).

Here I consider the two dimensional analogue in which n_1 is plotted along x and n_2 is plotted along y axis, as shown in figure (4.1). The representative points corresponding to the different modes of vibration are shown by the small circles, at the points of intersection of the straight lines drawn parallel to the n_1 and n_2 axes, unit distance apart. For large values of n_1 and n_2 number of representative point is the same as number of unit square. So the number of modes in vibration between frequency range ν to $\nu + d\nu$ can determined by counting the number of unit squares between two circular arcs of radii $r = 2\nu a/c$ and $r + dr = 2(\nu + d\nu)a/c$ in first quadrant. So the number is approximately equal to the area of the anulus divided by the area of each square.

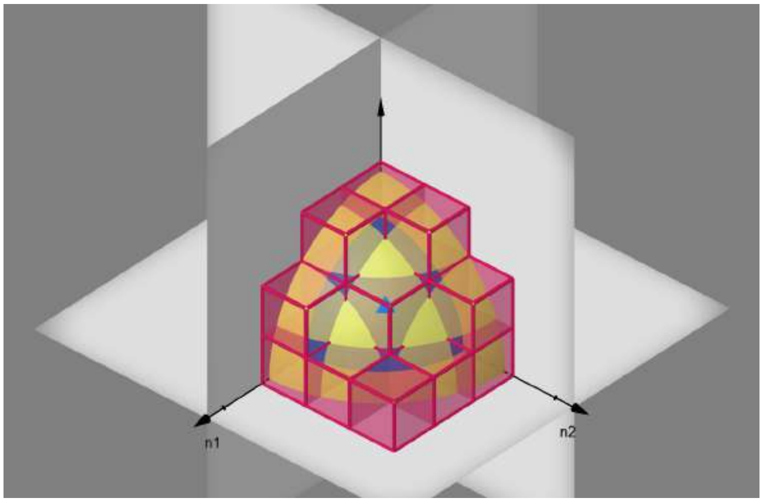


Figure 4.1: Three dimensional analogue of n_1 , n_2 and n_3

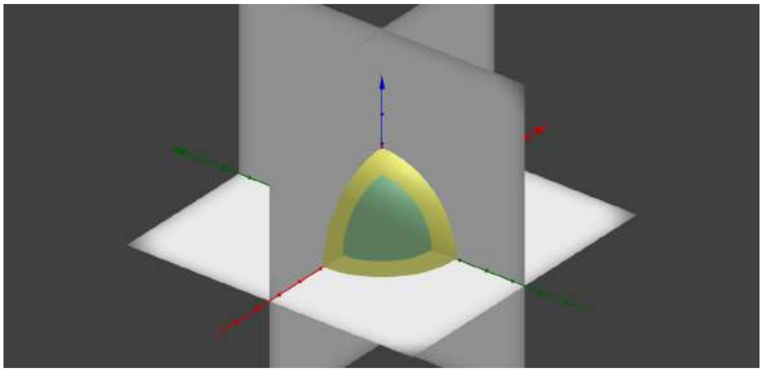


Figure 4.2: Volume of first octant of spherical shell on which the solutions are lying

But in $3-D$ case I have to draw straight line parallel to the n_1 n_2, n_3 axes unit distance apart. The point of intersection of these lines give the representative points corresponding the different modes of vibration see the figure (4.1). Hence the modes of vibration for frequencies ranging from ν to $\nu + d\nu$ is obtained by counting the number of unit cubes in the first octant of the three dimensional diagram between the two spheres of radii $r = 2\nu a/c$ and $r + dr = 2(\nu + d\nu)a/c$

Since we can associate one representative point with each unit cube (fig: 4.1) formed by the intersection of the three sets of mutually perpendicular straight lines parallel to the three axis, the number of such points $n_\nu d\nu$ is equals to the volume of the first octant of the spherical shell, as above divided by volume of each unit cube,

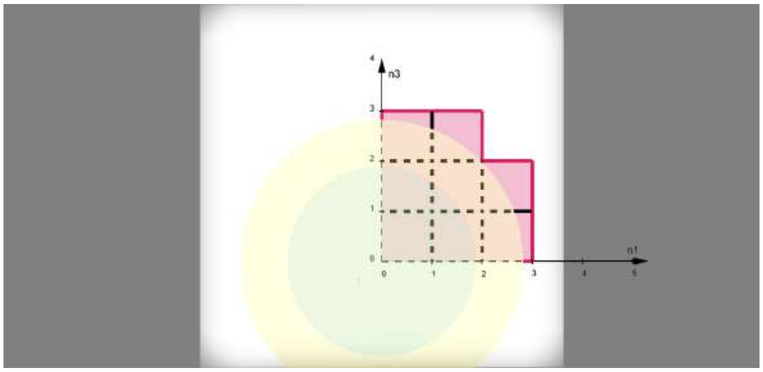


Figure 4.3: Detail view inside the spherical shell

$$n_\nu d\nu = \frac{4\pi r^2 dr}{8} = \frac{\pi}{2}\left(\frac{2a\nu}{c}\right)^2\left(\frac{2ad\nu}{c}\right)$$

$$\Rightarrow n_\nu d\nu = \frac{4\pi a^3}{c^3}\nu^2 d\nu = \frac{4\pi V}{c^3}\nu^2 d\nu \tag{4.17}$$

Here $V = a^3$ is the volume of the enclosure see (fig: 4.3). So the number of modes of vibration per unit volume of the enclosure for frequencies between ν to $\nu + d\nu$ is,

$$N_\nu(\nu)d\nu = \frac{n_\nu d\nu}{V} = \frac{4\pi}{c^3}\nu^2 d\nu \tag{4.18}$$

As electromagnetic radiation is transverse in nature with two possible directions of polarization, so we have to multiply above equation by two. Hence we get,

$$N_\nu(\nu)d\nu = \frac{8\pi}{c^3}\nu^2 d\nu \tag{4.19}$$

The average total energy of radiation leaving the cavity can be obtained by multiplying the average energy of the oscillators by the number of modes (standing waves) of the radiation in the frequency interval ν ***to*** $\nu + d\nu$***,*** [3]

$$N_\nu(\nu) = \frac{8\pi\nu^2}{c^3} \tag{4.20}$$

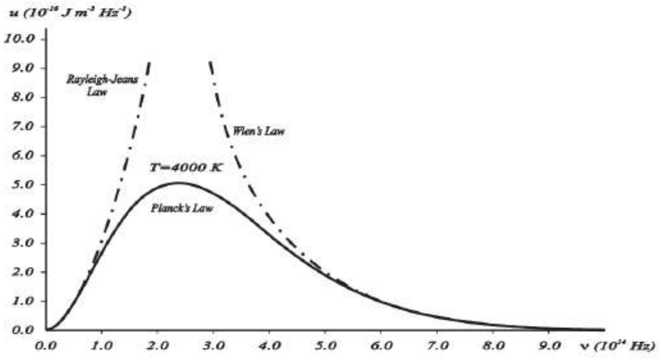


Figure 4.4: Comparison of various spectral densities: while the Planck and our experimental distributions match perfectly (solid curve), the Rayleigh-Jeans and the Wien's distributions (dotted curves) agree only partially with the experimental distributions.

here, $c = 3 \times 10^8 ms^{-1}$, the speed of light. So the electromagnetic energy density in the frequency range from ν to $\nu + d\nu$ is,

$$\rho(\nu, T) = N_\nu(\nu)\langle E\rangle = \frac{8\pi\nu^2}{c^3}\langle E\rangle \tag{4.21}$$

here $\langle E\rangle$ is the average energy of the oscillators present in the walls of the cavity. According to equipartition theorem of classical thermodynamics, all the oscillators in the cavity have same mean energy, this can't depend on their frequencies[2]

$$\langle E\rangle = \frac{\int_0^\infty Eexp(-E/kT)dE}{\int_0^\infty exp(-E/kT)dE} = kT \tag{4.22}$$

where $k = 1.3807 \times 10^{-23} JK^{-1}$ is the Boltzmann constant. Now substitute the value of $\langle E\rangle$ into our above equation leads to ***RAYLEIGH-JEANS*** expression, [3]

$$\rho(\nu, T) = \frac{8\pi\nu^2}{c^3}kT \tag{4.23}$$

Except for low frequencies, this law is in complete disagreement with experimental data. From above equation you can see $\rho(\nu, T)$ diverges for high values of ν , but in ***practical***

---

[2] Put $\beta = 1/kT$, then we have $\langle E\rangle = -\frac{\partial}{\partial\beta}ln(\int_0^\infty exp(-\beta E)dE) = -\frac{\partial}{\partial\beta}ln(1/\beta) = 1/\beta = kT$

situation it must be finite. Also if we integrate the above function over all frequencies then the integral ***diverges.This result is known as ultraviolet catastrophe.***

So our theory does not match with practical situation!!!

Chapter5

Planck's approach about the problem

In the series of five papers which PLANCK presented to Prussian Academy of Sciences in the years 1897 to 1899, he set forth his program to a theory of radiation. Actually MAX PLANCK was in a field of thermodynamics, so the program of theory of radiation arouse naturally from his earlier work of thermodynamics. He set his focus to finding out a basis in electrodynamics for irreversible approach of radiation to equilibrium. He started the work from the idea that ***the conservative system consisting of electromagnetic radiation in an enclosure, interacting with a collection of harmonic oscillators , could be shown to approach an equilibrium state, without the need for any assumptions beyond the laws of electromagnetism.*** Actually he thought, he saw the basic mechanism for the irreversible behaviour of the system in the way in which an ***oscillating dipole emits electromagnetic energy as a spherical wave, changing the character of the radiation incident upon the walls of hollow cavity, in irreversible manner.*** The harmonic oscillators were chosen not because they were thought to be as a simplest realistic model but rather for KIRCHHOFF'S theorem asserted that the equilibrium radiation distribution was independent of the system with which the radiation interacted. From RAYLEIGH-JEANS calculation we have already approach to an expression of spectral distribution $\rho(\nu, T)$ to the average energy $\langle E \rangle$ of an harmonic oscillator of frequency ν,

$$\rho(\nu, T) = \left(\frac{8\pi\nu^2}{c^3}\right)\langle E \rangle \tag{5.1}$$

So, PLANCK needed only to determine the $\langle E \rangle$, the average energy of a harmonic oscillator at temperature T. But PLANCK made no use of equipartition theorem, used in statistical mechanics, but referred as a "thermodynamic" approach where he was looking for a suitable relationship between the energy and entropy of the oscillator [9],

$$S = -\frac{E}{\beta\nu} ln \frac{E}{ae\nu} \tag{5.2}$$

where $E =$ average energy of the oscillator ($\langle E \rangle$) and β and a are constants and e is the base of natural logarithm. Using this expression PLANCK defined the entropy of an oscillator then he could determine the entropy of radiation in equilibrium and the total entropy was a monotonically increasing function of time. He was aware about his choice of a particular expression for entropy as a function of energy determines the resulting distribution law.

After the new experimental findings of RUBENS and KURLBAUM [12], PLANCK was in a problem to determine a distribution law which was consistent both with positive results of his own work and the new experimental findings . If WIEN'S distribution is valid then negative reciprocal of $\partial^2 S/\partial E^2$ is proportional to E and the next possibility to take $\partial^2 S/\partial E^2$, or rather its negative potential proportional to E^2, then $\rho(\nu, T)$ will proportional to E and T. The proper limiting forms for low and high frequencies be preserved by taking $-(\partial^2 S/\partial E^2)^{-1}$ proportional to $E(E+\gamma)$, where γ is frequency dependent constant. By standing upon this groundwork PLANCK proposed the distribution law,

$$\rho(\nu, T) = \frac{A\nu^3}{exp(B\nu/T) - 1} \tag{5.3}$$

where A and B are constants, this is the law follows from the assumption mentioned for $\partial^2 S/\partial E^2$, lastly the frequency dependence is fixed by displacement law [3]

---

[3] Taking $\partial^2 S/\partial E^2$ proportional to $-[E(E+\gamma)]^{-1}$ one can integrate it to find $\partial S/\partial E$ as a function of E. Also we know $\partial S/\partial E$ is equal to T^{-1} , from second law of thermodynamics. So you can obtain E in the form $d_1[exp(d_2/T) - 1]^{-1}$, where d_1 and d_2 are frequency dependent constants. Using Eq.(5.1) one can easily get $\rho(\nu, T) = (8\pi d_1/c^3)\nu^2[exp(d_2/T) - 1]^{-1}$ so d_1 and d_2 both are proportional to ν, in order to satisfy the displacement law.

Chapter6

Planck's statistical calculation and the physical picture

The physical ground of the electromagnetic radiation theory including the hypothesis of the "natural radiation" resists destructive criticism. The energy distribution law of the normal spectrum is totally defined if one succeeds in calculation of entropy S of irradiated monochromatic vibrating resonator as a function of its vibrational energy. Then from relation $dS/dE = T^{-1}$ one keeps the temperature T dependence of energy E and the energy E related with a radiation density of appropriate number of vibration so the temperature dependent on its radiation density is also obtained. So the normal distribution of energy is one for which the radiation densities of any different numbers of vibrations have the same temperature.

So Now the problem reduced to that of definition S as a function of E. Then PLANCK considered that WIEN"S law is necessarily the universal one, and one more condition must needed for calculating S uniquely. Eventually PLANCK had found one such condition that, ***infinitesimal irreversible alteration of the near thermal equilibrium being system of N uniform, just in stationary radiation field placed resonators,***the bound up with it alteration of the total entropy $S_N = NS$ depends only their total energy $E_N = NE$ and their alteration but not the energy E of particular resonators. This statement leads again to the WIEN"S energy distribution law.

The direction of those deliberate thoughts was indicated by the consideration of the fragility of early made supposition. The direction is described below.

6.1 The calculation of entropy of any resonator as a function of its energy

6.1.1 Total energy and total entropy of resonators

Entropy is conditioned by disorder, and this in accordance with electromagnetic theory of radiation which is based on monochromatic vibrations of any resonators if it remains in stationary field of radiation.Due to non regularity it permanently changes its amplitude and its phase. If the amplitude and the phase both are absolutely constant and also vibrations are homogeneous, no entropy could exist and the vibrational energy should be quite free convertible into work. E is constant energy of a particular vibrating resonator is therefore as a simultaneous average of energies of large number N of uniform resonators, which are placed into the stationary radiation field [13]. All the resonators are removed from one another to have no effect on each other directly. Therefore the total energy,

$$E_N = NE \tag{6.1}$$

of such system of N resonators is corresponded to to certain total entropy,

$$S_N = NS \tag{6.2}$$

of the same system. The average entropy of any separate resonator is represented by S and the entropy S_N is depend on the disorder with which total energy U_N is distributed among particular resonators.

Consider an entropy of a system is S_N with an arbitrary remaining additive constant to be proportional to logarithm of the probability Ω with which N resonators altogether possess the energy U_N, hence [14] [15],

$$S_N = k\ ln\ \Omega + constant \tag{6.3}$$

This supposition originates from the definition of probability[4] Ω which is put on the ground of the electromagnetic theory of radiation, enabling about this probability is a definite sense. It is the high time to find out the probability Ω of N resonators having a vibrational energy E_N. ***It is necessary for it to imagine U_N not as a continuous unlimited divided value but as a discreet one composed of integer number of finite equal parts.*** [13] Taking ϵ as a energy element so one can deduce that

$$E_N = P \cdot \epsilon \tag{6.4}$$

where P is an integer (0,1,2,3.....n), and the value for ϵ have to be defined.

The distribution P of energy elements among N resonators can happen in some limited number of ways, that's why we give a name "complexion" or microstates to every such manner following L. BOLTZMANN, who had used that idea for an expression [14]. So if we numbered the resonators and distribute it along a row by $1, 2, 3, ..N$ then number of energy elements fallen on it in an arbitrary distribution. As for example, consider ten resonators ($N = 10$) and the total distribution is $P = 50$ then,

$$N = 1 \rightarrow P = 7; N = 2 \rightarrow P = 10; N = 3 \rightarrow P = 0; N = 4 \rightarrow P = 3; N = 5 \rightarrow P = 15; N = 6 \rightarrow P = 4;$$

Therefore the number Ω for all possible complexions is equal to one of all possible forms which are shown above. Hence by following combinations the number of all possible complexions,

$$\Omega = \frac{N \cdot (N+1) \cdot (N+2) ... (N+P-1)}{1 \cdot 2 \cdot 3 ... P} = \frac{(N+P-1)!}{(N-1)!P!} \tag{6.5}$$

---

[4] Actually Ω is the the number of microstates to define a macrostate, or how the total energy is distributed among all resonators during the heating process.

According to Stirling's approximation,

$$N! = N^N \tag{6.6}$$

so the appropriate approximation [15],

$$\Omega = \frac{(N+P)^{N+P}}{N^N \cdot P^P} \tag{6.7}$$

Now according to the hypothesis the probability of that N resonators possess vibrational energy U_N is proportional to the number Ω of all possible complexions with energy U_N. The validity of the hypothesis will result in the conclutions on the special nature of vibration of resonators.

Therefore according to equation (6.3) the entropy of taken system of resonators with an additive constant is,

$$S_N = k\ ln\ \Omega \Rightarrow S_N = k[(N+P)\ ln\ (N+P) - N\ ln\ N - P\ ln\ P] \tag{6.8}$$

From Eq. (6.1) and Eq. (6.4)

$$S_N = kN\left[\left(1+\frac{E}{\epsilon}\right)\ ln\left(1+\frac{E}{\epsilon}\right) - \frac{E}{\epsilon}\ ln\frac{E}{\epsilon}\right] \tag{6.9}$$

Hence entropy of a resonator,

$$S = k\left[\left(1+\frac{E}{\epsilon}\right)\ ln\left(1+\frac{E}{\epsilon}\right) - \frac{E}{\epsilon}\ ln\frac{E}{\epsilon}\right] \tag{6.10}$$

Finally introducing yet more the entropy of resonator S,

$$\frac{dS}{dE} = \frac{1}{T} \tag{6.11}$$

Turns out to be,

$$\frac{dS}{dE} = \frac{1}{\nu} f \left(\frac{E}{\nu} \right) \tag{6.12}$$

Integrating[5]

$$S = f \left(\frac{E}{\nu} \right) \tag{6.13}$$

This is the entropy of vibrating resonator in an arbitrary diathermal medium depends on the variable E/ν and it keeps the universal constants.

Applying WIEN'S displacement law in Eq. (6.10) so then the energy element ϵ should be proportional to number of vibrations of resonators having frequency ν,

$$\epsilon = h \cdot \nu \tag{6.14}$$

Substituting into equation (6.10) we get [9],

$$S = k \left[\left(1 + \frac{E}{h\nu} \right) \ ln \ \left(1 + \frac{E}{h\nu} \right) - \frac{E}{h\nu} \ ln \ \frac{E}{h\nu} \right] \tag{6.15}$$

where h and k are universal constants. Now substituting the expression for S into Eq. (6.11) we get,

$$\frac{k}{h\nu} \ ln \ \left(1 + \frac{h\nu}{E} \right) = \frac{1}{T} \Rightarrow E = \frac{h\nu}{exp \left(\frac{h\nu}{kT} \right) - 1} \tag{6.16}$$

Recalling the Eq. (4.21) and substituting the energy distribution law from Eq. (6.16) to Eq. (4.21) and get [6]

$$\rho(\nu, T) = \frac{8\pi\nu^3}{c^3} \frac{h}{exp \left(\frac{h\nu}{kT} \right) - 1} \tag{6.17}$$

---

[5] Eq. (3.2) and Eq. (4.21) require that $E = \nu f(\nu/T)$. There is a same way to writing the result as $T = \nu F(E/\nu)$. As $\partial S/\partial E = 1/T$ then one can easily obtains $\partial S/\partial E = \nu^{-1}[F(E/\nu)]^{-1}$

[6] Eq. (6.17) is actually the Eq. (5.3) rewritten with the constants A and B.

6.2 How now the energy distribution is derived ?

MAX PLANCK postulated that the energy of the radiation (of frequency ν) emitted by the oscillating charges must come only integer multiples of $h\nu$ so,

$$E = h\nu \tag{6.18}$$

Assuming the energy oscillator is quantized, so then we can replace the integration of Eq. (4.22) corresponds to energy continuum- by a discreet summation,

$$\langle E\rangle = \frac{\sum_{n=0}^{\infty} nh\nu \ exp\left(\frac{-nh\nu}{kT}\right)}{\sum_{n=0}^{\infty} exp\left(\frac{-nh\nu}{kT}\right)} \tag{6.19}$$

Put $\beta = 1/kT$ then we can write [7],

$$\langle E\rangle = -\frac{d}{d\beta}\left[ln \sum_{n=0}^{\infty} exp(-nh\nu\beta)\right] \tag{6.20}$$

$$= -\frac{d}{d\beta}\left[ln\left(\frac{1}{1 - exp(\beta h\nu}\right)\right]$$

$$\langle E\rangle = \frac{h\nu}{exp\left(\frac{h\nu}{kT}\right) - 1} \tag{6.21}$$

Now substituting it into Eq. (4.22) I get [3],

$$\rho(\nu, T) = \frac{8\pi\nu^3}{c^3}\frac{h}{exp\left(\frac{h\nu}{kT}\right) - 1} \tag{6.22}$$

Same as Eq. (6.17).

Now looking at the behavior of Planck's distribution (6.17) and (6.21) in the limits of both low and high frequencies, and then try to establish its connection to the relations of RAYLEIGH–JEANS, Stefan–Boltzmann, and Wien. First, in the case of very low fre-

---

[7] To derive Eq. (6.21) I use $1/(1 - x) = \sum_{n=0}^{\infty} x^n$ and $x/(1 - x)^2 = \sum_{n=0}^{\infty} nx^n$

quencies $h\nu << kT$, we can show that both (6.17) and (6.22) reduces to the Rayleigh–Jeans law Eq.(4.23), we can also obtain total energy density which can be expressed in terms of STEFAN - BOLTZMANN'S total energy released per unit surface area, [8]

$$\int_0^\infty \rho(\nu,T) = \frac{8\pi h}{c^3}\int_0^\infty \frac{\nu^3 d\nu}{exp\left(\frac{h\nu}{kT}\right)-1} = \frac{8\pi k^4T^4}{h^3c^3}\int_0^\infty \frac{x^3dx}{e^x-1} = \frac{8\pi^5k^4}{15h^3c^3}T^4 = \frac{4}{c}\sigma T^4 \quad (6.23)$$

where $\sigma = \frac{2\pi^5k^4}{15h^3c^2} = 5.67\times 10^{-8}Wm^{-2}K^{-4}$ is the STEFAN-BOLTZMANN constant. From PLANCK'S relation (6.22) leads to a finite total energy density of the radiation emitted from a blackbody, and hence avoids the **ultraviolet catastrophe**.Also for high frequencies we can easily asertain that PLANCK'S distribution yields WIEN'S rule.

6.2.1 Quantization of energy and the Action Integral

In 1916 Wilson and Sommerfeld offered a scheme that included both quantization rules as special cases. In essence; their scheme, which applies only to systems with coordinates that are periodic in time, consists in quantizing the action variable $J = \oint p\,dq = nh$,[16], of classical mechanics,

$$\oint p\,dx = nh \quad (6.24)$$

Where n is a quantum number and p is the momentum conjugate associated with the coordinate q.This is WILSON-SOMMERFELD quantization rule and this ,may leads to PLANCK'S quantization rule. For an illustration, consider a one-dimensional harmonic oscillator where a particle of mass , m oscillates harmonically between $-a \le x \le a$ its classical energy is given by,

$$E(x,p) = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2 \quad (6.25)$$

---

[8] By performing the integration in eq. (6.23) I use the integral $\int_0^\infty \frac{x^3dx}{e^x-1} = \frac{\pi^4}{15}$

So $p(E, x) = \sqrt{2mE - m^2\omega^2 x^2}$ and at turning points, $x_{min} = -a$ and $x_{max} = a$, so the total energy is potential energy, $V = (a) = \frac{1}{2}m\omega^2 a^2$, and $a = \sqrt{2E/m\omega^2}$. From symmetry consideration we can write the action as,

$$\oint p\,dx = 2\int_{-a}^{a} \sqrt{2mE - m^2\omega^2 x^2}\ dx = 4m\omega \int_0^a \sqrt{a^2 - x^2}dx \tag{6.26}$$

Changing variables $x = asin\theta$ I get,

$$\int_0^a \sqrt{a^2 - x^2}dx = \int_0^{\pi/2} cos^2\theta \cdot d\theta = \frac{a^2}{2}\int_0^{\pi/2}(1 + cos2\theta)d\theta = \frac{\pi E}{2m\omega^2} \tag{6.27}$$

Since, $\omega = 2\pi\nu$ where ν is the frequency of the oscillators, I get

$$\oint p\,dx = \frac{2\pi E}{\omega} = \frac{E}{\nu} \tag{6.28}$$

Putting into Eq.(6.24)

$$\frac{E}{\nu} = nh \Rightarrow E_n = nh\nu \tag{6.29}$$

This simple calculation shows that PLANCK rule for energy quantization is equivalent to quantization of action [16].

Chapter7

Conclusion and further topics

In summary the spectrum of the black body radiation reveals the quantization of radiation notably the particle behaviour of electromagnetic waves. the introduction of the constant h had indeed heralded the end of classical physics and the dawn of new era: physics of the micro physical world. This era stimulated by success of PLANCK'S quantization of radiation. Cheers!!! Finally we able to see the ad hoc fashion of our nature. The revolutionary idea is not always recognized as such. But the way of theoritical physics is followed by its own way, how much ad-hoc it might be. Quantized concept of energy was unrecognized in physics over four years until ALBERT EINSTEIN proposed his **On a Heuristic Point of View about the Creation and Conversion of Light** in 1905. The theory of radiation was not the central topic in physics at 1900.Many times PLANCK'S egoistic movement rejects BOLTZMANN'S distribution but actually BOLTZMANN was the **Master** on this topic, without his statistical approach on **complexion** this mistry could not figured out.

RAYLEIGH and JEANS did some probing into the significance of PLANCK'S work in 1905. It is certainly fair to say, and both of them were sympathetic to the idea of energy quanta also they were not interested to deveoping the idea further. This revolutionary idea could take root and grew only in a thought. The mind was ready and fortunately for science and in the work of ALBERT EINSTEIN the full significance of PLANCK'S concept began to show itself.

In spring 1905 EINSTEIN pointed out the distribution law and then stressed both its inconsistency with experiment and its internal paradoxical nature, due to infinite radiation of energy which it implies. All those things as well as the deep gap between PLANCK'S quanta and classical physics wer, as EINSTEIN was to write many years later "quite clear to me shortly after the appearence of PLANCK'S fundamental work." Then EINSTEIN showed that for non-classical part of PLANCK'S radiation formula went to WIEN'S limit. It means as EINSTEIN demonstrated by characteristically simple argument **the radiation itself behaved as if it consisted of energy quanta whose magnitude was given by** $h\nu$ **.** Those phenomena can't be explicable on the basis of electromagnetic wave theory. Using that hypothesis on energy quanta Arthur Compton proved the X-ray are consisting of photons or light quanta. After that, NIELS BOHR composed his 'hydrogen atom model'. I want to say the last thing that this idea can't be visualized or can't feel on that regular way, and if you able to visualize the way of quantum mechanics then either you are lying or you have gone mad to read those topics.

> *Quantum theory is so shocking that EINSTEIN could not bring himself to accept it. Without it, we'd have no computers or internet, no science of molecular biology, no understanding of DNA, no genetic engineering.*
> - JOHN GRIBBIN

> *All the fifty years of conscious brooding have brought me no closer to the answer to the question, 'What are light quanta?' Of course today every rascal thinks he knows the answer, but he is deluding himself.*
> -ALBERT EINSTEIN

References

[1] John Gribbin. “In search of Schrodinger’s cat”. In: 1 (2012), pp. 27, 55, 57.

[2] B H Bransden and C J Joachain. *Quantum Mechanics ,(Harlow, United Kingdom.* 2000.

[3] Nouredine Zettili. *Quantum mechanics: concepts and applications.* 2003.

[4] Ajoy K Ghatak and S Lokanathan. *Quantum mechanics: theory and applications.* Macmillan, 2004.

[5] Max Planck. “1 (c) English Translation”. In: (), pp. 34–35.

[6] John Crepeau. “A brief history of the T4 radiation law”. In: *Heat Transfer Summer Conference.* Vol. 43567. 2009, pp. 59–65.

[7] Massimiliano Badino. “The odd couple: Boltzmann, Planck and the application of statistics to physics (1900–1913)”. In: *Annalen der Physik* 18.2-3 (2009), pp. 81–101.

[8] AI Fisenko and SN Ivashov. “Determination of the true temperature of emitted radiation bodies from generalized Wien’s displacement law”. In: *Journal of Physics D: Applied Physics* 32.22 (1999), p. 2882.

[9] Martin J Klein. “Max Planck and the beginnings of the quantum theory”. In: *Archive for History of Exact Sciences* 1.5 (1961), pp. 459–479.

[10] Lord Rayleigh. “LIII. Remarks upon the law of complete radiation”. In: *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science* 49.301 (1900), pp. 539–540.

[11] SN Ghoshal. *Atomic Physics.* S. Chand Publishing, 2007.

[12] H Rubens and F Kurlbaum. “On the heat radiation of long wave-length emitted by black bodies at different temperatures”. In: *The astrophysical journal* 14 (1901), p. 335.

[13] Max Planck. “On the law of distribution of energy in the normal spectrum”. In: *Annalen der physik* 4.553 (1901), p. 1.

[14] Raj Kumar Pathria and Paul D Beale. *Statistical mechanics.* 2011.

[15] Kerson Huang. “Statistical mechanics, john wily & sons”. In: *New York* (1963), p. 10.

[16] S Mukhopadhyay, K Bhattacharyya, and RK Pathak. “Wilson–Sommerfeld quantization rule revisited”. In: *International Journal of Quantum Chemistry* 82.3 (2001), pp. 113–125.

PREDATOR-PREY MODEL

*The project submitted, in partial fulfilment of the requirement for the assignments in **PHSA CC-XI, PHSA CC-XII, PHSA DSE-I, PHSA DSE-II** Papers (Semester 5th) in the Department of Physics*

Submitted by

RATUL SARKAR

Registration No: A01-1122-111-037-2019

Supervisor Teacher: Prof. Kalyan Chatterjee

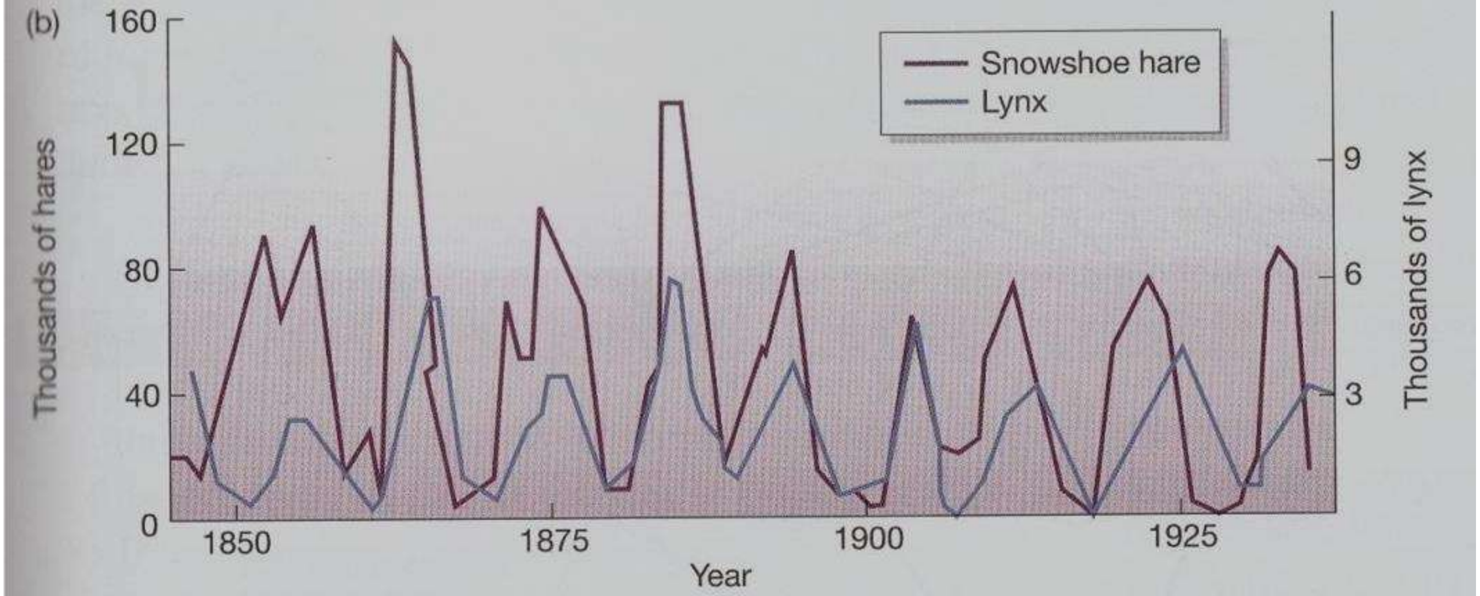
RAMAKRISHNA MISSION VIVEKANANDA
CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

INTRODUCTION

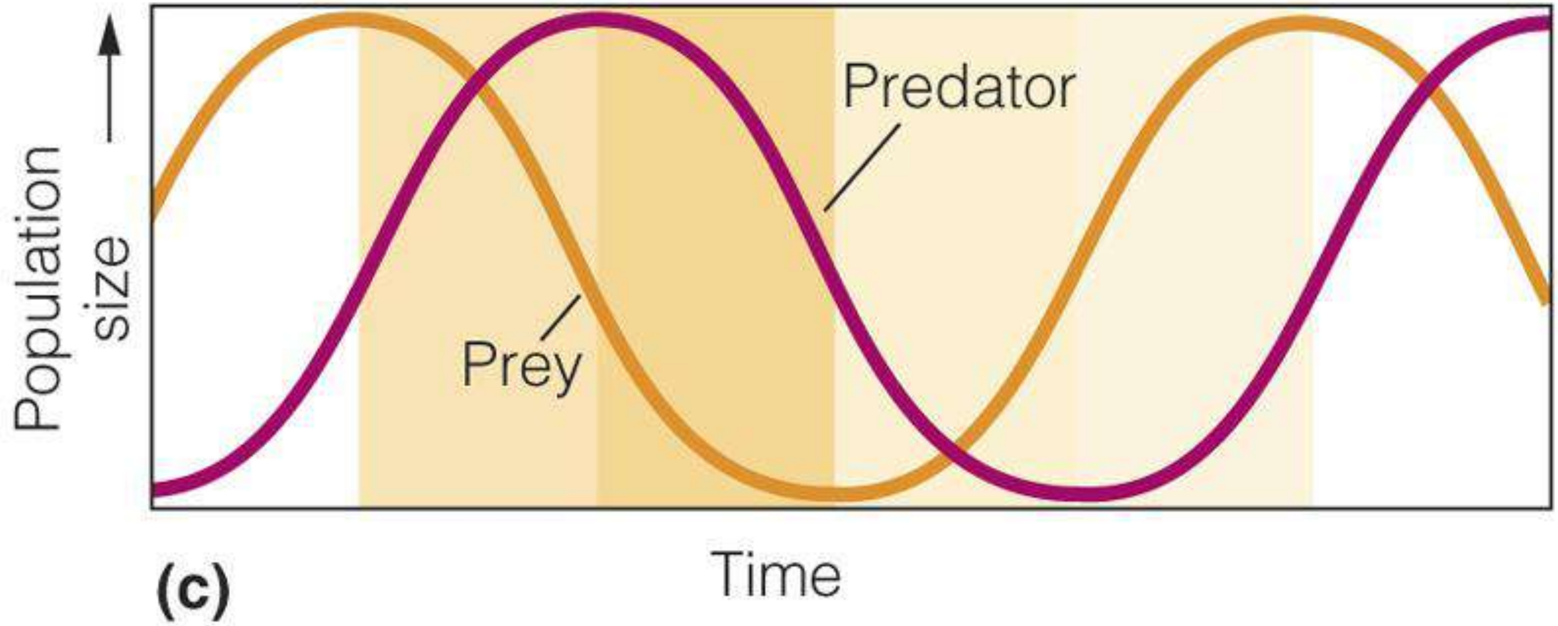
Predation is a straight-forward interspecies population interaction. One species uses another as a food resource. Predators play an important role in controlling prey population numbers in some systems. In simple systems, the predator-prey relationship results in coupled population oscillations.

Classic example of predator-prey dynamics:
Canadian lynx & snowshoe hare

(Coupled oscillation)



- prey numbers decreases, predator numbers increases to a point where the predation causes population decline in the prey item.

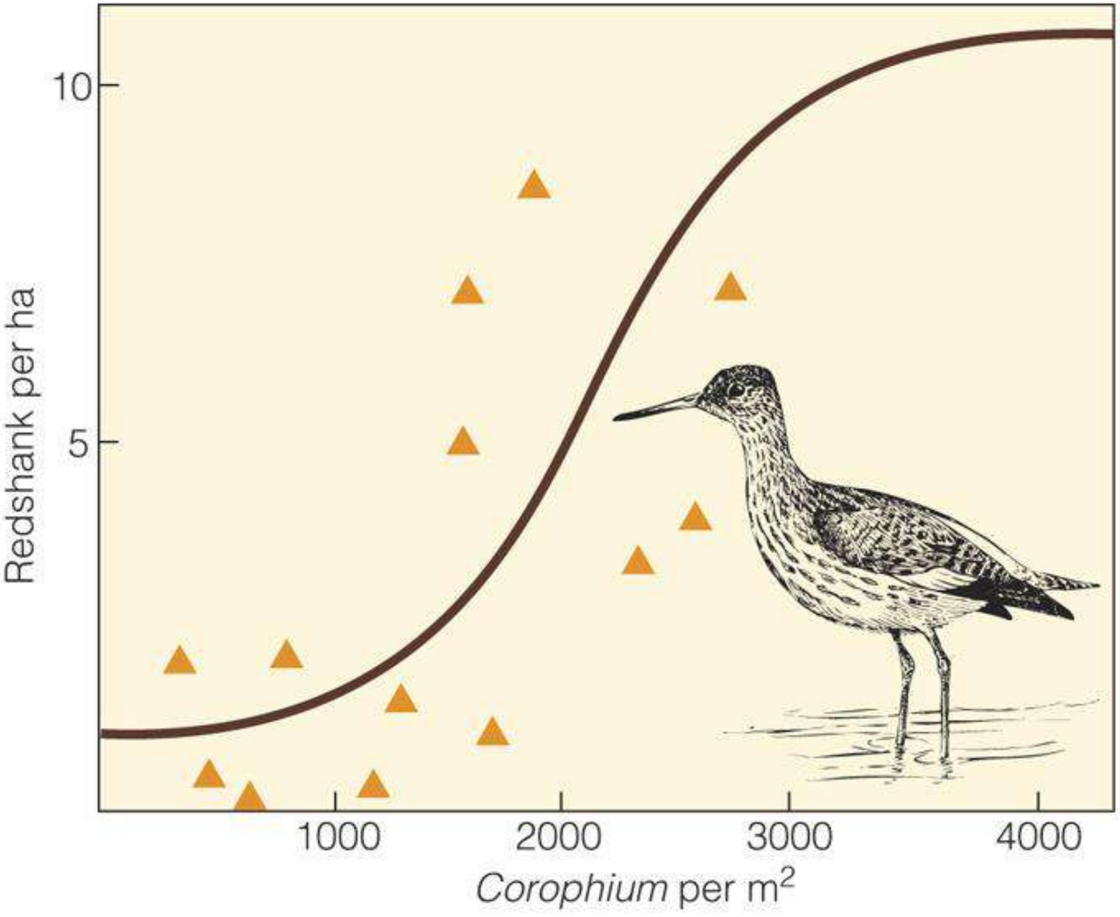


- Idealized predator-prey coupled dynamics.

•It is important to note that in most systems the **food web- the web of interactions among species-** is far more complex than just a single predator and single prey item. The relationships can become quite complex and the "coupled" nature of the interaction becomes much more vague.

In general, predator-prey relationships are much more complex than that displayed by lynx-hare oscillations.

Very often, but not always, an increase in **prey** density results in a straight-forward increase in **predator** population size.



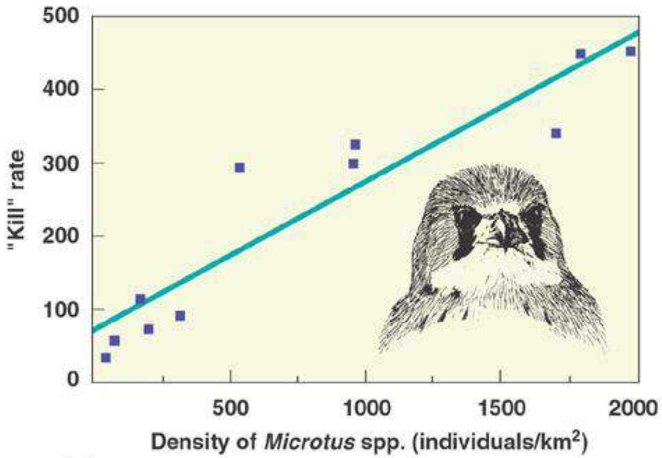
In this case often, but not always, an increase in **prey** density results in a straight-forward increase in **predator** population size.

The action of predators in the face of increasing prey availability can take different forms.

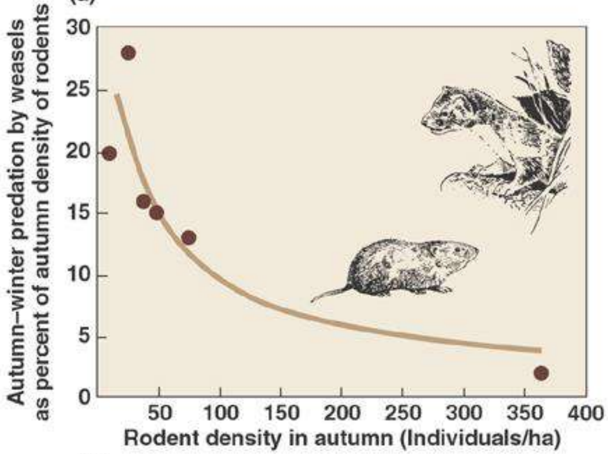
In the top panel, as the number of prey items (Microtus) increases, the number killed by the predator increases in a linear fashion.

In the middle panel- as the density of rodents increases, the *percentage* of the population killed by weasels declines in a curvilinear fashion

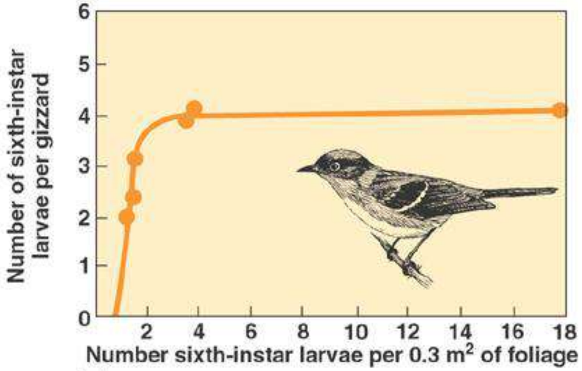
In the bottom panel, as the number of available prey items (sixth instar larvae) increases, the number of those found in the gut of the predator (bird) increases – the bird eats more- and then levels off.   There are various potential explanations of this, one being that the bird population is “satiated” at certain densities.



(a)



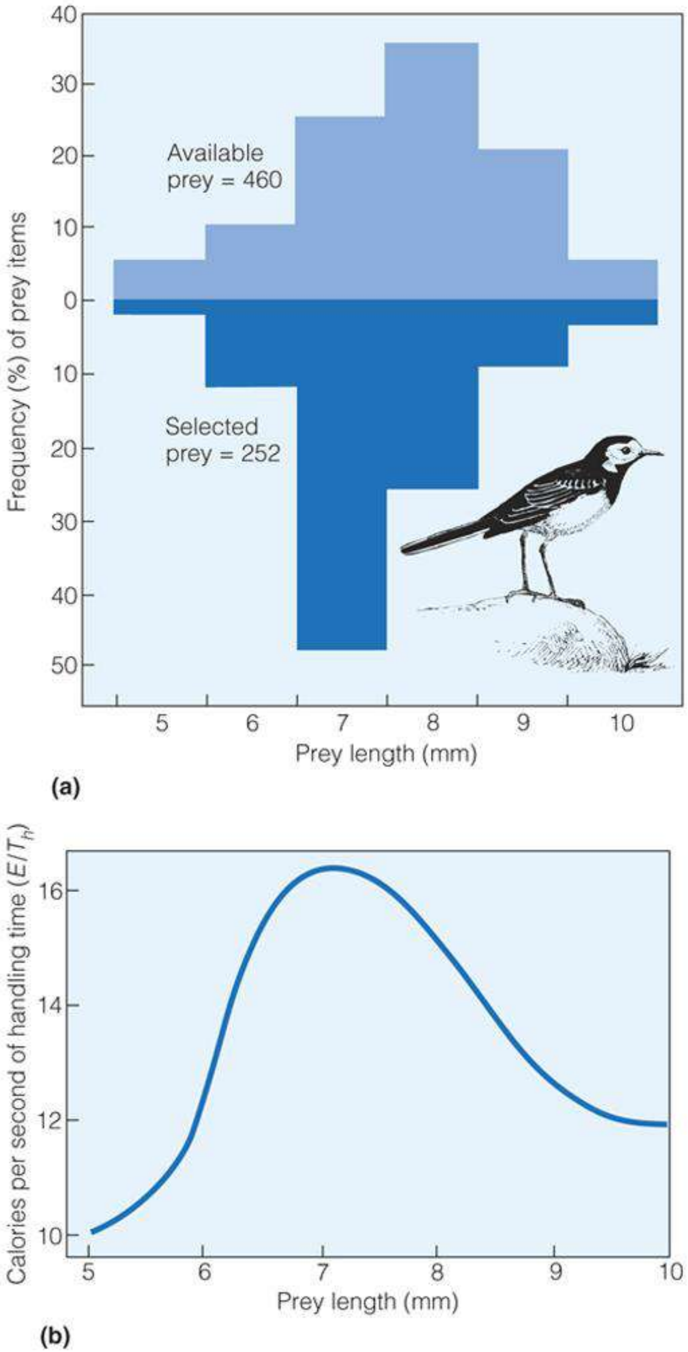
(b)

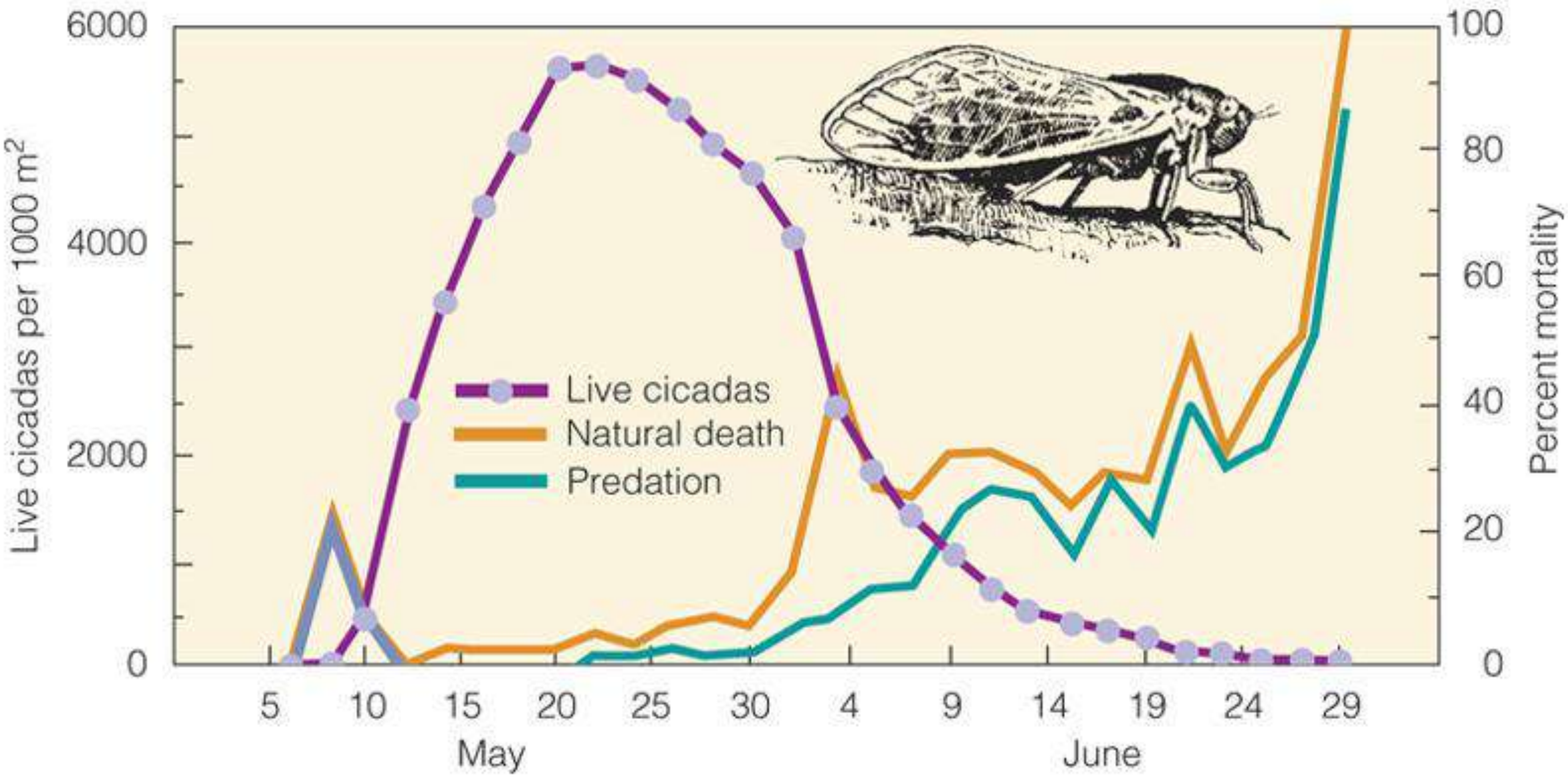


(c)

Prey are variable in value.

Don’t want to spend time & energy on prey items that are energetically expensive to process (if other options are available).

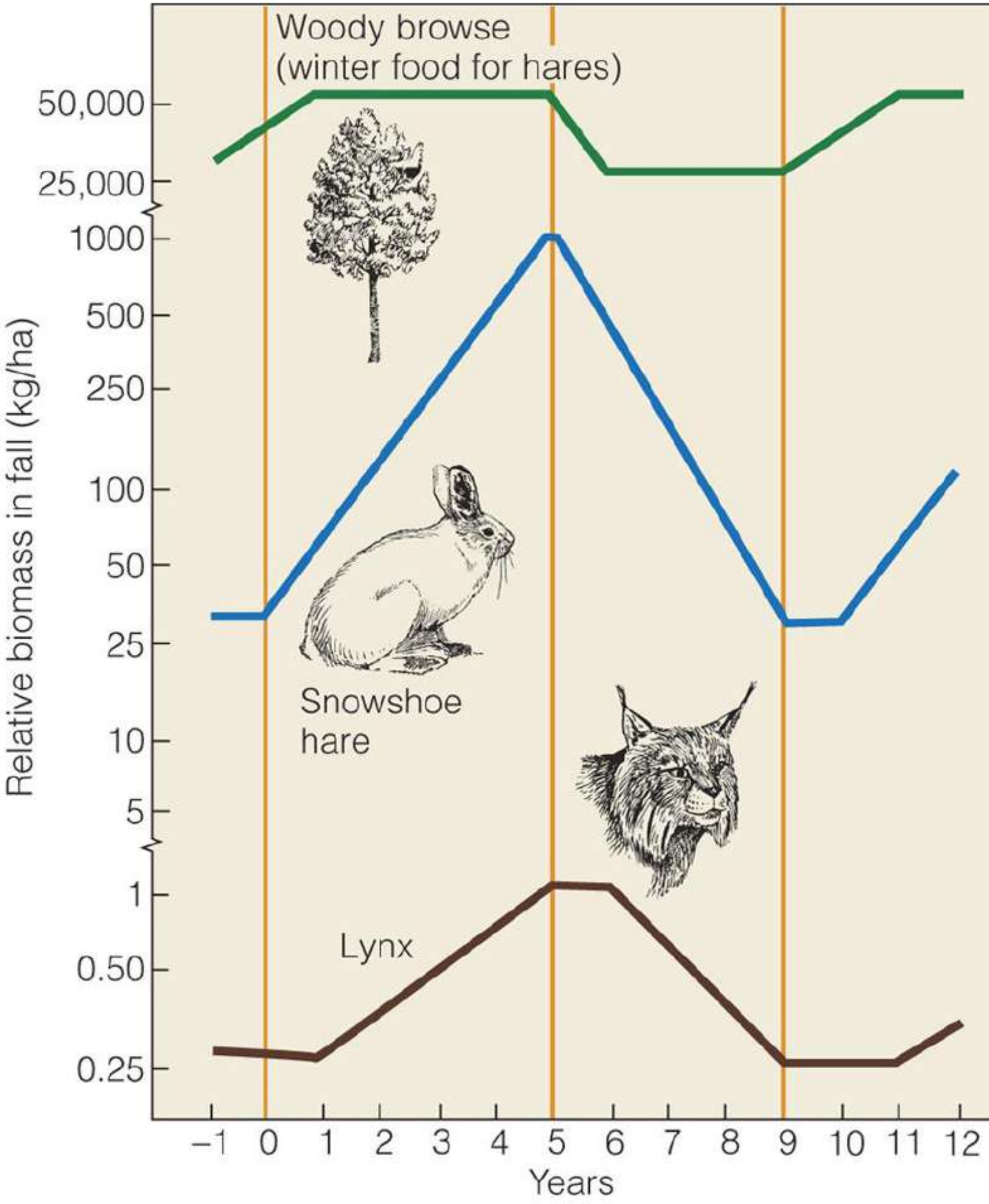




Predator-prey relationships can include:
 pulse, lag, response, lag, response- timing

Predator-prey relationships often have ramifications for other parts of the ecosystem.

The hare-lynx relationship is an example. Hares eat twigs, more hares = more damage to trees. More lynx = fewer hares and less damage to trees.



CONCLUSION

- Predator-prey relationships are dynamic

- They are influenced by climate dynamics, changes in food availability for the prey species, and dynamics in other areas of the food web (to be discussed later in the semester)

- Predator-prey relationships also are dynamic through evolutionary time.

- Often involve an evolutionary “arms race.” Natural selection simultaneously driving the predators toward greater hunting efficiency and the prey toward traits that help them avoid being eaten.

ACKNOWLEDGEMENT

Inspiration and motivation have always played a key role to complete any venture. Firstly, I like to express my gratitude to our institution **Ramakrishna Mission Vivekananda Centenary College, Rahara** and as well as to our revered **Principal Maharaj Swami Kamalasthananda** for including the project as part of our academic curriculum.

Then I would like to thanks, **Professor Kalyan Chatterjee** of our college who suggested me the topic and encouraged to take up this project as an introduction to the subject. I’m also grateful to my friends for their continuous support and motivation.

Construction and Application of Heat Sensor

The project submitted, in partial fulfilment of the requirement for the assignments in **PHSA CC XI, PHSA CC XII, PHSA DSE I, PHSA DSE II** *Paper (* **Semeste-V***) in the Department of Physics*

Submitted by

Tanay Routh

Registration No: A01-1122-111-038-2019

Supervisor Teacher: Prof. Anjan Kumar Chandra

RAMAKRISHNA MISSION VIVEKANANDA CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

CONSTRUCTION AND APPLICATION OF HEAT SENSOR

Tanay Routh

A01-1122-111-038-2019

B.Sc. Phys dept.(5th sem)

AIMS AND OBJECTIVES OF STUDY

The word “heat” is made manifest as a result of increase in temperature. Temperature is most often measured environmental quantities which correspond to primary sensations-hotness and coldness. This is due to the fact that most biological, chemical, electronic, mechanical and physical systems are affected by temperature. In many instances, some processes perform better within a range of temperatures. Also, certain chemical reactions, biological processes and even electronic circuits do better within limited temperature ranges.

When the needs to optimize these processes arose, the systems used for controlling the temperature within defined limits are then needed.

Semiconductor devices as well as LCDs (Liquid Crystal Displays) can be affected and get damaged by temperature extreme. As the temperature threshold gets exceeded, an immediate action should be taken so as to prolong the lifetime of the system. In these, temperature sensing helps to improve the reliability as well as the lifetime of the system.

The main aim of the research work is to examine the construction and application of heat sensor. Other specific objectives of the study include:

- To examine heat transfers in microprocessors.
- To determine the fundamentals of heat transfer.
- To examine the components of heat transfer sensors.
- To examine the temperature range in various heat sensors.

Key types of temperature sensors

There are two primary types of temperature sensors in use today:

1. **Contact temperature sensors**

These types need to touch the object that they're measuring the temperature of, whether it's a solid, liquid, or gas. They actually just measure their own temperature, but we infer that the temperature of whatever it's in contact with is in thermal equilibrium (i.e. are the same temperature).

Common types of contact temperatures sensors include thermocouples, RTD's, thermistors, thermostats, and semiconductor temperature sensors. They should be used when you are able to make good thermal contact between the device and what you're measuring. It's also easier to attain continuous monitoring and data collection with contact thermometers.

2. **Non-contact temperature sensors**

These determine temperatures from a distance, by measuring the thermal radiation emitted by an object or heat source. The applications for these are often in high temperatures or hazardous environments where you need to maintain a safe distance away from a particular body.

Thermal imaging and infrared sensors are the most common type of non-contact temperature sensors, and are used in the following circumstances: when the target object is moving (such as on a conveyor belt or within moving machinery), if it's a great distance away if there's a dangerous surrounding environment (such as high voltages) or at extremely high temperatures where a contact sensor would not function appropriately.

Sensors Covered

- Thermocouple
- Thermistors
- Resistance Temperature Detector(RTD)
- IR Sensor/ Infrared Sensor

THERMOCOUPLE

What is a Thermocouple

Thermocouple is used to measure the temperature at one specific point in the form of the EMF or an electric current. This sensor comprises two dissimilar metal wires that are connected together at one junction. The temperature can be measured at this junction, and the change in temperature of the metal wire stimulates the voltages. The amount of EMF generated in the device is very minute (millivolts), so very sensitive devices must be utilized for calculating the e.m.f. produced in the circuit. The common devices used to calculate the e.m.f are voltage balancing potentiometer and the ordinary galvanometer. From these two, a balancing potentiometer is utilized physically or mechanically

Thermocouple

Thermocouple Working Principle

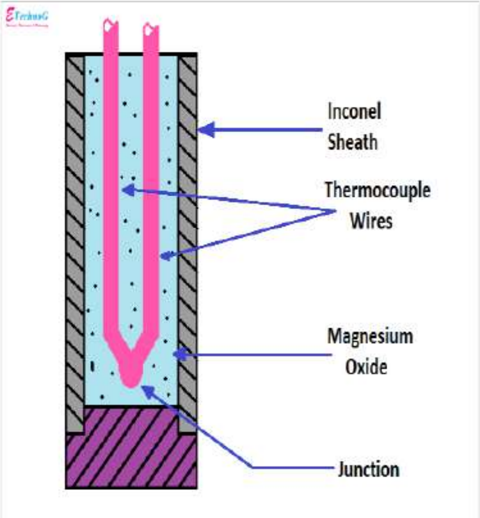
- The **thermocouple principle** mainly depends on the three effects namely Seebeck, Peltier, and Thompson.

See beck-effect

- This type of effect occurs among two dissimilar metals. When the heat offers to any one of the metal wires, then the flow of electrons supplies from hot metal wire to cold metal wire. Therefore, direct current stimulates the circuit.

Construction of Thermocouple

- The construction of the device is shown below. It comprises two different metal wires and that are connected together at the junction end. The junction thinks as the measuring end. The end of the junction is classified into three type's namely ungrounded, grounded, and exposed junction



Thermocouple Construction

Ungrounded-Junction

In this type of junction, the conductors are totally separated from the protecting cover. The applications of this junction mainly include high-pressure application works. The main benefit of using this function is to decrease the stray magnetic field effect.

Grounded-Junction

In this type of junction, the metal wires, as well as the protection cover, are connected together. This function is used to measure the temperature in the acidic atmosphere, and it supplies resistance to the noise.

Exposed-Junction

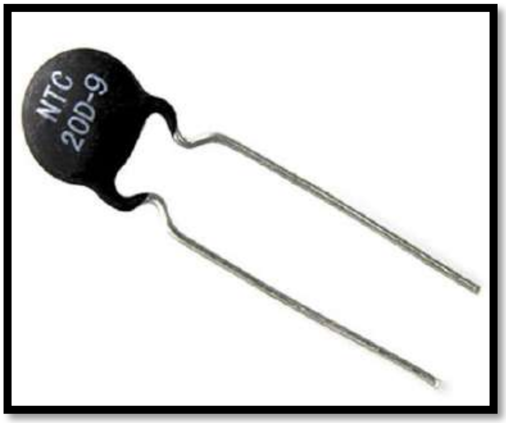
The exposed junction is applicable in the areas where a quick response is required. This type of junction is used to measure the gas temperature. The metal used to make the temperature sensor basically depends on the calculating range of temperature. Generally, a thermocouple is designed with two different metal wires namely iron and constantan that makes in detecting element by connecting at one junction that is named as a hot junction. This consist of two junctions, one junction is connected by a voltmeter or transmitter where the cold junction and the second junction is associated in a process that is called a hot junction.

Applications

Thermocouples are most suitable for measuring over a large temperature range, up to 1800°c. These are widely used in the steel industry, heating appliances, manufacturing of electrical equipments like switch, gears etc.

THERMISTORS

- Thermistors also referred to as thermal resistance i.e., the resistance depends upon temperature. These thermistors when subjected to temperature variations, there will be a change in their resistance. Thus if we measure the change in resistance value, the applied temperature can be determined.



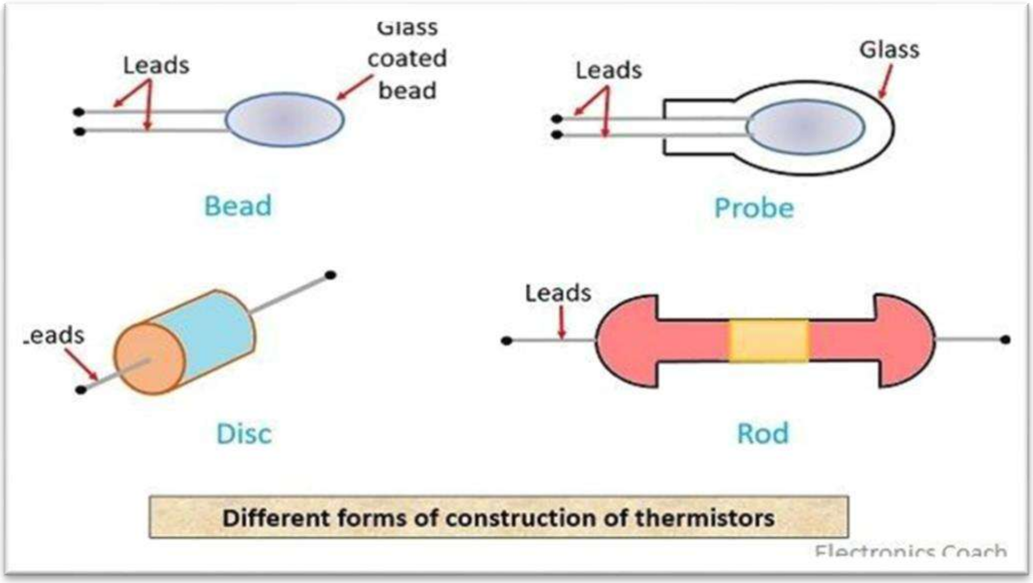
Thermistor

- Thermistors have a negative temperature coefficient and also a positive temperature coefficient. But usually, negative temperature coefficients thermistors are used. The property of the negative temperature coefficient is that the resistance of the thermistor increases with a decrease in temperature and decreases with an increase in temperature.
- This variation in the resistance due to variation in the temperature is known by the Wheatstone bridge circuit. Since the resistance changes with respect to the change in temperature, by measuring the resistance value temperature can be determined.

Construction of Thermistor

- The thermistor consists of a metal tube, leads, and temperature sensing element. The temperature sensing element is the main part of the thermistor, which senses temperature variations enclosed in a metal tube. The sensing element is basically a thermal resistor made with sintering (pressing) mixtures of metallic oxides like copper, nickel, cobalt, iron, manganese, and uranium.

- The sensing element is covered with an insulating material before enclosing it with the metal tube. Two leads are connected to the temperature sensing element and are brought out of the metal tube. The other end of the two leads is connected to one of the arms of the bridge circuit (generally Wheatstone bridge is used) which measures the resistance of the temperature sensing element.
- The commercial thermistors are made in the form of beads, probes, discs, and rods in a variety of sizes as shown below. Thermistors are highly sensitive to temperature variations, which makes them suitable for precise temperature measurements. They are used for measuring temperatures ranging from -100°C to +300°C



Different forms of construction of thermistors

Types

There are basically two broad types

1. NTC- Negative Temperature Coefficient: used mostly in temperature sensing

2. PTC- Positive Temperature Coefficient: used mostly in electrical current control

- A NTC thermistor is one in which the zero-power resistance decreases with an increase in temperature
- A PTC thermistor is one in which the zero-power resistance increases with an increase in temperature
- Assuming, a first order approximation, that the relationship between resistance and temperature is linear, then:

$$\Delta R = k\Delta T$$

Where

ΔR = is change in resistance

ΔT = change in temperature

k = first-order temperature coefficient of resistance For PTC k is postive while negative for NTC

Applications

- PTC thermistors can be used as current limiting devices for circuit protection, as replacement fuses. Current through the device causes a small amount of resistive heating.
- PTC thermistors can be used as heating elements in small temperature-controlled ovens. As the temperature rises, resistance increases, decreasing the current and heating, resulting in a steady state.

- NTC thermistors are used as resistance thermometer in low temperature measurements of the order of 10 K.
- NTC thermistors can be used as inrush-current limiting devices in power supply circuits. It also used in automotive applications.

RTD

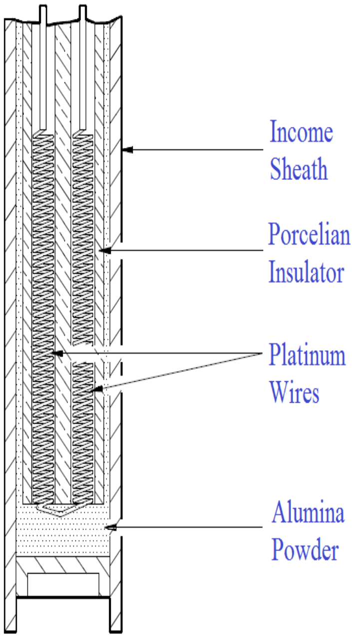
Resistance Temperature Detectors(RTD), as the name implies, are sensors used to measure temperature by correlating the resistance of the RTD element with temperature

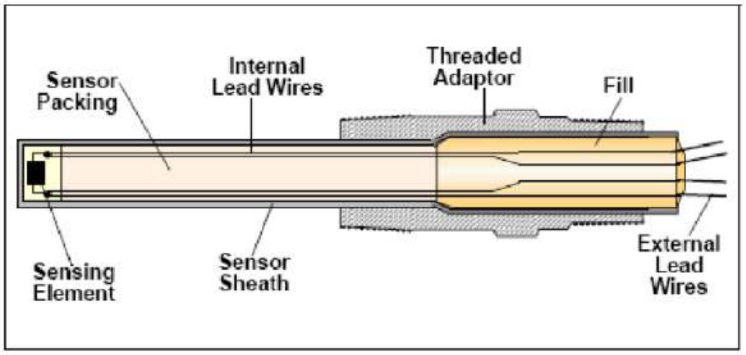
As they are almost invariably made of platinum, they are often called platinum resistance thermometers(PRTs)

Constructions

Common resistance materials for RTD: Platinum(most popular and accurate) Nickel, Copper, Tungsten(rare)

Internal Construction of a RTD





RTD elements consist of a length of fine coiled wire wrapped around a ceramic or glass core

The element is usually quite fragile, so it is often placed inside a sheathed probe to protect it.

The RTD element is made from a pure material whose resistance at various temperatures has been documented. The material has a predictable change in resistance as the temperature changes; it is this predictable change that is used to determine temperature.

Types

There are two broad categories, "film" and "wire-wound" types.

Film thermometers have a layer of platinum on a substrate; the layer may be extremely thin, perhaps 1 micrometer.

Wire-wound thermometers can have greater accuracy , specifically for wide range temperatures.

Application of RTD

- RTD sensor is used in automotive to measure the engine temperature, an oil level sensor, intake air temperature sensors.
- RTD is used in power electronics, computer, consumer electronics, food handling and processing, industrial electronics, medical electronics, military, and aerospace.

	Thermocouple	**Thermistor**	**RTD**
Temp. range	-270 to 1800º C (-454 to 3272º F)	-80º to 150º C (-112 to 302ºF) (Typical)	-260 to 850º C (-436 to 1562º F)
Sensor cost	Low	Low	Moderate
System cost	High	Moderate	Moderate
Stability	Low	Moderate	Best
Sensitivity	Low	Best	Moderate
Linearity	Moderate	Poor	Best
Specify for	Highest temperatures	▪ Best sensitivity ▪ Narrow ranges ▪ Point sensing	▪ General purpose sensing ▪ Highest accuracy ▪Temperature averaging

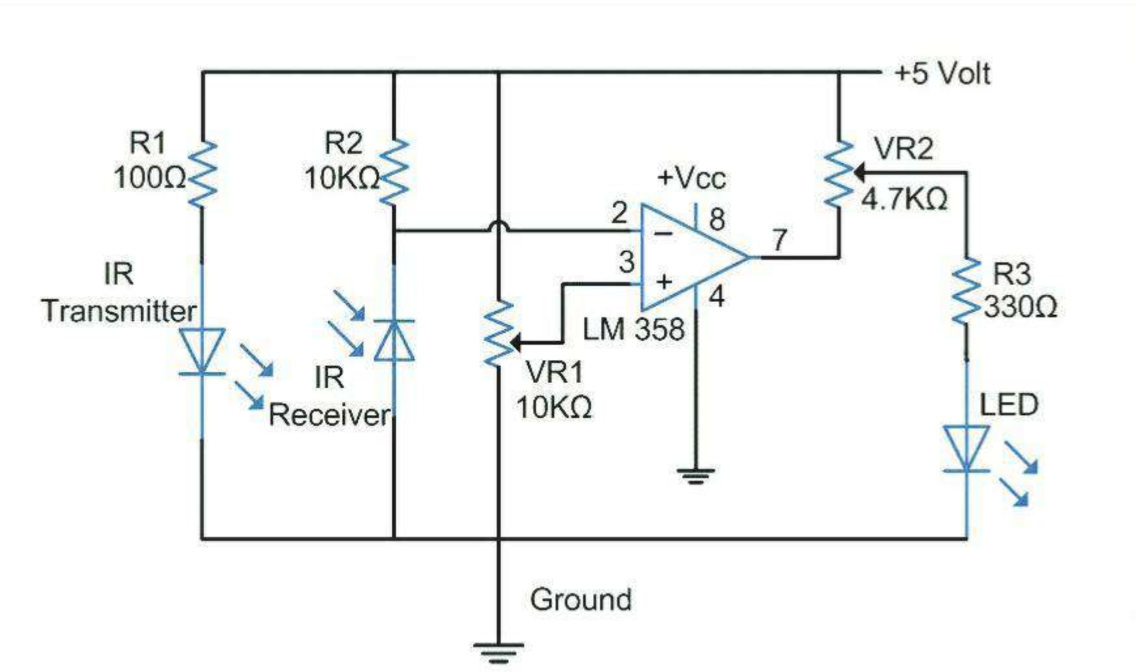
IR SENSOR/ INFRARED SENSOR

An infrared sensor is an electronic device, that emits in order to sense some aspects of the surroundings. An IR sensor can measure the heat of an object as well as detects the motion. These types of sensors measure only infrared radiation, rather than emitting it that is called a passive IR sensor. Usually, in the infrared spectrum, all the objects radiate some form of thermal radiation.

These types of radiations are invisible to our eyes, which can be detected by an infrared sensor. The emitter is simply an IR LED (Light Emitting Diode) and the detector is simply an IR photodiode that is sensitive to IR light of the same wavelength as that emitted by the IR LED. When IR light falls on the photodiode, the resistances and the output voltages will change in proportion to the magnitude of the IR light received.

IR Sensor Circuit Diagram

An infrared sensor circuit is one of the basic and popular sensor modules in an electronic device. This sensor is analogous to human’s visionary senses, which can be used to detect obstacles and it is one of the common applications in real-time. This circuit comprises the following components



In this project, the transmitter section includes an IR sensor, which transmits continuous IR rays to be received by an IR receiver module. An IR output terminal of the receiver varies depending upon its receiving of IR rays. Since this variation cannot be analyzed as such, therefore this output can be fed to a comparator circuit. Here an operational amplifier (op-amp) of LM 339 is used as a comparator circuit.

When the IR receiver does not receive a signal, the potential at the inverting input goes higher than that non-inverting input of the comparator IC (LM339). Thus the output of the comparator goes low, but the LED does not glow. When the IR receiver module receives a signal to the potential at the inverting input goes low. Thus the output of the comparator (LM 339) goes high and the LED starts glowing.

Resistor R1 (100), R2 (10k), and R3 (330) are used to ensure that a minimum of 10 mA current passes through the IR LED Devices like Photodiode and normal LEDs respectively. Resistor VR2 (preset=5k) is used to adjust the output terminals. Resistor VR1 (preset=10k) is used to set the sensitivity of the circuit Diagram. Read more about IR sensors.

Applications

- Heating and air conditioning – Detection insulation breakdown, heat loss and gain and furnace and duct leakage
- Industrial/Electrical – Monitoring motor/engine cooling systems performance, boiler operations, steam systems and detection of hot spots in electrical systems and panels
- Food safety – Checking equipment performance, sanitation and process temperature conditions, and scanning refrigerated display cases, trucks, storage areas and cooling systems
- Agriculture – Monitoring plant temperatures for stress and animal bedding to detect spoiling.

Acknowledgement

The success and final outcome of this project required a lot of guidance and assistance from many people and I am extremely privileged to have got this all along the completion of my project. All that I have done is only due to such supervision and assistance and I would not forget to thank them.

I respect and thank our Principal, Swami Kamalasthananda and Vice Principal, Swami Vedanuragananda for providing me an opportunity to do the project work. I am extremely thankful to the Head of the Department of Physics, Prof. Asok Kumar Pal for providing such a nice support and guidance although he had a very busy schedule.

I owe my deep gratitude to our project guide Prof. Anjan Kumar Chandra, who took keen interest on my project work and guided me all along, till the completion of the project work by providing all the necessary information for developing a good system. I heartily thank all my friends and classmates for their encouragement and more over for their timely support and guidance till the completion of my project work. I am thankful to and fortunate enough to get constant encouragement, support and guidance from all Teaching staffs of RKMVCC Physics Department which helped me in successfully completing my project work.

Sources

https://www.slideshare.net/harshadd/temperature-sensors-828567

https://www.pyrosales.com.au/blog/thermocouple-information/what-are-temperature-sensors-used-for/

https://www.iprojectmaster.com/physics/final-year-project-materials/construction-and-application-of-heat-sensor

Title of the Project

The Stars: Life of the Giant Nuclear Reactors

The project submitted, in partial fulfilment of the requirement for the assignments in ***CC-XI, CC-XII, DSE-I, DSE-II*** *Paper (Semester* ***V*** *) in the Department of Physics*

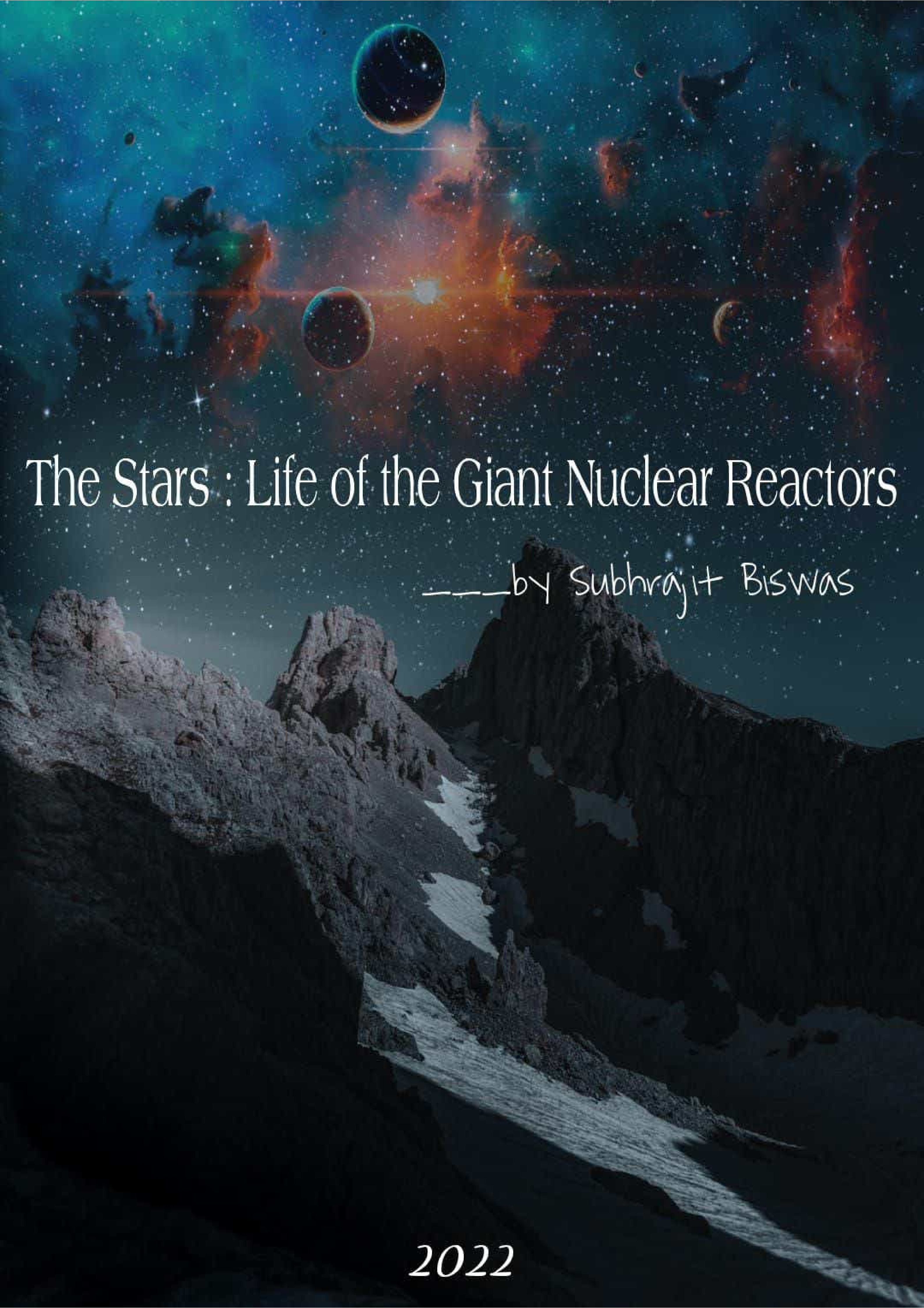
Submitted by

SUBHRAJIT BISWAS

Registration No: A01-1122-111-039-2019

Supervisor Teacher: DR. PALASH NATH

RAMAKRISHNA MISSION VIVEKANANDA
CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA



The Stars : Life of the Giant Nuclear Reactors

___by Subhrajit Biswas

2022

Acknowledgement

I greatfully acknowledge our respected Principal Maharaj for giving inspiration and motivation.

I'm Greatful to my advisor, Dr. PALASH NATH, Associate Professor, Department of Physics, Ramakrishna Mission Vivekananda Centenary College, Kolkata-700118 for his guidance on the related area of this project work and continuous support.

I'm also very much thankful to all our respected teacher, whose valuable teaching and research ideas have continuously motivated me. I'm also thankful to all other respected staff members of our department.

Finally, my deepest admiration goes to my parents for their all-out support thourg out my life.

Subhrajit Biswas

Department of Physics
Ramakrishna Mission Vivekananda Centenary College
Rahara, Kolkata - 700118

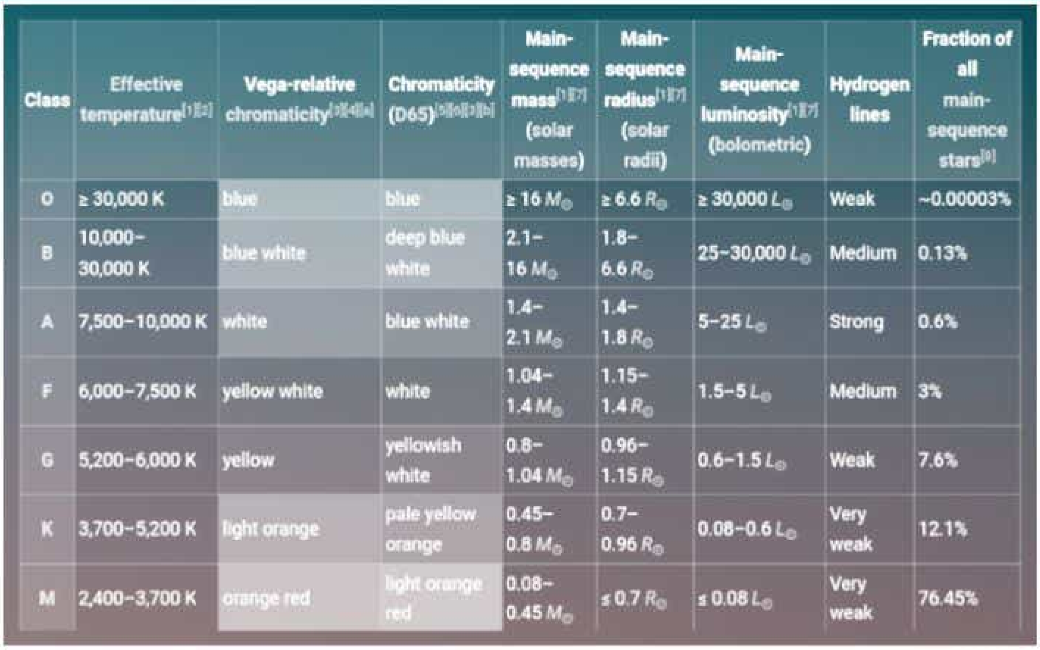
Date : 15/01/2022
Place : Kharad

Stellar Classification :

~stellar classification, scheme for assigning stars to types according to their temperatures as estimated from their spectra. The generally accepted system of stellar classification is a combination of two classification schemes: the Harvard system, which is based on the star's surface temperature, and the MK system, which is based on the star's luminosityluminosity :

1.Harvard spectral classification :

The Harvard system is a one-dimensional classification scheme by astronomer Annie Jump Cannon, who re-ordered and simplified the prior alphabetical system by Draper (see next paragraph). Stars are grouped according to their spectral characteristics by single letters of the alphabet, optionally with numeric subdivisions. Main-sequence stars vary in surface temperature from approximately 2,000 to 50,000 K, whereas more-evolved stars can have temperatures above 100,000 K. Physically, the classes indicate the temperature of the star's atmosphere and are normally listed from hottest to coldest.



Class	Effective temperature[1][2]	Vega-relative chromaticity[3][4][a]	Chromaticity (D65)[5][6][3][b]	Main-sequence mass[1][7] (solar masses)	Main-sequence radius[1][7] (solar radii)	Main-sequence luminosity[1][7] (bolometric)	Hydrogen lines	Fraction of all main-sequence stars[9]
O	≥ 30,000 K	blue	blue	≥ 16 $M_{\odot}$	≥ 6.6 $R_{\odot}$	≥ 30,000 $L_{\odot}$	Weak	~0.00003%
B	10,000–30,000 K	blue white	deep blue white	2.1–16 $M_{\odot}$	1.8–6.6 $R_{\odot}$	25–30,000 $L_{\odot}$	Medium	0.13%
A	7,500–10,000 K	white	blue white	1.4–2.1 $M_{\odot}$	1.4–1.8 $R_{\odot}$	5–25 $L_{\odot}$	Strong	0.6%
F	6,000–7,500 K	yellow white	white	1.04–1.4 $M_{\odot}$	1.15–1.4 $R_{\odot}$	1.5–5 $L_{\odot}$	Medium	3%
G	5,200–6,000 K	yellow	yellowish white	0.8–1.04 $M_{\odot}$	0.96–1.15 $R_{\odot}$	0.6–1.5 $L_{\odot}$	Weak	7.6%
K	3,700–5,200 K	light orange	pale yellow orange	0.45–0.8 $M_{\odot}$	0.7–0.96 $R_{\odot}$	0.08–0.6 $L_{\odot}$	Very weak	12.1%
M	2,400–3,700 K	orange red	light orange red	0.08–0.45 $M_{\odot}$	≤ 0.7 $R_{\odot}$	≤ 0.08 $L_{\odot}$	Very weak	76.45%

2.Morgan-Keenan (MK) classification :

Most stars are currently classified under the Morgan-Keenan (MK) system using the letters O, B, A, F, G, K, and M, a sequence from the hottest (O type) to the coolest (M type). Each letter class is then subdivided using a numeric digit with 0 being hottest and 9 being coolest (e.g., A8, A9, F0, and F1 form a sequence from hotter to cooler). The sequence has been expanded with classes for other stars and star-like objects that do not fit in the classical system, such as class D for white dwarfs and classes S and C for carbon stars.

In the MK system, a luminosity class is added to the spectral class using Roman numerals. This is based on the width of certain absorption lines in the star's spectrum, which vary with the density of the atmosphere and so distinguish giant stars from dwarfs.

Luminosity class 0 or Ia+ is used for hypergiants, class I for supergiants, class II for bright giants, class III for regular giants, class IV for subgiants, class V for main-sequence stars, class sd (or VI) for subdwarfs, and class D (or VII) for white dwarfs. The full spectral class for the Sun is then G2V, indicating a main-sequence star with a surface temperature around 5,800 K.

Class	Star
Ia-O	extremely luminous supergiants
Ia	luminous supergiants
Ib	less luminous supergiants
II	bright giants
III	normal giants
IV	subgiants
V	main sequence dwarf stars
VI, or sd	subdwarfs
D	white dwarfs

The total Stellar Luminosity (energy emitted per second) is therefore:

[$L = F \times Area = 4\pi R^2 \sigma T^4$]

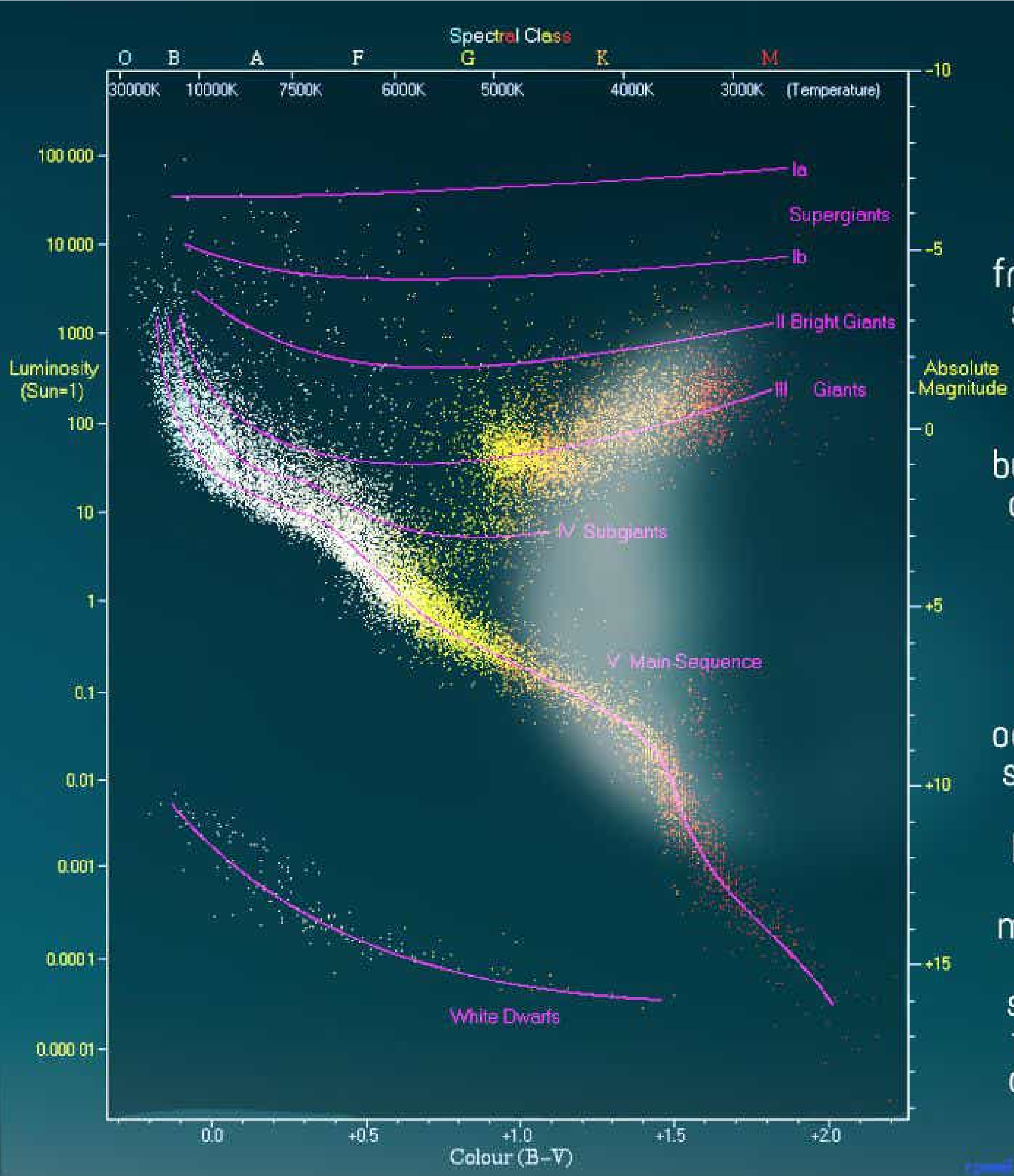
This is the Luminosity-Radius-Temperature Relation for stars.

In words:
'The Luminosity of a star is proportional to its Effective Temperature to the 4th power and its Radius squared'

The Hertzsprung-Russell Diagram :

The Hertzsprung-Russell diagram shows the relationship between a star's temperature and its luminosity. It is also often called the H-R diagram or colour-magnitude diagram. The chart was created by Ejnar Hertzsprung and Henry Norris Russell in about 1910. It is a very useful graph because it can be used to chart the life cycle of a star.

plotting stars' surface temperatures in degrees Kelvin on the x-axis (horizontal axis) and plotting stars' luminosity (or absolute magnitude) along the y-axis (vertical axis) : The x-axis of the H-R diagram can use different data. It can show the star's temperature, its spectral class (OBAFGKM), or its colour. All these types of data show the same relationship with a star's luminosity.



There are 3 main regions (or evolutionary stages) of the HR diagram :

1. The Main Sequence stretching from the upper left (hot, luminous stars) to the bottom right (cool, faint stars) dominates the HR diagram. It is here that stars spend about 90% of their lives burning hydrogen into helium in their cores. Main sequence stars have a Morgan-Keenan luminosity class labelled V.

2. Red Giant and supergiant stars (luminosity classes I through III) occupy the region above the main sequence. They have low surface temperatures and high luminosities which, according to the Stefan-Boltzmann law, means they also have large radii. Stars enter this evolutionary stage once they have exhausted the hydrogen fuel in their cores and have started to burn helium and other heavier elements.

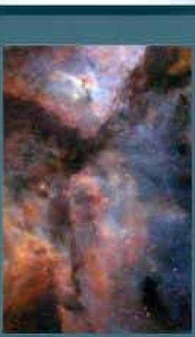
3. White Dwarf stars (luminosity class D) are the final evolutionary stage of low to intermediate mass stars, and are found in the bottom left of the HR diagram. These stars are very hot but have low luminosities due to their small size.

The Stellar Birth :

~Star Formation

Stars are born within the clouds of dust and scattered throughout most galaxies. A familiar example of such as a dust cloud is the Orion Nebula. Turbulence deep within these clouds gives rise to knots with sufficient mass that the gas and dust can begin to collapse under its own gravitational attraction. As the cloud collapses, the material at the center begins to heat up. Known as a protostar, it is this hot core at the heart of the collapsing cloud that will one day become a star. Three-dimensional computer models of star formation predict that the spinning clouds of collapsing gas and dust may break up into two or three blobs; this would explain why the majority the stars in the Milky Way are paired or in groups of multiple stars.

As the cloud collapses, a dense, hot core forms and begins gathering dust and gas. Not all of this material ends up as part of a star — the remaining dust can become planets, asteroids, or comets or may remain as dust.



Powerful Stellar Eruption

The observations of Eta Carinae's light echo are providing new insight into the behavior of powerful massive stars on the brink of detonation.

Credit: NOAO, AURA, NSF, and N. Smith (University of Arizona)

Astronomers have been able to obtain the first image of a dusty disc closely encircling a massive baby star, providing direct evidence that massive stars do form in the same way as their smaller brethren — and closing an enduring debate. The flared disc extends to about 130 times the Earth-Sun distance — or astronomical units (AU) — and has a mass similar to that of the star, roughly twenty times the Sun. In addition, the inner parts of the disc are shown to be devoid of dust. Credit: ESO/L. Calçada/M. Kornmesser

In some cases, the cloud may not collapse at a steady pace. In January 2004, an amateur astronomer, James McNeil, discovered a small nebula that appeared unexpectedly near the nebula Messier 78, in the constellation of Orion. When observers around the world pointed their instruments at McNeil's Nebula, they found something interesting – its brightness appears to vary. Observations with NASA's Chandra X-ray Observatory provided a likely explanation: the interaction between the young star's magnetic field and the surrounding gas causes episodic increases in brightness.

Source of Stellar Energy :

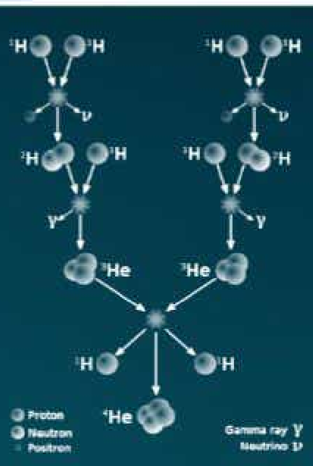
`Nuclear Fusion :

An important fusion process is the stellar nucleosynthesis that powers stars, including the Sun. In the 20th century, it was recognized that the energy released from nuclear fusion reactions accounts for the longevity of stellar heat and light. The fusion of nuclei in a star, starting from its initial hydrogen and helium abundance, provides that energy and synthesizes new nuclei. Different reaction chains are involved, depending on the mass of the star (and therefore the pressure and temperature in its core).

The primary source of solar energy, and that of similar size stars, is the fusion of hydrogen to form helium (the proton–proton chain reaction), which occurs at a solar-core temperature of 14 million kelvin. The net result is the fusion of four protons into one alpha particle, with the release of two positrons and two neutrinos (which changes two of the protons into neutrons), and energy. In heavier stars, the CNO cycle and other processes are more important. As a star uses up a substantial fraction of its hydrogen, it begins to synthesize heavier elements. The heaviest elements are synthesized by fusion that occurs when a more massive star undergoes a violent supernova at the end of its life, a process known as supernova nucleosynthesis.

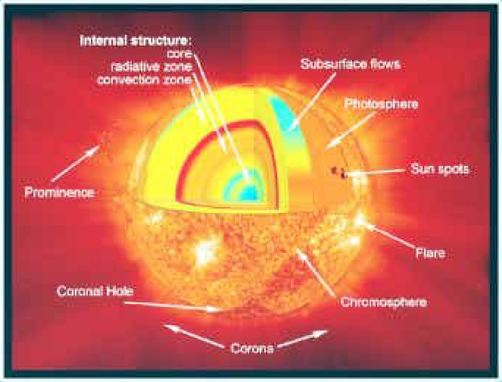
The proton-proton nuclear fusion cycle in a star containing only hydrogen begins with the reaction $H + H \rightarrow D + \beta+ + \nu$; $Q = 1.44$ MeV, where the Q-value assumes annihilation of the positron by an electron

Nuclear Fusion



The proton–proton chain reaction, branch I, dominates in stars the size of the Sun or smaller.

The CNO cycle dominates in stars heavier than the Sun.



Dynamical Equilibrium :

Stars are held together by gravity. Gravity tries to compress everything to the center. What holds an ordinary star up and prevents total collapse is thermal and radiation pressure. The thermal and radiation pressure tries to expand the star layers outward to infinity.

In any given layer of a star, there is a balance between the thermal pressure (outward) and the weight of the material above pressing downward (inward). This balance is called hydrostatic equilibrium. A star is like a balloon. In a balloon the gas inside the balloon pushes outward and the elastic material supplies just enough inward compression to balance the gas pressure. In a star the star's internal gravity supplies the inward compression. Gravity compresses the star into the most compact shape possible: a sphere. Stars are round because gravity attracts everything in an object to the center.

Stars are born from a contracting cloud of interstellar matter. As the cloud contracts, gravitational potential energy is released. Part of this energy is used to heat up the gas; in this way the cloud becomes hotter than its surroundings and starts to radiate energy away. Hence the first nuclei to react are those with the lowest charge, i.e., hydrogen, starting when the temperature reaches a few million degrees.

$$4\ 1H \rightarrow 4He + 2e+ + 2ve$$

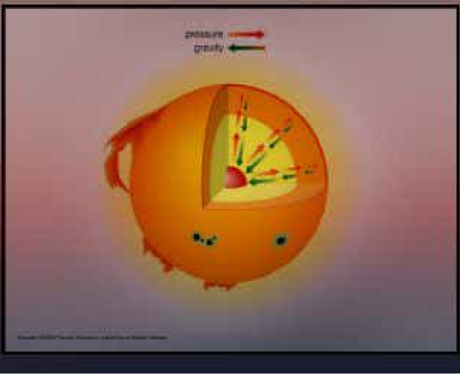
Once, hydrogen burning in the core pha has been established, the contraction of the stars stops. Stars in this phase of their evolution are said to be on the main sequence.
Now, after long time, hydrogen burning stops in the core, the core contracts, releasing gravitational energy and heating up. The star is now proceeding towards the Red Giant phase.

STARS

are

STABLE

Deeper layers have more gravity compression, so they have greater outward pressure to compensate.



The Stellar Death :

~Several billion years after its life starts, a star will die. How the star dies, however, depends on what type of star it is :

1. Stars Like the Sun :

When the core runs out of hydrogen fuel, it will contract under the weight of gravity. However, some hydrogen fusion will occur in the upper layers. As the core contracts, it heats up. This heats the upper layers, causing them to expand. As the outer layers expand, the radius of the star will increase and it will become a red giant. The radius of the red giant sun will be just beyond Earth's orbit. At some point after this, the core will become hot enough to cause the helium to fuse into carbon. When the helium fuel runs out, the core will expand and cool. The upper layers will expand and eject material that will collect around the dying star to form a planetary nebula. Finally, the core will cool into a white dwarf and then eventually into a black dwarf. This entire process will take a few billion years.

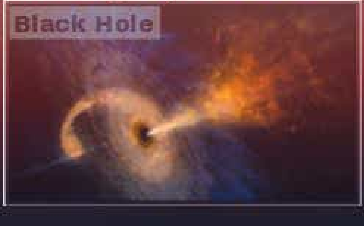
Planetary Nebula

White Dwarf

Black Dwarfs

2. Stars More Massive Than the Sun :

When the core runs out of hydrogen, these stars fuse helium into carbon just like the sun. However, after the helium is gone, their mass is enough to fuse carbon into heavier elements such as oxygen, neon, silicon, magnesium, sulfur and iron. Once the core has turned to iron, it can burn no longer. The star collapses by its own gravity and the iron core heats up. The core becomes so tightly packed that protons and electrons merge to form neutrons. In less than a second, the iron core, which is about the size of Earth, shrinks to a neutron core with a radius of about 6 miles (10 kilometers). The outer layers of the star fall inward on the neutron core, thereby crushing it further. The core heats to billions of degrees and explodes (supernova), thereby releasing large amounts of energy and material into space. The shock wave from the supernova can initiate star formation in other interstellar clouds. The remains of the core can form a neutron star or a black hole depending upon the mass of the original star.

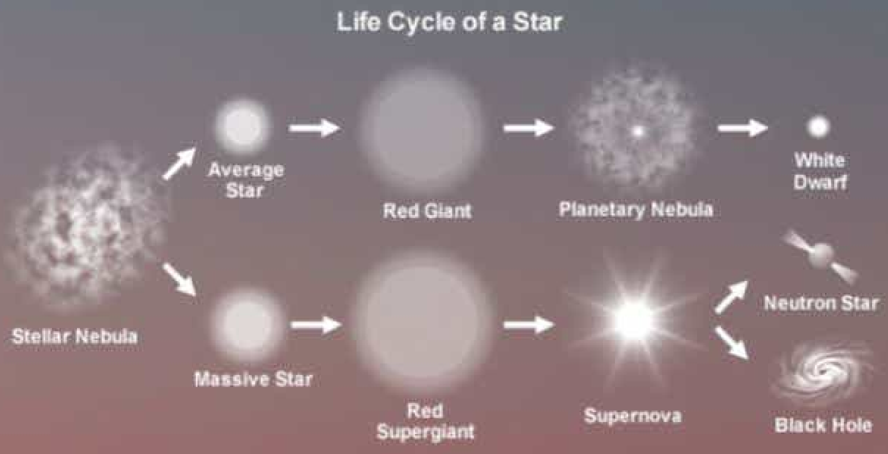


The Conclusion :

.....at the end of the day, we find that,

Stellar evolution is the process by which a star undergoes a sequence of radical changes during its lifetime. Depending on the mass of the star, this lifetime ranges from only a few million years for the most massive to trillions of years for the least massive, which is considerably longer than the age of the universe. The table shows the lifetimes of stars as a function of their masses.[1] All stars are born from collapsing clouds of gas and dust, often called nebulae or molecular clouds. Over the course of millions of years, these protostars settle down into a state of equilibrium, becoming what is known as a main-sequence star.

All stars eventually run out of their hydrogen gas fuel and die. The way a star dies depends on how much matter it contains—its mass. While most stars quietly fade away, the supergiants destroy themselves in a huge explosion, called a supernova.... The death of massive stars can trigger the birth of other stars.



Noted Be : Stellar evolution is not studied by observing the life of a single star, as most stellar changes occur too slowly to be detected, even over many centuries. Instead, astrophysicists come to understand how stars evolve by observing numerous stars at various points in their lifetime, and by simulating stellar structure using computer models.

REFERENCES :

1. Lecture Notes on Stellar Structure and Evolution
Jørgen Christensen-Dalsgaard
Institut for Fysik og Astronomi, Aarhus Universitet

2. Stellar Evolution: Birth, Life, and Death of Stars
John R. Percy
International Astronomical Union
University of Toronto, Canada

3. Image Courtesy: NASA, Caltech

4.www.wikipedia.com

The End

Thank You.

....by, Subhrajit Biswas

CHAOS IN THE SOLAR SYSTEM

The project submitted, in partial fulfilment of the requirement for the assignment in PHSA CC-XI, PHSA CC-XII, PHSA DSE-I, PHSA DSE-II Paper (Semester 5^{th}) in the department of physics

Submitted by

Animesh Mandal

Registration No: A01-1122-111-040-2019

Supervisor Teacher : Prof. Sankhasubhra Nag

RAMAKRISHNA MISSION VIVEKANANDA CENTENARY COLLEGE

P.O. RAHARA , KOLKATA-700118

CHAOS IN THE SOLAR SYSTEM

- **Abstract :** The implications of the chaotic evolution of the Solar System are briefly reviewed, both for the orbital and rotational motion of the planets. In particular, Why Venus spins backward? can be now understood while considering the transition through a highly chaotic state during its history; chaotic state that the Earth itself would have experienced in absence of the Moon, while the large variations of Mars' obliquity were probably at the origin of considerable climate variations that may have left some geological traces on its surface. The limits of predictability for a precise solution of the planetary orbits is an obstruction to the use of the astronomical insolation computations as an absolute geological time scale through paleoclimates reconstructions beyond a few tens of millions of years. On the opposite, as the paleoclimate geological records increase in duration and quality, they may provide an ultimate constraint for the dynamical model of the Solar system.

INTRODUCTION

Newton certainly believed that the Solar System is topologically unstable. In his view the perturbations among the planets were strong enough to destroy the stability of the Solar System. He even made the hypothesis that God controls the instabilities so as to insure the existence of the Solar System: “but it is not to be conceived that mere mechanical causes could give birth to so many regular motion. . .. This most beautiful system of the sun, planets, and comets, could only proceed from the counsel and dominion of an intelligent powerful Being”. The problem of Solar System stability was (and for many aspects still is) a real one: Halley was able to show, by analyzing the Chaldean observations transmitted by Ptolemy, that Saturn was moving away from the Sun while Jupiter was getting closer. A crude extrapolation leads to a possible collision in 6 million years in the past.

From a mathematical point of view arguments supporting the long-time stability of the orbits of the planets were given by Lagrange, Laplace and Poisson who proved the absence of secular evolution (polynomial increase in time) of the semi -major axis of the planets up to third order in the planetary masses.

On the contrary the researches of Poincare and Birkhoff showed that instabilities might occur in the dynamics of the planets and that the phase space must have a quite complicated structure.

Many people were discouraged by Poincare’s discovery. It seemed to imply that it wasn’t worth pursuing the problem; nothing would be gained. For years few people worked on it, but the few who did carried the torch forward. Even though the problem couldn’t be solved directly, it could be approximated closely using perturbation techniques. Over the years perturbation techniques improved significantly, until finally astronomers could, in theory, get answers to any degree of accuracy.

Still, there was a problem: Long hours of tedious calculation were needed to get high accuracy, and few wanted to indulge themselves. Months and even years of long routine calculation were required. And after all the work the results were sometimes questionable because of the approximations that were used.

In the last few year, our vision of the dynamics of the solar system has notably changed and the picture of the planets moving around the sun in a regular quasi-periodic motion has suffered many outrages. In particular, Sussman and Wisdom (1988) have shown by direct numerical integration of the outer planets over nearly 10^9 years with the Digital Orrery that the motion of Pluto presents a positive Lyapunov exponent of 1/20 myr^{-1}. The chaotic nature of the solution of the secular system for the Solar System excluding Pluto (the mass of Pluto is only 1/130,000,000 of the solar mass) with a maximum Lyapunov exponent reaching the value of 1/5 myr^{-1}.

From a rapid analysis of the resonances in the secular system, it was possible to forecast that this high value of the Lyapunov exponent was mainly due to the existence of secular resonances among the inner planets, but this needed more explanation. One of the major questions, faced with the appearance of chaos in the Solar System, was to know where this chaos comes from, and what the sizes of the chaotic zones are. Do very small changes in the initial conditions or in the approximation modify the behaviour of the solutions from chaotic to quasi-periodic, or do we stay in a large chaotic zone? The construction of the secular system and the accuracy of this system. In particular, we use a little trick to improve the length of validity of the solution of the outer planets which becomes very close to the results of direct numerical integrations. The secular resonances which are present in the secular system and analyze their effects on the appearance of the chaos. A new method for a numerical estimation of the size of the chaotic zones, based on the variation with time of the main frequencies of the system. This analysis confirms that the chaos comes mainly from the inner Solar System and in particular from the resonance $2(g_3 – g_4) - (s_3 – s_4)$ between Mars and Earth. The method used for the analysis of the size of the chaotic zones is well suited for the dynamical system of many degrees of freedom when the classical surface of section method is no longer possible. It can be used only when the chaos is not too large (i.e., when the Lyapunov exponent is small with respect to the main frequencies of the system).

The chaotic motion of the solar system

The orbital motion of the planets in the Solar System is chaotic. After Pluto (Sussman and Wisdom, 1988), the evidence that the motion of the whole Solar system is chaotic was established with the averaged equations of motion, and confirmed later on by direct numerical integration (Sussman and Wisdom, 1992). For a review, see, and for some historical considerations. The most immediate expression of this chaotic behaviour is the exponential divergence of trajectories with close initial conditions. Indeed, the distance of two planetary solutions, starting in the phase space with a distance $d(0) = d_0$, evolves approximately as $d(T) \approx d_0 e^{T/5}$ or in a way which is even closer to the true value,

$$d(T) \approx d_0 10^{T/10} \tag{1}$$

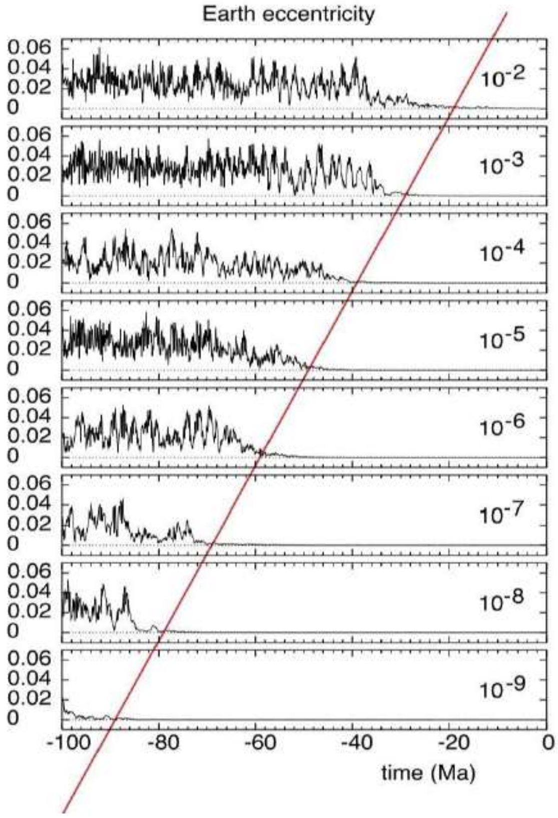


Figure 1: Error in the eccentricity of the Earth resulting from an initial change of 10^n radian in the perihelion of the Earth at the origin. After about n×10 millions of years, the exponential divergence of the orbits dominates, and the solutions are no longer valid. Error in eccentricity is plotted versus time (in Ma).

where T is expressed in million of years. Thus, an initial error of 10^{10} leads to an indeterminacy of ≈ 10^9 after 10 millions of years, but reaches the order of 1 after 100 millions of years. When specific planets are considered, these results may vary, as the coupling between some of the planets is small, and the outer planets (Jupiter, Saturn, Uranus, Neptune) are more regular than the inner ones (Mercury, Venus, Earth, Mars). For the Earth the above formula is a very good approximation of the divergence of the orbits (Fig.1).

Secular resonances

In, it was demonstrated that this chaotic behaviour arise from multiple secular resonances in the inner solar system. In particular, the critical argument associated to

$$(s_4 - s_3) - 2(g_4 - g_3) \qquad (2)$$

where g3, g4 are related to the precession of the perihelion of the Earth and Mars, s3, s4 are related to the precession of the node of the same planets, is presently in in a librational state, but can evolve in a rotational state, and even move to libration in a new resonance, namely

$$(s_4 - s_3) - (g_4 - g_3) = 0\ , \qquad (3)$$

showing that a region of relatively strong instability extend over these two resonances. This result was questioned by Sussman and Wisdom

(1992) which did not recover such a large variation for the secular frequencies, but reached only $2(s_4 – s_3) − 3(g_4 – g_3) = 0$ instead of (??). Recently, new numerical integrations of the complete equations of motion of the Solar System, including all 9 main planets, the Moon as a separate object, Earth and solar oblateness, tidal dissipation in the Earth Moon system, and the effect of general relativity. This new solution was also adjusted and compared to the JPL ephemeris DE406 (Standish, 1998). Different numerical integrations were conducted, with inclusion or not of the oblateness of the Sun ($J_2 = 10^7$) (num2002 and num2001 in Fig.2). The secular equations were also slightly readjusted and integrated over the same time (sec2001 in Fig.2). In all cases, the resonant argument corresponding to $(s_4 −s_3)−2(g_4 −g_3)$ was in libration over 30 Myr. The transition to circulation occurred at 30 Myr and about 40 Myr for sec2001 and num2002, while no transition was observed for num2001 (over 100 Myr). In both cases when the transition occurs, the orbit reaches the resonance $(s_4 – s_3) − (g_4 – g_3)=0$ before 100 Myr, comforting the conclusions of. The other important resonant argument $((g_1 – g_5) − (s_1 – s_2))$ that is at the origin of the chaotic behaviour of the inner planets presents a similar behaviour.

The chaotic motion of the solar system is mainly due to the interactions of the secular resonances in the precessing motion of the inner planets, while the secular motion of the outer planets is very regular. In order to evaluate the possible chaotic diffusion of the planetary orbits over the age of the Solar system, numerous numerical integrations have been conducted over several billions of years (Gyr), extending even the age of the Solar system. From these integrations, it appears that Venus and the Earth present some moderate chaotic diffusion, while the lightest planets, Mars and Mercury, can experience very large changes in their orbit eccentricity, allowing even for collision between Mercury and Venus in less than 5 Gyr. From these integrations, it appears that the chaotic diffusion of the orbits in the earlier stages of the solar system formation could provide some clues for the planetary distribution in semi major axis.

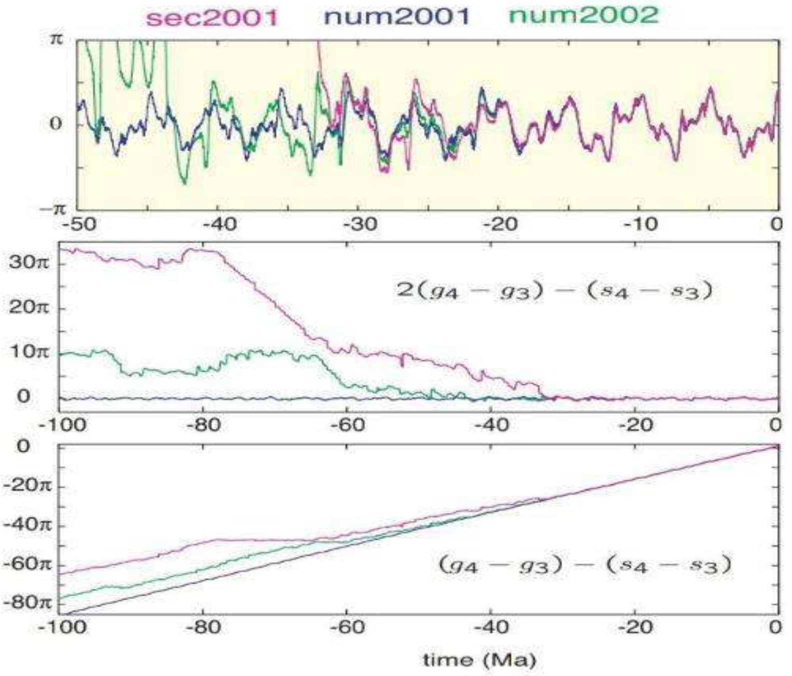


Figure 2: Evolution of the critical angle related to $2(g_4 - g_3) - (s_4 - s_3)$ (top and middle) and $(g_4 - g_3) - (s_4 - s_3)$ (bottom) for three different solutions for the Solar system over 100 Myr. sec2001 is obtained with an averaged system, while num2001 and num2002 are two direct numerical integrations with very close dynamical models.

The outer planets

The secular motion of the outer planets is very regular. Nevertheless, there exists also some instabilities among these planets, due to resonances of high order in their orbital motion around the Sun (mean motion), although these instabilities do not lead to significative changes in the orbits (Murray and Holman, 1999).

A global view of the short period dynamics of the outer planets is given in figure 3 using frequency analysis. In each plot, three of the planets initial conditions are fixed, and the initial conditions of the remaining one (respectively Saturn, Uranus, or Neptune) are changed in a large area of the phase space, spanning semi-major axis and excentricity in a regular mesh around their current position. In each case, an integration is conducted over 1 Myr, and a stability index, obtained as the variation of the mean motion frequency with time is reported, from very stable (blue) to very unstable (red), while close encounter or ejection is denoted by white dots. The resonances are identified and reported in black. It is thus very clear that although the motion of the outer planets are essentially stable, they are numerous mean motion resonances and small chaotic zones that in the vicinity of the present solar system solution.

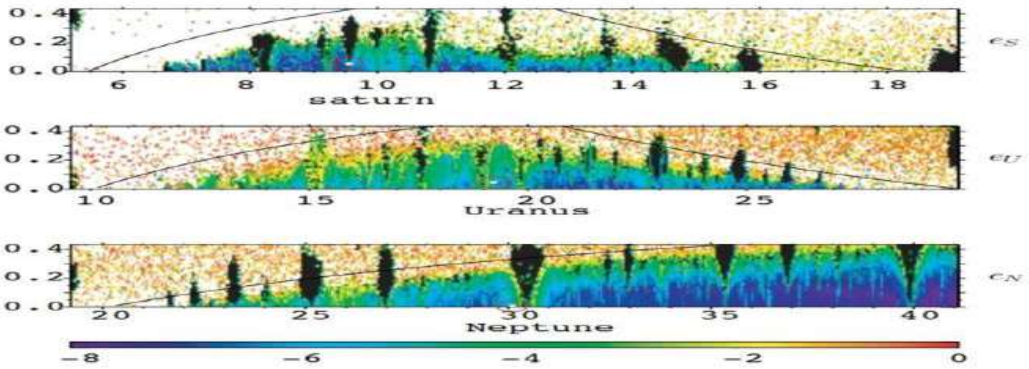


Figure 3: Global dynamics of the giant planets. The abscissa and ordinate correspond to the initial semi major axis and eccentricity of respectively Saturn (top), Uranus (middle) and Neptune (bottom), while in each case the initial conditions of the other planets are taken at their actual values. The current position of each planet is given by a circle. The colours correspond to a stability index obtained by frequency analysis on two consecutive time interval of 1 Myr, and the black regions the mean motion resonances locations.

Obliquity of the planets

The planetary perturbations induce also some effect on the tilt (obliquity) of their equatorial plane over their orbital plane . For the Earth, as the precession frequency of the axis is far from the main orbital secular frequencies of the precession motion of the ecliptic, the obliquity presents only small variations of about 1.3 degrees around the mean value of 23.3 degrees. In the absence of the Moon, the situation would be very different, as multiple resonances then occur between the precession of the axis and the precession of the orbital plane, and a very large chaotic zone would exist, ranging from 0 to about 85 degrees. This is also what will happen as the Moon recedes from the Earth due to tidal dissipation. This situation is very similar for all inner planets.

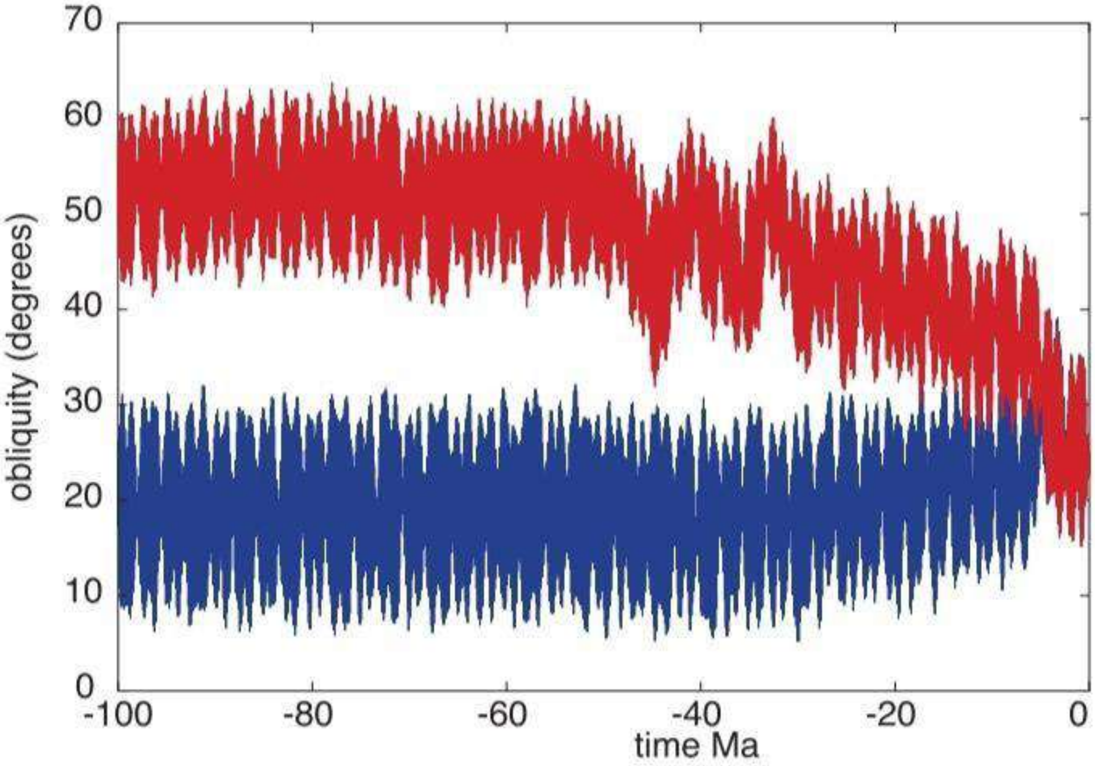


Figure 4: Two different solutions for Mars obliquity over 100 Myrs, obtained with direct integration of the planetary orbits, and with initial conditions of the planet spin within the uncertainty of the most recent determinations.

▪ Mars obliquity

Mars presents a very large chaotic zone for its obliquity ranging from 0 to more than 60 degrees (Touma and Wisdom, 1993). On Figure 4 are plotted two possible solutions for the past evolution of the obliquity of Mars, obtained using our latest numerical integration for the orbital motion of all the planets, and initial conditions and parameters within the uncertainty of the best known values for Mars rotational parameters. It is clear that the climatic history of the planet would be very different in the two situations. It is presently particularly important to understand what are the possible past evolutions of the obliquity (and thus climate) of Mars in the past, as the recent Martian spacecrafts provide very detailed observations of the Martian surface, and give some accurate account of the weather on the planet. In particular, the polar caps of Mars present crevasses that reveal the layered features of the ice (Fig. 5). This layers are thought to be induced by climate variations on Mars surface, and the succession of layers can indeed be related to the insolation variations on its surface, although the possible relation from insolation to surface feature on Mars remains very uncertain.

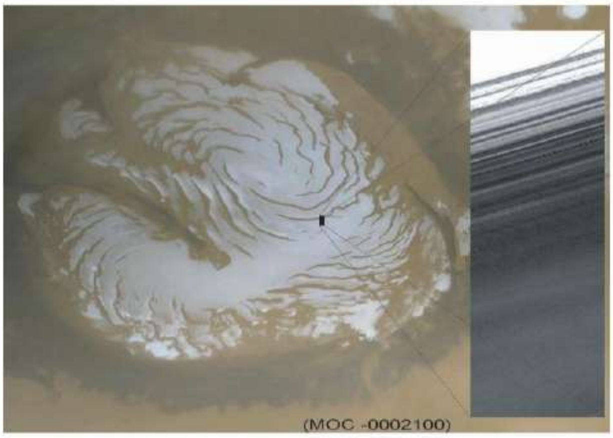


Figure 5: Mars North polar ice cap, and detailed view of a crevasse, showing the layered nature of the ice. The picture was taken by the Mars Orbital Camera (MOC) on april 13, 1999.

- ## Venus final states

In 1962, using radar measurements, the slow retrograde rotation of Venus was discovered. Since, the understanding of this particular state becomes a challenge as many uncertainties remain in the dissipative models of Venus’ rotation. Various hypothesis were proposed for its evolution, aiming to search weather Venus was born with a direct or retrograde rotation. The most favoured scenario assumes that its axis was actually tilted down during its past evolution as a result of core mantle friction and atmospheric tides (for a detailed review,2003). Nevertheless, this requires high values of the initial obliquity, and it was proposed that Venus was strongly hit by massive bodies which would have tilted it significantly or started its rotation backward.

The discovery of a very large chaotic zone in the obliquity evolution of Venus allowed new possible scenario for driving Venus to 180 degrees obliquity, but some difficulties remained. Recently, we have shown that due to the dissipative effects, there are only 4 possible final states for Venus’ rotation, and only 3 of them are really reachable. When the planetary perturbations are added, most of the initial conditions lead to the two states corresponding to the present configuration of Venus, one with period −243.02 days and nearly 0 degree obliquity, and the other with opposite period and nearly 180 degree obliquity. We thus demonstrate that a large impact is not necessary to have a satisfying scenario for the reverse rotation of Venus (Fig. 6), and that most initial conditions can lead to the present reverse rotation. On the opposite, we demonstrate that the present observed retrograde spin state of Venus can be attained by two different processes that cannot be discriminated by the observation of the present rotational state of Venus. In the first scenario, the axis is tilted towards 180 degrees while its rotation rate slows down, while in the second one, the axis is driven towards 0 degree obliquity and the rotation rate decreases, stops, and increases again in the reverse direction, being accelerated by the atmospheric tides, until a final equilibrium value.

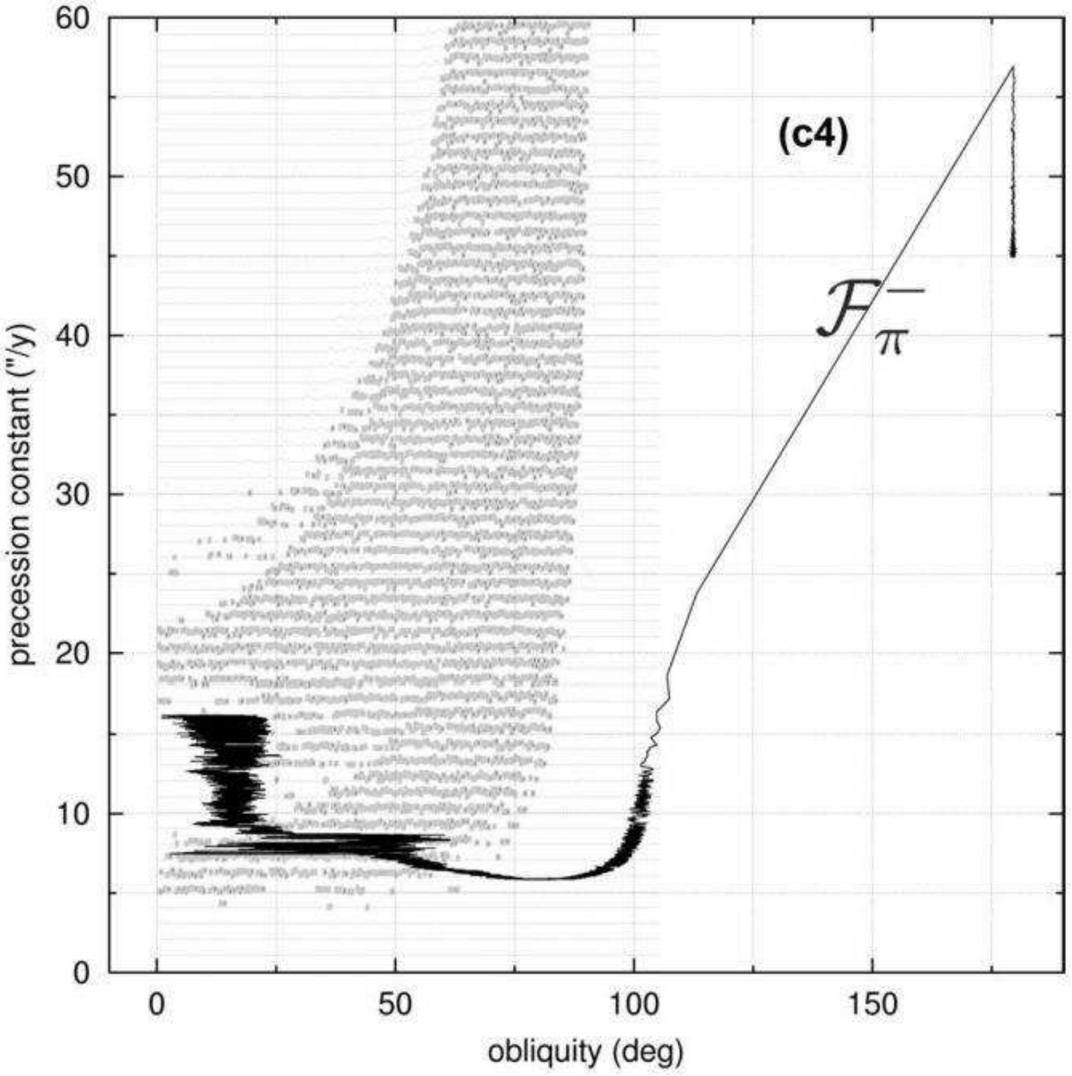


Figure 6: examples of the possible evolution of Venus spin axis during its history. The precession frequency (in arcsec/years) is plotted versus obliquity (in degrees). The initial obliquity is 1 degree, and initial period 3 days. The initial precession frequency is about 16 arcsec/year, but due to tidal dissipation and core-mantle friction, the planet slows down and the precession frequency decreases. The obliquity then enters a zone of larger chaos (in grey) where the variations of the obliquity increases, until the dissipations drives the obliquity out of the chaotic zone, for a high obliquity. The various dissipative effects can then drive the axis towards 180 degrees.

Earth paleoclimates and chaos

For Mars, the chaotic variations of the obliquity are large, and certainly induce some dramatic effects on its climate. For example, it is probable that at large obliquity (> 45 degrees), the polar caps entirely sublimate, the ice been eventually redisposed in the equatorial regions. For the Earth, the situation is very different. The variation of the obliquity is also chaotic, but mainly because it is driven by the chaotic orbital motion of the planet that acts as a forcing term on the obliquity. The variations of the obliquity and orbital parameters are small, but they induce significative changes of the insolation on Earth at a given latitude, that are thought to be at the origin of large climatic variations in the past (Hays et al., 1976). Even more, the astronomical solution is used to provide an absolute time scale for the geological paleoclimate records, over a few tens of millions of years, when the effect of the chaotic behaviour of the solution is not yet sensitive. The quality of the geological data is now becoming sufficient to allow on the other hand to constrain some geophysical parameters of the Earth that are not well known, by comparison of the computed astronomical evolution of the insolation with the geological data (Lourens et al., 2001). As the age of good quality geological records, obtained over long period of time, is now exceeding 30 to 40 millions of years, it becomes interesting to search whether it would be possible to use these geological data to trace back the chaotic diffusion of the solar system in the past.

In order to tackle such a challenging problem it is important, as the quality of geological records decreases with age, to search in the solution some features that would have an important implication on the observed data. Such factor could actually be the resonant argument

$$(s_4 - s_3) - 2(g_4 - g_3)\ , \qquad (4)$$

and its evolution with time. It would be indeed fascinating to be able to retrieve in the geological data the transitions from libration to circulation of this argument, as well as its eventual transition to the resonance

$$(s_4 - s_3) - (g_4 - g_3)=0 \ . \qquad (5)$$

The direct observation of the individual arguments related to g3, g4, s_3, s_4 is certainly out of reach. But it may be possible to follow the evolution in time of the differences $g_3 - g_4$ and $s_3 - s_4$. Indeed, $s_3 - s_4$ appears as a beat of about 1.2 million of years in the solution of obliquity, as the result of the beat between the $p + s_3$ and $p + s_4$ components of the obliquity, where p is the precession frequency of the axis (Fig. 7). In a similar way, g3 − g4 appears as a beat with period of about 2.4 Myr in the climatic precession (Fig. 7). One understands that because of the occurrence of these beats, the detection of the resonant state in the geological data can be possible, as one has to search now for phenomena of large amplitude in the geological signal. Indeed, in the newly collected data from Ocean Drilling Program Site 926, the modulation of 1.2 Myr of the obliquity appears clearly in the spectral analysis of the paleoclimate record.

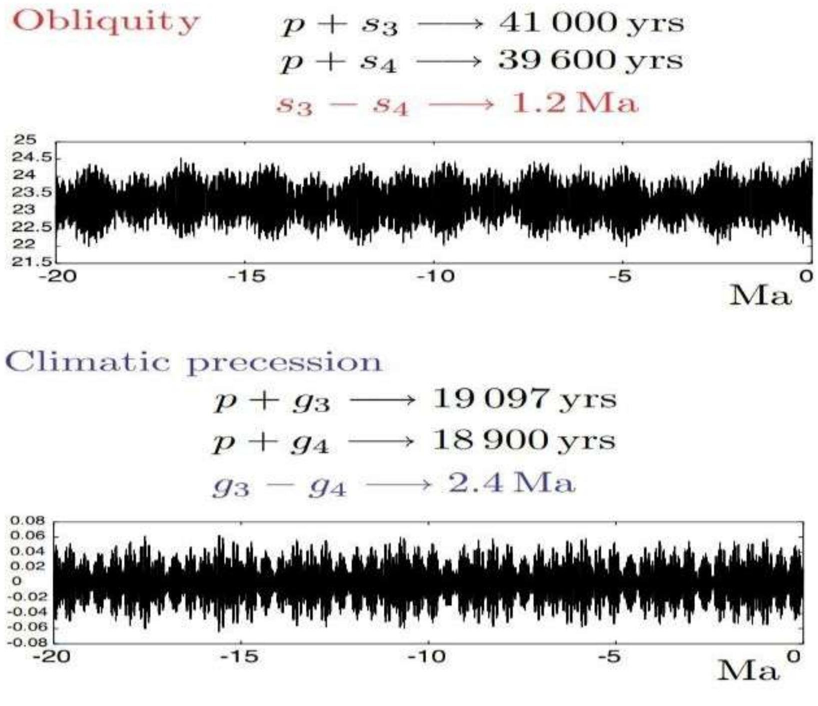


Figure 7: The main components of the obliquity of the Earth are given by two terms of period close to 40000 years ($p + s_3$ and $p + s_4$) than induce a beat of about 1.2 Myr period. For the climatic precession ($e \sin \omega$, where e is the excentricity and ω the longitude of perihelion from the moving equinox), the main terms are of period close to 20 000 years, and the beat $g3 - g4$ has a period of about 2.4 Myrs.

The search for the determination of these resonant angles in the geological data is just starting, but we may expect that a careful analysis, and new data spanning the last 60 to 100 Myr, will allow to determine the possible succession of resonant states in the past, allowing, for example, to discriminate between the solutions displayed in figure 2. Of particular importance would be the detection of the first transition from libration to circulation of the resonant argument $(s_4-s_3)- 2(g_4-g_3)$. This program, if completed, will provide some extreme constraint for the gravitational model of the Solar System. Indeed, the observation of a characteristic feature of the solution at 40 to 100 Myr in the past, because of the exponential divergence of the solutions, will provide a constraint of 10^4 to 10^{10} on the dynamical model of the Solar System.

CONCLUSION

The evolution of the entire planetary system has been numerically integrated for a time span of nearly 100 million years. This calculation confirms that the evolution of the solar system as a whole is chaotic, with a time scale of exponential divergence of about 4 million years. Additional numerical experiments indicate that the Jovian planet subsystem is chaotic, although some small variations in the model can yield quasiperiodic motion. The motion of Pluto is independently and robustly chaotic.

Advances in computer technology have made it possible to begin to directly address the age-old question of the nature of the long-term evolution of the solar system, with startling results. Sussman and Wisdom presented numerical evidence that the motion of Pluto is chaotic, with a time scale for exponential divergence of nearby trajectories of only about 20 million years. Subsequently, Laskar found numerical evidence of the chaotic evolution of the solar system excluding Pluto, with a time scale for exponential divergence of only about 5 million years. Laskar's calculation was feasible because he analytically averaged the equations of motion to remove the rapid variations with time scales of the order of the orbital period. The averaged equations are perturbative and necessarily truncated after a particular order in eccentricity, inclination, and mass ratio. An integration of the whole solar system without these approximations was required

Direct integrations of the whole planetary system are computationally expensive. Notable long-term integrations of the outer solar system include: the classic 1-million-year integration of Cohen, Hubbard, and Oesterwinter, the 5-million-year integration of Kinoshita and Nakai, the 210-million-year integration performed on the Digital Orrery, the 100-million-year integration of the LONGSTOP project, and the 845-million-year Digital Orrery integration of Sussman and Wisdom. Studies of the long-term evolution of the whole solar system have been more limited because the computational resources required are significantly larger, by about two orders of magnitude. Integrations of the whole solar system include: the 3-million-year Digital Orrery integration (which excluded Mercury), the 2-million-year integration of Richardson and Walker, and recent [+ or -]3-million-year integration of Quinn, Tremaine, and Duncan (hereafter QTD).

Acknowledgement

Primarily I would like to profound gratitude to Ramakrishna Mission Vivekananda Centenary College., Rahara for including the project as part of our academic curriculum. This has provided me an opportunity to learn something beyond the text book. I wish to express my deep gratitude and sincere thanks to revered principal Maharaj Swami Kamalasthananda, Ramakrishna Mission Vivekananda Centenary College, Rahara, for his encouragement and the facilities provided for this project work.

I extend my sincere thanks to Dr. Sankhasubhra Nag, Professor, Ramakrishna Mission Vivekananda Centenary College, Rahara and my mentor who guided me to successful completion of the project.

I take this opportunity to express my deep sense of gratitude for his valuable support and constructive comment.

I can’t forget to offer my sincere thanks to my family and friends whose immense motivation and support had helped me to complete the project in limited time frame.

ANIMESH MANDAL

PHYSICS HONOURS (5TH SEMESTER)

COLLEGE ROLL NO : 154

References

Study material :

- Correia, A., Laskar, J.: 2001, The Four final Rotation States of Venus, Nature, 411, 767–770, 14 june 2001
- Correia, A., Laskar, J., N´eron de Surgy, O.: 2003, Long term evolution of the spin of Venus - I. Theory. Icarus, 123, 1–23
- Correia, A., Laskar, J.: 2003, Long term evolution of the spin of Venus - II. Numerical simulations. Icarus, 123, 24–45
- Dones, L., Tremaine, S.: 1993, Why does the Earth spin forward?, Science, 259, 350–354
- Folkner, W.M., Yoder, D.N., Yuan, E.M., Standish, E.M. and Preston, R.A.: 1997, Interior structure and seasonal mass redistribution of Mars from radio tracking of Viking, Science, 278, 1749–1752
- Hays, J.D., Imbrie, J. & Schackleton, N.J., 1976. Variations of the Earth's Orbit: Pacemaker of the ice ages, Science, 194,1121–1132.
- Laskar, J.: 1989, A numerical experiment on the chaotic behaviour of the Solar System, Nature, 338, 237–238

A SIMPLE MODEL FOR BONDING IN SOLIDS WITH CHEMICAL BONDS AND LATTICE ENERGY

The project submitted, in partial fulfilment of the requirement for the assignments in ***PHSA CC -XI, PHSA CC -XII, PHSA DSE-I, PHSA DSE-II*** *Paper (Semester 5th.) in the Department of Physics*

Submitted by

Porikshit Mondal

Registration No: A01-1122-111-041-2019

Supervisor Teacher: Prof. Asok Kumar Pal

RAMAKRISHNA MISSION VIVEKANANDA
CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

INDEX

❖ Introduction

Solid is one of the four fundamental states of matter. The molecules in a solid are closely packed together and contain the least amount of kinetic energy. Bonding in solids can be understand by understanding interatomic forces. The force acting between the atoms in a solid are electrostatic in nature.

One of the fundamental problems in the theory of solids is the calculation of the binding energy of a crystal. This evidently requires a knowledge of the forces acting between the composing particles. We shall begin with a discussion of perfect crystals.

❖ Bonding in Solids:

Bonding in solids can be understand by understanding interatomic forces. The forces acting between the atoms in solids are electresitedica in nature; they are determined esentially by the way in which the outer elections of the composing atoms are distributed in space.

From the very existence of solids, one may draw two conclusions:

1. There must act attractive forces between the atoms or molecules in a solid which keep them together.
2. There must be repulsive forces acting between the atoms as well, since large external pressures are required to compress a solid to any appreciable extent. (Both conclusions also apply to liquids)

In order to illustrate the importance of both types of forces, let us consider a simple, system in this respect, viz, a single pair of atoms A and B which form a stable chemical compound Without paying attention to the physical origin of the forces between the two atoms, let us assume that the potential energy of atom B due to the prosence of atom A is given by an expression of the type;

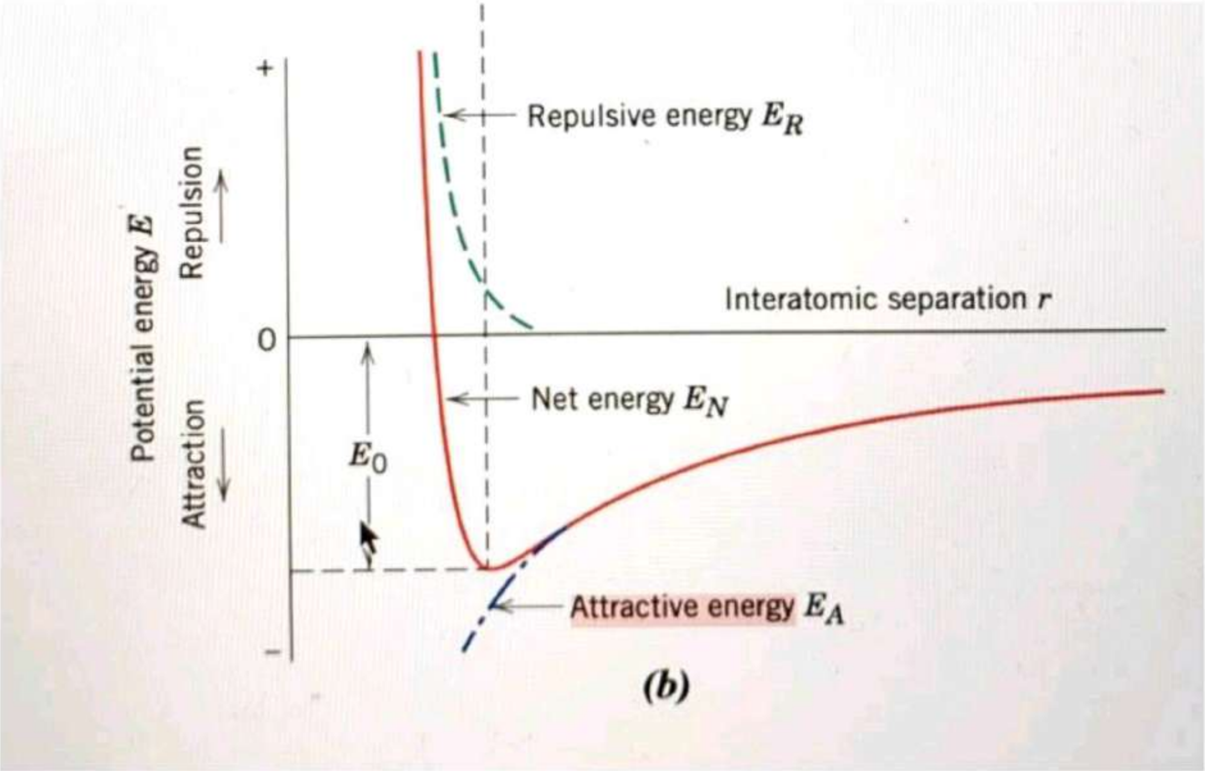
$$E(r) = -\frac{\propto}{r^n} + \frac{\beta}{r^n}$$

Where, r: distance bethen the nuclei of the two atoms α,β, m, n are constant characteristic for the A,B melecule.

here,

$-\frac{\propto}{r^n}$ is the negative energy. It corresponds to energy associated with force of attraction.

$+\frac{\beta}{r^n}$ the positive energy. It corresponds to energy associated with force of repulsion.



Fg1: Schematic representation of the energy between two atoms as function of their seperation r .

Here,

The dashed curved are is the sum of the attractive and repulsive curves.

Now, if r_0 be the equilibrium seperation betwerk atoms A and B in the stable molecule AB, the E(r) must have a minimum at r=r_0

So, at r=r_0 , $\frac{dE}{dr}$=0 & $\frac{d^2E}{dr^2}$ >0

Now, at r=r_0 , $\boldsymbol{\frac{dE}{dr} = -\frac{-n\alpha}{{r_0}^{n+1}} + \frac{-m\beta}{{r_0}^{m+1}}}$

Or, $\frac{{r_0}^{n+1}}{{r_0}^{m+1}} = \frac{n\alpha}{m\beta}$

Or, $r_0 = (\frac{n\alpha}{m\beta})^{\frac{1}{n-m}}$

This is the equilibrium seperation.

- The force between A & B is given by,

$$F(r) = -\frac{dE}{dr}$$

Or, $F(r) = -\frac{n\alpha}{r^{n+1}} + \frac{m\beta}{r^{m+1}}$

where 1st term is for attractive force and 2nd term is for repulsive force.

Now, at equilibrium point So, at equilibrium point, at $r = r_0$, $\frac{dE}{dr} = 0$

So, at $r = r_0$, $F(r) = -\frac{dE}{dr} = 0$

So, at equilibrium point, net force will be zero.

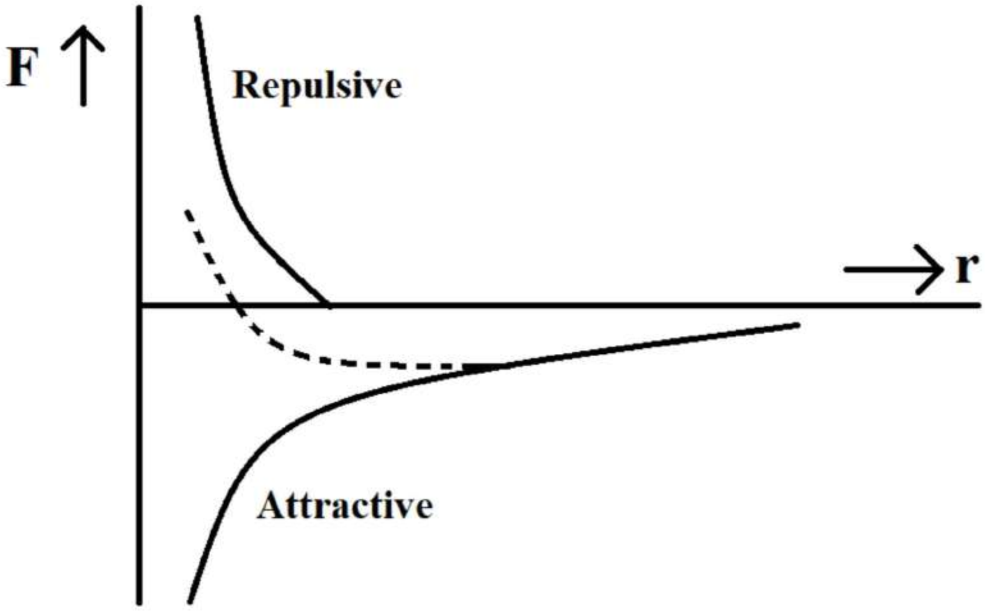


Fig2: Schemalic representation of the force between two atoms as function of their seperation r. The dashed curves is the sum of the attractive and repulsive curves.

- So, at equilibrium seperation, the energy will be,

$E(r_0) = -\frac{\alpha}{r_0{}^n} + \frac{\beta}{r_0{}^m}$ by, , $r_0 = (\frac{n\alpha}{m\beta})^{\frac{1}{n-m}}$

$E(r_0) = -\frac{\alpha}{r_0{}^n} + \frac{\alpha}{r_0{}^n} * \frac{n}{m}$ Get,, $\frac{\beta}{r_0{}^m} = \frac{\alpha}{r_0{}^n} * \frac{n}{m}$

$E(\boldsymbol{r_0}) = -\frac{\alpha}{r_0{}^n}(1- \frac{n}{m})$

Note: although the attractive and repulsive forces are equal in equilibrium, the attractive and repulsive energies are not equal since n≠m. In fact, if m>> n, the total binding energy is essentially determined by the energy of attraction $-\frac{\alpha}{r_0{}^n}$

- Formation of chemical bond:

Chemical bond will form, when the energy curve will be like fig(1). Here, a minimum in the energy curve is possible only if m>n. Let's see how it will come:

from their Curve, we can say,

at $r = r_0$, $\frac{d^2E}{dr^2} > 0$

Now,

$E = -\frac{\propto}{r^n} + \frac{\beta}{r^n}$

Taking the 1st derivative w.r.t r in both side ,

$$\frac{dE}{dr} = \frac{n\alpha}{r^{n+1}} - \frac{m\beta}{r^{m+1}}$$

Taking the 2nd derivative,

$$\frac{d^2E}{dr^2} = \frac{-(n+1)n\alpha}{r^{n+2}} + \frac{(m+1)m\beta}{r^{m+2}}$$

Or, $[\ \frac{d^2E}{dr^2}\ ]\, r^{n+2} = \frac{-(n+1)n\alpha}{r^{n+2}} r^{n+2} + \frac{(m+1)m\beta}{r^{m+2}} r^{n+2}$

Or, [$\frac{d^2E}{dr^2}$] r^{n+2} = -n(n+1)α+m(m+1)β r^{n-m}

at r=r_0 , $\frac{d^2E}{dr^2}$>0

So, -n(n+1) α +m(m+1) β r^{n-m}>0

Or, -n(n+1) α +m(m+1) β [$(\frac{n\alpha}{m\beta})^{\frac{1}{n-m}}]^{n-m}$ >0

Or, -n(n+1) α +m(m+1) β [$\frac{n\alpha}{m\beta}$] >0

Or, -n(n+1) α +n(m+1) α >0

Or, α (m+1)n>n(n+1) α

Or,(m+1)>(n+1)

So, m>n

So, we may conclude that a minimum in the energy curve is possible only if m>n ; thus the formation of a Chemical bond requires that the repulsive force be of shorter range than the attractive ones.

❖ LATTICE ENERGY

One of the fundamental problems in the theory of solids is the calculation of the binding energy of a crystal. This evidently requires a knowledge of the forces acting between the composing particales. We shall begin with a discussion of perfect, crystals, assuming that all ions occupy the lattic points. However, perfect crystals do not exist, and even if a crystal is perfectly grown and chemically pure, there are always a (relatively small) number of lattices decfects present. So, the simplest group of crystals to deal with in this respect are the lonic crystals, for which calculations of the cohesive energy were made in 1910 by Born and Madelung.

The basic assumption in the theory of the cohesive energy of ionic crystal is that the solid may be considered as a system of positive and negative ions.

- ELECTROSTATIC OR MADELUNG ENERGY:

The main contribution to the binding energy of ionic crystals is electrostatic and is called Madelundg energy

The long range interaction between ions with charge $\pm$q is electrostatic intraction $\pm \frac{q^2}{4\pi\epsilon_0 r}$

Say, U_{ij} : interaction energy between ith-ion and jth ion.

We suppose that U_{ij} may be written as the sum of a central field repulsive potential of the form ‘ $\boldsymbol{\lambda e}^{\frac{-r_{ij}}{\rho}}$, where λ and ρ are empirical parameters, and a coulomb potential $\pm \frac{q^2}{4\pi\epsilon_0} * \frac{\mathbf{1}}{\boldsymbol{r_{ij}}}$

Thus $U_{ij} = \boldsymbol{\lambda e}^{\frac{-r_{ij}}{\rho}} \pm \frac{q^2}{4\pi\epsilon_0} * \frac{\mathbf{1}}{\boldsymbol{r_{ij}}}$

Note: Central field repulsive potential is on the basis of quantum mechantes, non-classical in nature. It is restrict, overlap of electronic cells of neighbouring ions.

Now,

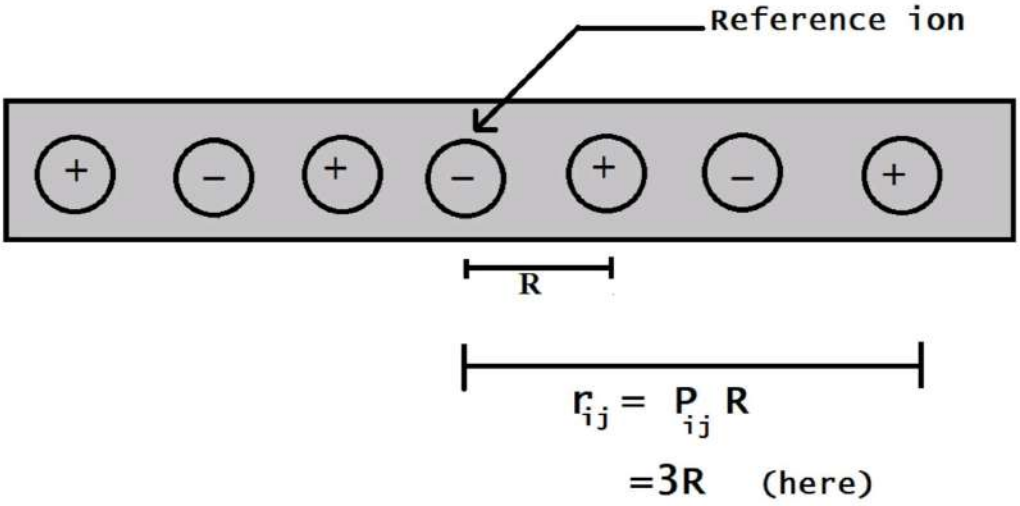
Interaction energy of ith-ion due to all other ions is,

$$U_i = \sum_{j(j \neq i)} U_{ij}$$

So, total Lattic energy of a crystal composed of N molecules (i.e 2N-ions),

U_{total}=NU_i

Line of ions of alternating signs, with distance R between ions



r_{ij}=p_{ij}R

Where, r_{ij}: distance between ith ion and jth ion

R: nearest - neighbour separation in the crystal

Now, Restricting the repulsive interaction among only nearest neighbours we may have,

U_{ij}= $\boldsymbol{\lambda e^{\frac{-R}{\rho}}} - \frac{q^2}{4\pi\epsilon_0 R}$ (nearest neighbours)

$= \pm \frac{q^2}{4\pi\epsilon_0 R} * \frac{\mathbf{1}}{\boldsymbol{P_{ij}}}$ (otherwise)

Electrostatic energy of a reference ith ion (taken as -ve) is

$$- \sum_{j(j \neq i)} \pm \frac{q^2}{4\pi\epsilon_0 R} * \frac{\mathbf{1}}{\boldsymbol{P_{ij}}}$$

$$= -\frac{q^2}{4\pi\epsilon_0 R} \sum_{j(j\neq i)} (\pm)\frac{\mathbf{1}}{\boldsymbol{P_{ij}}}$$

$$= -\frac{q^2\alpha}{4\pi\epsilon_0 R}$$

where, $\alpha = \sum_{j(j\neq i)} (\pm)\frac{\mathbf{1}}{\boldsymbol{P_{ij}}}$ = Madelung Constant

So, U_{total}= NU_i= N$\sum_{j(j\neq i)} U_{ij}$

$$=\mathrm{N}(\mathrm{z}\ \boldsymbol{\lambda e}^{\frac{\boldsymbol{-R}}{\boldsymbol{\rho}}} - \sum_{j(j\neq i)} \pm\frac{q^2}{4\pi\epsilon_0 R} * \frac{\mathbf{1}}{\boldsymbol{P_{ij}}})$$

$$= \mathrm{N}(\mathrm{z}\ \boldsymbol{\lambda e}^{\frac{\boldsymbol{-R}}{\boldsymbol{\rho}}} - \frac{q^2\alpha}{4\pi\epsilon_0 R}) \text{ ....(1)}$$

Where, z is the number of nearest neighbour of ith ion.

Now suppose,

R = R_0 is the equilibrium separation,

Therefore,

at R = R_0 , $\frac{dU_{total}}{dR}$=0

at R = R_0 , $\frac{dU_{total}}{dR}$= N(z $\boldsymbol{\lambda e}^{\frac{\boldsymbol{-R}}{\boldsymbol{\rho}}} - \frac{q^2\alpha}{4\pi\epsilon_0 R}$)=0

or, $\frac{Nq^2\alpha}{4\pi\epsilon_0 R} = \frac{N}{\rho}$z $\boldsymbol{\lambda e}^{-\frac{R_0}{\rho}}$

or, z $\boldsymbol{\lambda e}^{-\frac{R_0}{\rho}} = \frac{\rho q^2\alpha}{4\pi\epsilon_0 R}$ ....(2)

Now, Total lattice energy of the crystal of N molecules (i.e. 2N- cons) at their equilibrium Separation R_0 will be,

$$U_{total}= \text{N}(\text{z}\, \boldsymbol{\lambda e}^{\frac{-R_0}{\rho}} - \frac{q^2\alpha}{4\pi\in_0 R_0})$$

Or, $U_{total}= \text{N}(\frac{\rho q^2\alpha}{4\pi\in_0 R^2{}_0} - \frac{q^2\alpha}{4\pi\in_0 R_0})$

So , $\boldsymbol{U_{total}}= -\frac{\boldsymbol{Nq^2\alpha}}{\boldsymbol{4\pi\in_0 R_0}}(1\ -\ \frac{\boldsymbol{\rho}}{\boldsymbol{R_0}})$

The term ($-\frac{\boldsymbol{Nq^2\alpha}}{\boldsymbol{4\pi\in_0 R_0}}$) is the Madelung energy of electrostatic energy. We shall find that is the order of 0.1 Ro, so that the repulsive interaction has a very short range.

Evaluation of Madeling constant :

$\alpha = \sum_{j(j\neq i)} (\pm)\frac{\boldsymbol{1}}{\boldsymbol{P_{ij}}}$ (take reference ion as -ve ion; then,(+)sign for +ve ion and (-)sign for -ve ion)

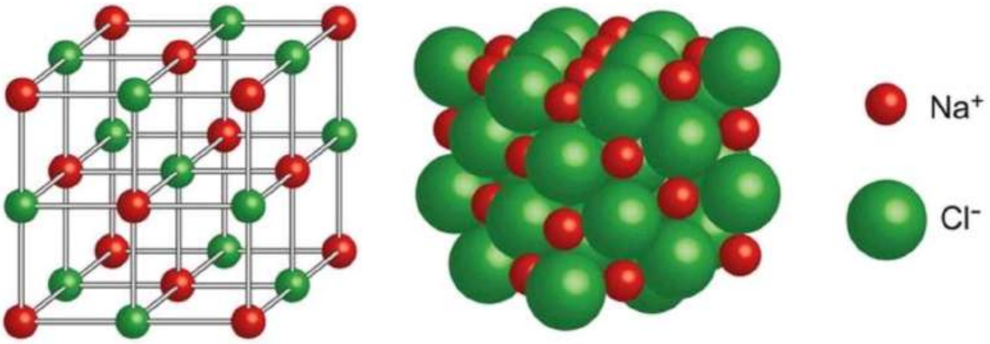
or, $\alpha = \sum_{j(j\neq i)} (\pm)\frac{\boldsymbol{R}}{\boldsymbol{r_{ij}}}$

or, $\frac{\boldsymbol{\alpha}}{\boldsymbol{R}} = \sum_{\boldsymbol{j(j\neq i)}} (\pm)\frac{\boldsymbol{1}}{\boldsymbol{r_{ij}}}$ ($\boldsymbol{r_{ij}}=\boldsymbol{P_{ij}R}$)

#As an example, we may calculate the Madelung constant for NaCl crystal (simple cubic)

for a given Na+,

No of ions	Distance
6(Cl-)	R
12(Na+)	$R\sqrt{2}$
8(Cl-)	$R\sqrt{3}$
6(Na+)	$R\sqrt{4}$
24(Cl-)	$R\sqrt{5}$



$$\frac{\alpha}{R} = \frac{6}{R} - \frac{12}{R\sqrt{2}} + \frac{8}{R\sqrt{3}} - \frac{6}{R\sqrt{4}} + \frac{24}{R\sqrt{5}} - \cdots$$

or, $\frac{\alpha}{R}=\frac{1}{R}*1.747558$

$\alpha_{NaCl} \approx 1.747558$

❖ Conclusion

We have tried to understand a simple model for bonding in solids with chemical bonds and lattice energy. We came to know that there must act attractive forces between the atoms or molecules in a solid which keep them together. Similarly, there must be repulsive forces acting between the atoms as well since large external pressures are required to compress a solid to any appreciable extent. We determined the energy for their equilibrium separation. We got some condition (as $m > n$) by which we may conclude that the repulsive force be shorter range than the attractive ones.

For lattice energy we get total energy of the crystal of N molecules at their equilibrium separation. So, as a result we get an idea about bonding in solids.

❖ Acknowledgement

The success and final outcome of this project required a lot of guidance and assistance from many people and I am extremely privileged to have got this all along the completion of my project. All that I have done is only due to such supervision and assistance and I would not forget to thank them.

I respect and thank our Principal, Swami Kamalasthananda and Vice Principal, Swami Vedanuragananda for providing me an opportunity to do the project work. I am extremely thankful to the Head of the Department of Physics, Prof. Asok Kumar Pal for providing such a nice support and guidance although he had a very busy schedule.

I owe my deep gratitude to our project guide Prof. Asok Kumar Pal, who took keen interest on my project work and guided me all along, till the completion of the project work by providing all the necessary information for developing a good system. I heartily thank all my friends and classmates for their encouragement and more over for their timely support and guidance till the completion of my project work. I am thankful to and fortunate enough to get constant encouragement, support and guidance from all Teaching staffs of RKMVCC Physics Department which helped me in successfully completing my project work.

❖ Source:

(1) Introduction to Solid State Physics by Charles Kittel

(2) Solid State physics by AJ Dekker

(3) Atomic physics by Dr. SN Ghoshal

Nature of Light

The project submitted, in partial fulfilment of the requirement for the assignments in {CC-XI, CC-XII, DSE-I & DSE-II} Paper, (Semester V) in the Department of Physics

Submitted by
SK SUHAYEL MAMUN

Registration No: A01-1142-111-042-2019

Supervisor Teacher: **Dr. Chandan Kumar Das**

RAMAKRISHNA MISSION VIVEKANANDA CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

ACKNOWLEDGEMENT

Inspiration and motivation have always played a key role to complete any venture.
I wish to express my deep gratitude and sincere thanks to for his encouragement and for all the facilities that he provided for this project work. I sincerely appreciate this magnanimity by taking me into his fold for which I shall remain indebted to him. I extend my hearty thanks to ***Prof. Dr. Chandan Kumar Das****, Physics Teacher who guided me to do this project successful completion of this project. I take this opportunity to express my deep sense of gratitude for his invaluable guidance, constant encouragement, constructive comments, sympathetic attitude and immense motivation which has sustained my effort at all stages of this project work.*

CONTENT

1.INTRODUCTION

Optics is ‘‘science of sight, or of the medium of sight, i.e., light; that is a branch of physics which deals with the properties and phenomena of light’’, but this definition poses questions like ‘‘what is light?’’ and ‘‘what causes sight?’’. In this project we will see the major milestones in development of modern optics and learn the simultaneous search for an answer to the nature of light. Maybe it's not possible to be able to provide a completely satisfactory answer to this question, although the nature of light may not be fully known, we at least have come to know agreat deal more about how light behaves in a variety of situations.

The first notions of light, which can be traced back to several ancient cultures, were of a religious nature. The sun was considered to be divine as the sun god (Ra) of the ancient Egypt, from whose eye the light ofday was believed to emanate. The bible too presents us with an understanding of light as of divine origin already in the first day of genesis: ‘‘God said, ‘Let there be light,’ and there was light; and God sawthat the light was good, and he separated light from darkness. He called the light day, and the dark night.’’

2.PHILOSOPHICAL THEORY OF LIGHT

Then around 5-3 century BC **Pythagoreans** imagined that something came forth from the eye to the object, the followers of **Democritus**believed that something solid and extended like a husk, carrying information about the objects shape and colour, reached the eye from the object, and the followers of **Empedocles** thought that seeing was a combined effect of both something coming forth from the eye to the object and vice versa.

The Ancient Greek mathematician **Euclid (300 B.C.)** was the first to state a number of important properties, and described the law of reflection in about 300 BCE. This states that light travels in straight lines and reflects from a surface at the same angle at which it hit it. When this law combined with **Hero's principle** (100 A.D.), (that **light follows the shortest path**,) and the **law of refraction,** whose origin dates back to **Ptolemy** (170 A.D.), one has the essential basis of so-called **geometrical optics**.

But ancient Greeks never succeeded in discovering the mechanisms involved in vision presumably since they consider sight to be restricted to visual cones or pyramids with their apex located at the eye, rather than actual studies of the eye.

Galen (200 A.D.) described the anatomy of the eye on the basis of dissections although he erroneously concluded that vision was produced at the surface of the crystalline lens. Long after this an Arab scientist **Alhazen (1000 A.D.)** resolved the puzzle of light sensation of the eye and concluded that the eye causes the sensation of vision. Alhazen's conclusions are mostly based on experimental observations, in contrast to the works of the ancient Greeks, but nevertheless in the west, where the older texts were of stronger influence, the ancient Greeks concept of visual rays that originated in the eye held sway for a long time yet.

Later an Iraqi Mathematician **Ibn Sahl** discovered the full **law of refraction** (known as **Snell's law**) in **984**. Sahl showed that the angle of incidence is related to the angle of refraction using the law of sines **(Sin α1/ Sine α2 = constant).**

Sahl couldn't use this method to measure the actual speed of light, however, and could only determine the ratios.

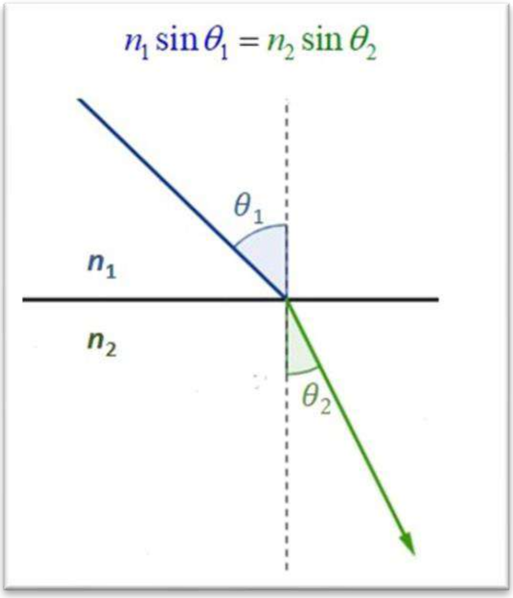


Fig 1: Snell's law

Towards the end of the **13th century Italian glassworkers made lenses** that were used for spectacles and thus to correct for poor sight caused by presbyopia. Centuries later, **Kepler (1604)** took up the challenge and studied the transmission of rays through lenses with geometrical optics, and he applied his studies to the eye. Thus, he was able to describe how spectacle lenses could correct for poor sight and he realised that the external world was seen via the inverted image that was formed on the retina by comparing the eye with **da Vinci's camera obscura.** That was the start of geometrical optics.

Towards the end of the 16th century Dutch spectacle makers had combined two lenses which resulted in the invention of the telescope. **Galileo (1609)** was the first to see the scientific importance of this tool, and he began to produce his own telescopes that had **magnifications up to 30 times** and used telescopes to observe planetary motion. That's another branch of physics which we are not discussing here.

The first law about the behavior of light was discovered by **Fermat** in about **1650,** and it is called the **principle of least time**, or **Fermat's principle**. His idea is this: that out of all possible paths that it might take to get from one point to another, light takes the path which requires the shortest time. It has a great significance in discovering true behavior of light in that time.

3. WHAT IS THE TRUE NATURE OF LIGHT? – HISTORY AND DEBATES:

The scientists of that time understood the use of light. But civilization came to a standstill in this question **'what is the light?'**

3.1 Corpuscular or particle theory of light –

Scientists were trying to understand why light shows these characteristic behaviors of straight-line motion, reflection, refraction, colours, etc.

In that time the great French philosopher **Rene Descartes** (1596-1650) on **1637** had used amechanical analogy of light by assuming that it consists of tiny particles, and from this hypothesis he had derived the correct expression of the refraction formula what Snell found often known as **Snell's law of refraction** experimentally.

With regard to colour, Descartes suggested that it could be a manifest of different angular velocities of rotation of the light particles, and this correctly foreshadowed a kind of periodicity and that colour is a property inherent to the light itself. Previously thinkers had mostly heldthat light and colour were two completely different entities with colour as a property of the objects that were carried forward to the observerby light.

Then it comes to Newton, who was respected by all who had interest in science, came out with what can becalled "**particle model of light**". According to this model when you put on a bulb, a candle or any other source of light, the source emits special kinds of particles—the particles of light. These particles were commonly called corpuscles and the description of light given by Newton, corpusclemodel of light. The model was very simple in nature and was able to explain the observations available in those periods. Let us see how.

Newton's first law of motion tells that every particle moves in a straight line with a constant speed if no force acts on it. So, the particles of light should also move in straight lines in free space---a simple explanation for rectilinear motion of light.

Reflection is also simple to understand using the corpuscle model. A rubber ball hitting a smooth, hard surface rebound. For a perfectly elastic oblique collision from a hard plane surface, the angle of incidence is equal to the angle of reflection, and the two velocity vectors and the normal to the surface are in the same plane. Similarly, a particle of light when strikes a smooth surface, say a mirror, reflects obeying the known laws of reflection.

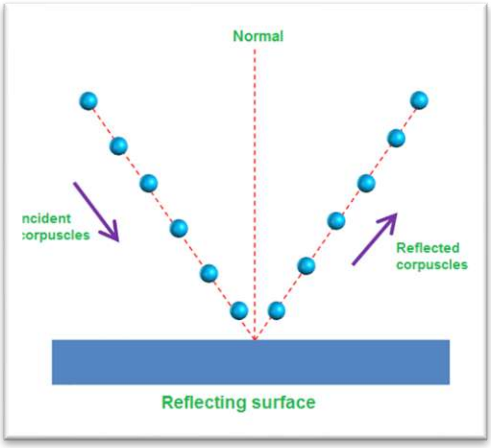


Fig 2: Reflection of corpuscles.

If light goes from air to another transparent medium such as glass, the particles of the second medium attract the particles of light, resulting in deflection in their paths. Once the particle is well within the second medium, force is exerted on it from all sides. This makes the resultant force zero and the particle moves along a straight line. The deflection at the surface causes bending of the light ray. The force of attraction on the particles of light by the medium will also increase their speed and according to the Newton's model, speed of light in water or glass shouldbe larger than that in air. However, in Newton's times there was no wayto measure the speed of light in a medium and this prediction could notbe verified.

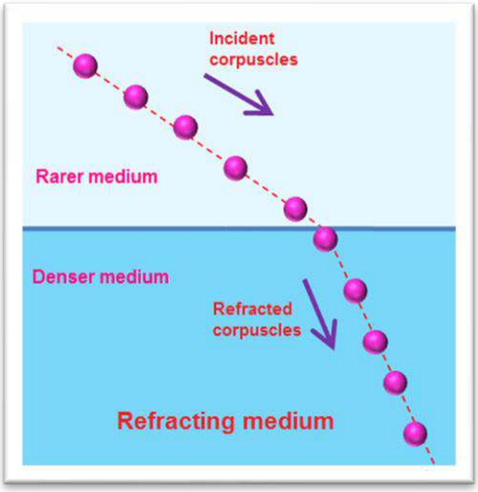


Fig 3: Refraction of corpuscles

But to explain colours, Newton assumed that there are different types of particles of light, each corresponding to a particular colour. He had performed a number of experiments to study the behavior of light. The book **Optiks** written by him, gives a classic account of these experiments.

He also made a series of wonderful experiments in order to reveal the nature of light. And the best-known one is where he found that spectral colours can be extracted from white light by making use of the refraction of light when transmitted through a prism. In consequence, he found that white **light is constituted by light of various Colours and that different colours have distinct refrangibility** (i.e., the law of refraction depends on colour). He believed that light was of a material origin consisting of minute particles and that an attraction of these towards the larger body (prism) was the cause of the refraction phenomenon. In such a mechanical picture the different refraction experienced by rays ofdistinct colour corresponded to light particles of different size. He also studied the partial reflection of light from transparent materials and argued that it could be accounted for by introducing a curious propertyof light: that it could be in fits of easy transmission or reflection that made it prone to be either transmitted or reflected.

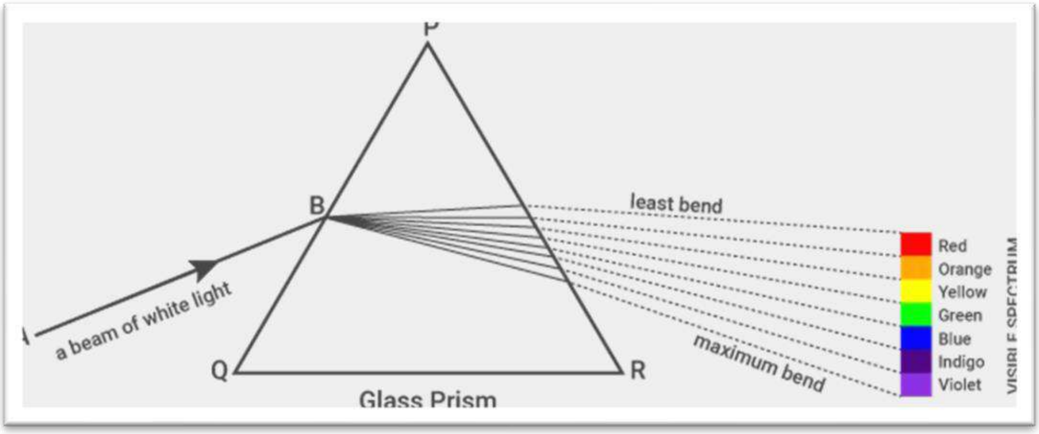


Fig 4: Dispersion of light through prism

Then the observation of **Newton's ring** and **Polarization** showed that light might have some other sides. This, however, he finally left among anumber of queries for others to investigate in further detail.

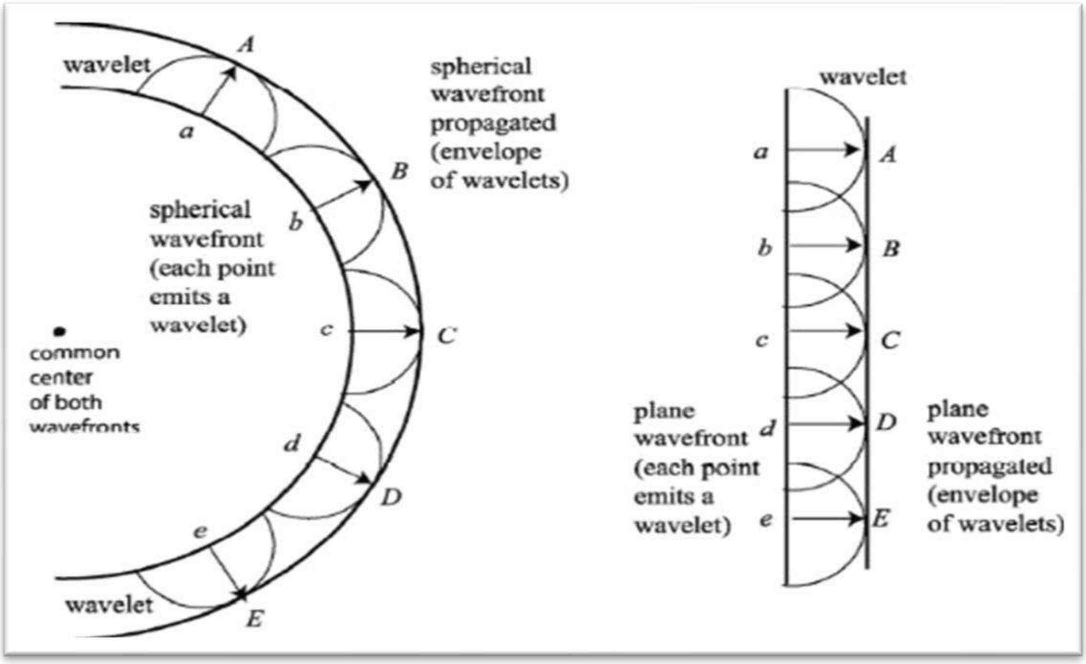
Newton's theory, though very successful in understanding the known behavior of light, did not go unchallenged. In **1678** the Dutch physicist **Christian Huygens (1629-1695)**, a contemporary of Newton, had a very different view. He suggested that light travels as a wave motion in a medium, more like a sound wave.

3.2 Wave theory of light -

Huygens' proposal remained in a dump foralmost about a century. The scientific community then had great faith in Newton's writings and the particle theory remained in chair for a long time when it was seriously challenged by the **double-slit experiment** of **Thomas Young** (1773-1829) in **1801.** And that was the starting of the web theory of light.

Huygens (1690) believed in the wave-like nature of light and considered it to propagate as small spherical waves that add up to forma wavefront that afterwards acts as a source for new secondary waves known as **Huygens principle.** A continuous repetition of this principle suffices to account for the propagation of light. With this model he was able to explain both the reflection and refraction of light, but a crucial feature was that the speed of light had to be slower in the denser medium. This was in contradiction with the corpuscular theory of light although both had given the correct expression for the refraction formula.

Fig 3: Huygens principle



Then Young **(1803)** came up with an experiment known as **Young's double-slit experiment** where he showed that two monochromatic light superose to form a pattern of redistribution of energy take place**.** To explain this phenomenon, he follows **Huygens principle** and publish his theory on **principle of interference**, and gave an explanation of the phenomenon of Newton's rings in terms of either cancellation or addition of wave amplitudes of light reflected from thetwo interfaces.

Interference is a phenomenon of light where two waves superpose to form a resultant wave with a non-uniform redistribution of energy. He studied various interference phenomena, including the well-known experiment with transmission of light through two closely-spaced small holes, that allowed him to estimate the light wavelength. For utmost red he found 0.71 mm and for blue 0.42mm (the visible range is now often identified with 0.40–0.70 mm).

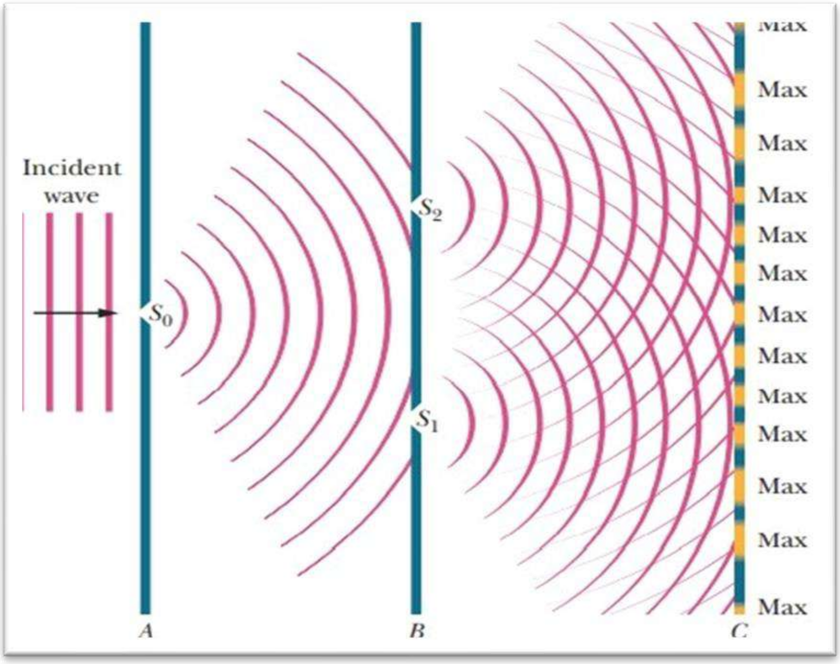


Fig 4: Interference of two coherent waves

He also studied vision and found that accommodation was caused by changes in the shape of the eye lens. Moreover, he suggested that the eye has a discrete number of light sensitive elements with only three kinds of colour responses (**red, green and blue–violet**) so that other colours are seen via a proper combination of those.

Along with interference there is another phenomenon that was unableto explain through Corpuscular theory this phenomenon is known as **Diffraction.**

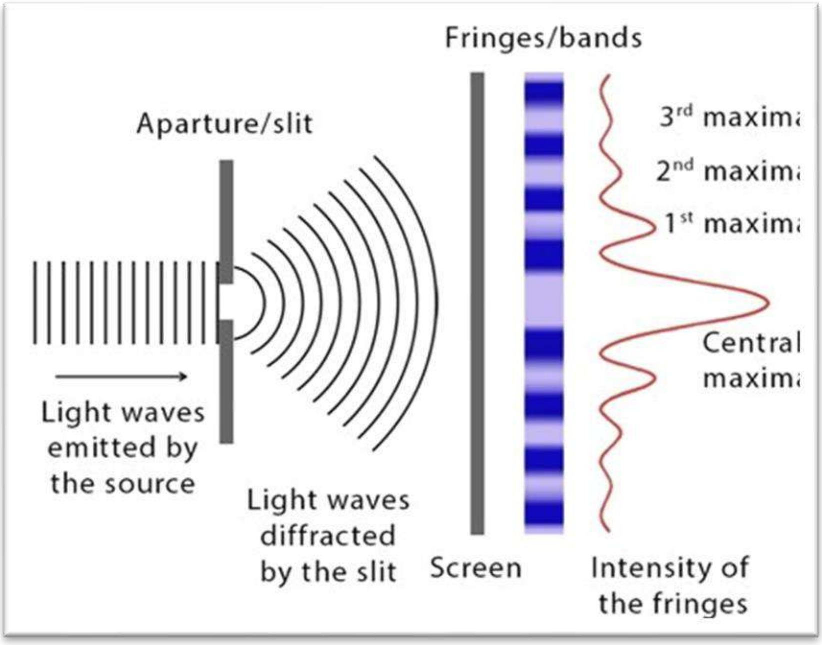


Fig 5: Diffraction of light wave

It is the bending of light around corners such that it spreads out and illuminates’ regions, where a shadow is expected.

Then a solid basis for the wave theory of light was formulated by **Fresnel (1819)** who also did experiments on diffraction of light.He considered the light to propagate as a sum of Huygens waves that produce both diffraction and interference effects, and he submitted a mathematical description of his model to the French Academy of Sciences where he won over particle theory and this proved the end to the former particle description of light.

Maxwell (1873) is known for the great achievement of combining all the phenomena of electricity and magnetism in a single theoretical framework and a set of four famous equations—the **Maxwell equations**:

$$\nabla.E = \frac{\rho}{\epsilon_0} \qquad (1)$$

Gauss' law

$$\nabla.B = 0 \qquad (2)$$

Magnetic monopoles

$$\nabla \times E = -\frac{\partial B}{\partial t} \qquad (3)$$

Faraday's law

$$\nabla \times H = J + \frac{\partial D}{\partial t} \qquad (4)$$

Ampere-Maxwell law

From those expressions a wave equation for an electromagnetic field can be derived and he found that the combination of two constants, the vacuum permittivity (ε_0) and the vacuum permeability (μ_0), led to a remarkable number that coincided (within experimental accuracy) to the speed of light in vacuum. As a consequence, he suggested that light waves must be of an electromagnetic nature.

Also, **Lorenz (1867)** too had reached this conclusion on the basis of his work on electrodynamics. Later, **Hertz (1888)** made experimental work on the emission and receiving of electromagnetic waves (at MHz frequencies and thus well below that of light) and he found that they obey the same reflection and refraction formulas as light and indeed do propagate with the speed of light as previously predicted. In consequence, light had now been found to belong to a broader class of transverse electromagnetic waves is often identified with a broader spectrum beyond what is visible to the human eye so as to include also infrared and ultraviolet radiation discovered as extensions to the visible spectrum already in the beginning of the 19th century.

3.3 Dual nature of light-

At the turn of the century the wave nature of light was considered an indisputable fact, and the description of matter moved more into focus. The wave theory of light fails when light properties are to be explained, specially the interaction of light with matter. **Hertz,** in a famous experiment in **1887** discovered the photoelectric effect which was unable to describe by wave theory.

Planck (in 1900), however, studied a subject at the borderline between light and matter as he tried to reconcile classical theory with a description of temperature-dependent electromagnetic radiation from a **blackbody** (i.e., a perfectly absorbing material). He postulated that the internal energy of the blackbody material could only change as an integer number of small **quanta**. This meant that the electromagnetic field would also change energy in discrete jumps with the size of this energy quantum when radiation was either emitted or absorbed by the blackbody.

Einstein (1905) made the bold assumption that light then too consisted of tiny energy **quanta**, equal in magnitude to those of the **blackbody** problem. On this assumption he was able to explain the **photoelectric effect** that is emission of electrons from metallic plates when exposed to light. That curiously had been discovered by **Hertz** in his work to confirm the electromagnetic waves predicted by **Maxwell**. Thus, light interacts with materials as if it consists of particles each carrying a tiny **lump of energy.** These light quanta were later named **photons** by **Lewis(1926)** in analogy with elementary particles.

The wave theory and the particle theory of light were again positioned on a collision course! It remained dilemma until theory of **dual nature of light** proposed.

The basis of **wave–particle duality** is that light can behave as a wave in certain situations and as a particle under other conditions. Einstein was the first to face the duality concept. His equation for explaining the photoelectric effect recognized both aspects: E, a property of light as a particle, and ν, a property of light as a wave.

Then it comes to **Louis de Broglie** who states that a matter can act as waves much like light and radiation. According to de Broglie’s momentum–wavelength equation (**p=h/λ**) all materials having momentum should have some wave associated with it.This means even an electron, photon or neutron will have not only momentum and energy attributes, but also wavelength attributes.

The latest major contribution to an understanding of the nature of light has been the **quantum mechanical picture of light** contained in quantum electrodynamics developed by **Feynman and others**. It is considered to propagate as a wave function and thereby requiring that all possible propagation paths shall be considered from the emission to the detection of the light. When detecting the light its wave function collapses and the interaction is therefore seen as if light consists of individual particles of light. This dual nature of light is still a highly active area of research being explored, e.g., in single photon experiments.

6.CONCLUSION

We have now obtained a deep understanding of the nature of light. So, live is no appropriate nature of light, the general consensus of opinion is that it behaves as both a wave and a particle, depending on the situation—for example, the colours in a soap bubble are attributed to the wave properties of light whereas when it enters a camera, it acts as a particle. Light comprises packets of energy called photons and can also be considered as a wave with periodically oscillating electric and magnetic fields that are perpendicular to each other and perpendicular to the direction of propagation of the wave—it can undergo reflection, refraction, interference, polarization and diffraction when behaving as a wave. Also the speed of light under vacuum is the highest in the universe.

7. REFERENCES

Sites:

- *http://www.thestargarden.co.uk/19th-Century-wave-theories.html*
- *https://www.researchgate.net/publication/228620003_The_nature_of_light_II*

Books:

- *Ajoy Ghatak, Optics, 6th Edition*
- *Olivier Darrigol, A History of Optics from Greek Antiquity to the Nineteenth Century,*
- *Eugene Hecht, Optics, Pearson*

Runge-Lenz Vector In Kepler Problem

The project submitted, in partial fulfilment of the requirement for the assignments in (PHSA CC-XI,CC-XII,DSE-I & DSE-II) Paper, (Semester v) in the

Department of Physics

Submitted by

(**SOUBHIK PRAMANIK**)

Registration No: A01-1112-111-044-2019 of 2019-20

Supervisor Teacher: ATISDIPANKAR CHAKRABARTI

RAMAKRISHNA MISSION VIVEKANANDA
CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

ACKNOWLEDGEMENT

I would like to express my special thanks of gratitude to my professor ATISDIPANKAR CHAKRABARTI for his able guidance and support in completing project. I would like to extend my gratitude to him to give me this opportunity to do this project in this interesting topic and he also encourage me for this project work.

▪ Prehistory of the “Runge-Lenz” Vector:

Heintz’s exposition of the “Runge—Lenz” vector perhaps provides an occasion to remark on the earlier his-tory of this vector invariant of the Kepler problem, and to appeal for further information about the various eponymic changes it has undergone. When Lenz' made use of this vector to calculate the energy levels of perturbed Kepler motions on the basis of the old quantum theory, he de- scribed it as “little known,” and referred to a then popular text by Runge on vector analysis. In his discussion of the vector invariant, Runge makes no claim for originality.

The vector appears in a section intended to illustrate procedures for differentiating and integrating vectors by application to central force motion. After proving the in- variance of the angular momentum vector Runge goes on briefly to show that, if the central force varies inversely as the square of the distance, another constant vector can be obtained from the equation of motion. In turn, from this constant vector the orbit equation is then derived. No suggestion is made in the text that Runge was the first to discover this vector constant, any more than the in- variance of the angular momentum vector. **Pauli, in** his pioneer 1926 paper‘ on the derivation of the levels of the hydrogen atom on the **basis of** the new matrix mechanics, starts again from the vector invariant, which he notes as “previously utilized by Lenz.” Nevertheless, the vector has entered almost all subsequent **physics** literature as the **“Runge—Lenz vector.”**

The earliest appearance of the constants of motion making up this vector invariant that I know of is in Vol. 1 of Laplace’s *Traire* ***de mecanique celeste,*** **which** appeared in 1799. Laplace’s physical reasoning is clear, explicit, and reasonably complete. It deserves to be described in some detail. Laplace collects seven integrals of the motion for the reduced Kepler problem, not explicitly dependent on time. The first three of these, in modern notation, are respectively l_z/μ $—l_y/\mu$ and l_x/μ, where μ is the reduced mass and **L** is the total angular momentum. The next three, again in modern notation, are the cartesian components of A/μ^2, where **A** is the Runge—Lenz vector in the form

$$A=P\times L-\mu K\frac{r}{r}$$

Since then, the LRL vector has been quite popular as a simple example of hidden symmetries. People
have gone on to make generalizations of it to apply to cases with electric and magnetic fields, and relativistic versions as well. For an introduction to the generalizations, see the review by Leach and Flessas.

▪ Kepler Problem:

The Laplace-Runge-Lenz (LRL) vector has its origins in the peculiarities of the Kepler problem. The vector itself

$$A = \mathbf{P} \times \mathbf{L} - \mu \mathbf{K} \hat{\boldsymbol{r}}$$

is an additional conserved quantities for central force problem9s with an inverse square potential. Here p is the momentum, L=r×p the angular momentum,
m the mass (or for the two body problem, replace $\mu = m_1 m_2/(m_1 + m_2$ the reduced mass). And k is a constant characterizing the strength of the potential. We are interesting in the general Kepler Problem, i.e. the Hamiltonian

$$H = \frac{|P|^2}{2m} - \frac{k}{r}$$

This problem is a very interesting one, marked by a great deal of symmetry.Besides the usual conserved energy E, since the Hamiltonian is rotationally invariant, we know to expect conservation of the angular momentum $L = r \times p$.

- ## Derivation of LRL Vector:

In order to derive the conservation of the LRL vector, we can follow the development
in Goldstein . Given that our force is central, we can restrict Newton's law to the form

$$\dot{p} = f(r) \frac{\boldsymbol{r}}{r}$$

I.e. the force should point in the radial direction and depend only on r. This allows us to find a form for

$$\dot{p} \times L = f(r) \frac{\boldsymbol{r}}{r} \times (r \times p) = f(r) \frac{\boldsymbol{r}}{r} \times (r \times m\dot{r})$$
$$= \frac{mf(r)}{r} [r \times (r \times \dot{r})] = \frac{mf(r)}{r} [r(r \cdot \dot{r}) - r^2 \dot{r}]$$

where we have used Laplace's identity in the last equality. Next we need to be clever and further simplify the expression by noticing

$$r \cdot \dot{r} = \frac{1}{2}(\dot{r} \cdot r + r \cdot \dot{r}) = \frac{1}{2}\frac{d}{dt}(r \cdot r) = \frac{1}{2}\frac{dr^2}{dt^2} = r\dot{r}$$

i.e. the radial component of the velocity is just $\dot{r}$. Using this, we obtain

$$\dot{p} \times \mathrm{L} = \frac{mf(r)}{r}[\mathrm{r}\dot{r}r - r^2\dot{r}] = mf(r)r^2\left[\frac{r\dot{r}}{r^2} - \frac{\dot{r}}{r}\right]$$

Since L is a conserved quantity, we can write

$$\frac{d}{dt}(\mathrm{P}\times\mathrm{L}) = -mf(r)r^2[\frac{\dot{r}}{r} - \frac{\dot{r}r}{r^2}] = -mf(r)r^2\frac{d}{dt}\left[\frac{\boldsymbol{r}}{r}\right]$$

So we’ve made some progress, found and interesting expression. Unfortunately, at this point we are stuck, unless of course $f(r) \propto r^{-2}$ such as for the Kepler problem. This peculiarity enables us to take $f(r) = -k/r^2$ and we obtain

$$\frac{d}{dt}(p \times L) = \frac{d}{dt}\left(\frac{mk\boldsymbol{r}}{r}\right)$$

which is to say we have

$$\frac{d}{dt}\left[p \times L - mk\frac{\boldsymbol{r}}{r}\right] = 0$$

i.e. we’ve found that

$$\mathrm{A} = p \times L - mk\frac{\boldsymbol{r}}{r}$$

Is conserved.

Discussion on LRL vector:

Now that we have found a new conserved vector for the Kepler problem, the question remains as to its utility. In particular, we should already be a little concerned. The one body Kepler problem has 6 degrees of freedom, but we already have E and L and now A, suggesting 1+3+3 = 7 conserved quantities. Surely this cannot be the case. In particular, we know that for the Kepler problem, there is nothing that should tell us the initial time of our motion, leaving, at most, 5 degrees of freedom that can be conserved. This implies that there should be some relations between A and E; L. In particular, we notice that if we take the dot product

$$A \cdot L = \left[p \times L - mk\frac{\boldsymbol{r}}{r}\right] \times L = (p \times L) \times L - mk\frac{\boldsymbol{r}}{r} \times (r \times P) = 0$$

i.e. A is perpendicular to L at all points of the motion, i.e. it lies in the plane of motion. In particular, we can compute the LRL vector at various points in the orbit. Borrowing from Goldstein. we notice that A lies in the plane of the orbit, as discussed, is a constant of the motion, and appears to point in the direction of the symmetry axis of our ellipse.

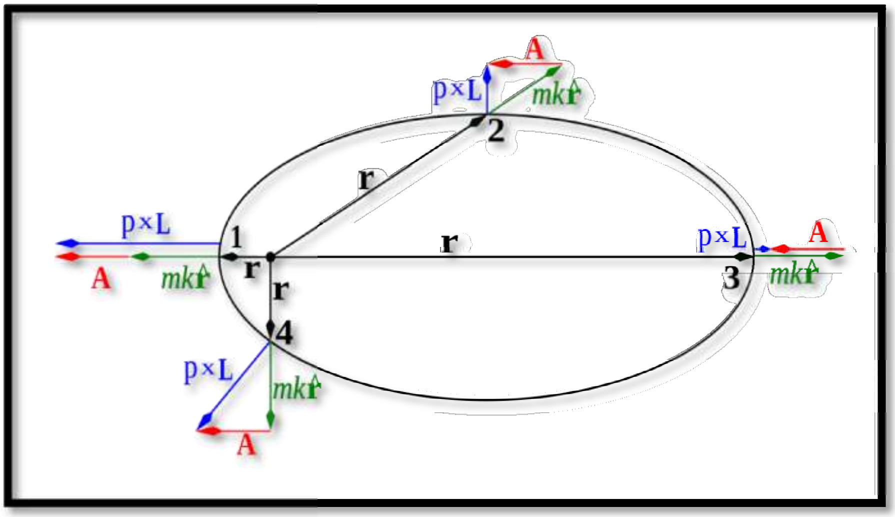


Figure 1: LBL vector at various points in the orbit. Borrowed from Goldstein [3].In order to find the other relation amongst E;L; A we need to look at the direction A points in detail. We can take the dot product with the radius vector

$$A \cdot r = Ar\cos\theta = r \cdot (p \times L) - mkr$$

and permuting the triple product we get the form

$$A \cdot r = (r \times p) \cdot L = L^2$$

and we can obtain the equation

$$Ar\cos\theta = L^2 - mkr$$

$$\frac{1}{r} = \frac{mk}{L^2}\left(1 + \frac{A}{mk}\cos\theta\right)$$

which has the form of a conic section. Interestingly enough, we have found the orbits of the Kepler problem in terms of the LRL vector. Comparing this with the standard solution for the orbit .

$$\frac{1}{r} = \frac{mk}{L^2}\left(1 + \sqrt{1 + \frac{2EL^2}{mL^2}}\cos(\theta - \theta')\right)$$

we find that we can write

$$\frac{A^2}{m^2L^2} = 1 + \frac{2EL^2}{mk^2}$$

which gives us

$$A^2 = m^2k^2 + 2mEL^2$$

So, it would appear as though our peculiar symmetry, the LRL vector seems to be tied with the fact that our orbits are closed. With this realization, we might expect something akin to the LRL vector for different problems, in particular Bertand's Theorem would suggest an analog to the LRL vector for the harmonic oscillator in particular, the other potential that admits closed orbits. We will leave the investigation of the corresponding harmonic oscillator case until later.

- ## Application:

- ## Hodographs:

So, from our pictorial investigation, we are lead to believe that our LRL vector points in the direction of the symmetry axis of our orbit. In fact, as Goldstein recounts , Hamiltonian may have been the first to utilize the LRL conservation. He referred to it as the eccentricity vector, where if we rearrange our expression as

$$mk\frac{\mathbf{r}}{r} = p \times L - A$$

and take the dot product of it with itself, we obtain

$$(mk)^2 = A^2 + p^2L^2 + 2L \cdot (p \times A)$$

and now choosing L to be along the z axis and the major semiaxis along the x we obtain

$$p_x^2 + (p_y - A/L)^2 = (mk/L)^2$$

but this gives us an interesting representation for the momentum vector. In fact it illustrates that the momentum should just travel around a circle of radius mk/L centered (A/L)away from the center of force perpendicular to A.

Hydrogen Atom:

Another interesting problem that fits the bill for the above description (1) central force, and (2) inverse square force, is the Hydrogen atom. Just as before we have the Hamiltonian

$$H = \frac{p^2}{2m} - \frac{e^2}{r}$$

which has the same form. As we know, borrowing from Shankar, we have energy levels

$$E_{nlm} = -\frac{me^4}{2\hbar^2}\frac{1}{n^2}$$

which we notice depends only on n and not l or m. We have a great deal of degeneracy in the orbitals of the hydrogen atom. Not only do the energy levels not depend on m, corresponding to the rotational symmetry, but neither do they depend on l, a much deeper symmetry, which we might suspect to be linked to our LRL vector.

Quantum LRL:

So, we suspect the high level of symmetry should derive from the LRL vector just as it did in the classical case. As such, we begin by attempting to look for a quantum analog to:

$$\mathrm{A} = p \times L - mk\frac{\boldsymbol{r}}{r}$$

or in this case we will use a slight modification

$$N = \frac{p \times L}{m} - \frac{e^2}{r}\boldsymbol{r}$$

which suggests a particular quantum form, taking care to ensure Hermiticity

$$\hat{N} = \frac{1}{2m}\left[\hat{p} \times \hat{L} - \hat{L} \times \hat{p}\right] - \frac{e^2}{r}\hat{R}$$

using hats to denote the quantum operators.

Lucky for us, this operator manages to commute with the Hamiltonian. With a conservative force $\hat{p}$ we know commutes, and the rotational symmetry ensures that so do $\hat{R}$;$\hat{L}$. So we have [$\hat{N}$;$\hat{H}$] = 0. This implies that it should act as the generator of some symmetry. Having found the corresponding operator, it would be nice to see how it generates the degeneracy. This can be accomplished following the procedure in Shankar [7]. We need to express $\hat{N}$ in spherical form

$$\hat{N}_1^{\pm 1} = \mp \frac{N_x \pm i N_y}{\sqrt{2}} \qquad \hat{N}_1^0 = N_z$$

Having done this we can consider having our $\hat{N}_1^1$ operator act on a state $|nll\rangle$ We know that this should produce a state with the same energy since our operator commutes with the Hamiltonian $[\hat{N}_1^1, H] = 0$,
So what does our operator due to our state? Well, we know that $\hat{N}_1^1|nll\rangle$ should behave like $|11\rangle \otimes |ll\rangle = |l+1, l+1\rangle$ i.e. we have

$$\hat{N}_1^1\, |n, l, m\rangle = c\, |n, l+1, l+1\rangle$$

And we've done it. We can use the quantum analogy to the LRL vector to demonstrate the degeneracy in the different l levels.

- ## Poisson Brackets:

Drawing from our experience with the Quantum application, it might be useful to look into the algebraic structure of the LRL vector. First lets revisit the Poisson brackets for the angular momentum vector. Again drawing from Goldstein we know:

$$\{L_{i,} L_j\} = \epsilon_{ijk} L_k$$

But this structure is the same as for the generators of rotation in three dimensional space. i.e. the group of transformations generated by $L_{i,}$ can be identified with SO(3). Or, to be clear, lets follow the notation of Rogers and write

$$L = (r \times p)$$

$$D = \frac{1}{\sqrt{-2mE}} \left[\frac{p \times L}{mk} - \frac{\boldsymbol{r}}{r}\right]$$

In order to discover the SO(4) structure that these imply, consider looking at the two linear combinations

$$M = \frac{1}{2}(L + D) \qquad N = \frac{1}{2}(L - D)$$

These new guys have the following Poisson structure

$$\{M_{i,}M_j\} = \epsilon_{ijk}M_k$$

$$\{N_{i,}N_j\} = \epsilon_{ijk}N_k$$

$$\{N_{i,}M_j\} = 0$$

But at this point, it should be clear that the structure created by $L; D$ is that of $SO(3)\otimes\ SO(3)$ or in other words $SO(4)$. So we've done it. We have investigated the structure defined by the angular momentum and **LRL** vector in concert and now see that we should expect a great deal of symmetry in the Kepler problem, namely $SO(4)$ or a 4 dimensional rotational symmetry. Unfortunately for us, this deeper symmetry is not clear in our usual coordinates. In order to better appreciate its appearance

▪ Conclusion:

Having demonstrating the appearance and utility of the LRL vector to some physical problems, hopefully an appreciation for the depth of the corresponding symmetry was conveyed.

▪ References:

I. H. Goldstein. Prehistory of the"Runge–Lenz"vector. American Journal of Physics, 43:737, 1975.

II. H. Goldstein. More on the prehistory of the Laplace or Runge–Lenz vector. American Journal of Physics, 44:1123, 1976.

III. H. Goldstein, C. Poole, J. Safko, and S.R. Addison. Classical mechanics. American Journal of Physics, 70:782, 2002.

IV. https://en.wikipedia.org/wiki/Laplace%E2%80%93Runge%E2%80%93Lenz_vector

DOPPLER EFFECTS AND EXPANDING PICTURE OF THE UNIVERSE

The project submitted, in partial fulfilment of the requirement for the assignments in PHSA CC-XI,PHSA CC-XII,PHSA DSE-I,PHSA DSE-II. Paper (Semester V) in the Department of Physics

Submitted by

Sayan Maity

Registration No:A01-1112-111-045-2019

Supervisor Teacher:Prof Rajen Kundu

RAMAKRISHNA MISSION VIVEKANANDA CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

Introduction

The Doppler effect was named after Christian Doppler, who first came up with the idea in 1842. He learned that sound waves would have a higher frequency if the source was moving toward the observer and a lower freqency if the source was moving away from the observer. A commonly used example of the Doppler effect is observed in the context of a train.. When a train is approaching, the whistle has a higher pitch than normal. we can hear the change in pitch as the train passes.
The same is true with sirens on police cars and the engines of race cars.

One way to visualize the Doppler effect is to think of sound waves as pulses emitted at regular intervals. Imagine that each time we take a step, we emit a pulse. Each pulse in front of Us would be a step closer than if we were standing still and each pulse behind Us would be a step further apart. In other words, the frequency of the pulses in front of we are higher than normal and the frequency of the pulses behind us lower than normal.

The Doppler effect doesn't just apply to sound. It works with all types of waves, which includes light. Edwin Hubble used the Doppler effect to determine that the universe is expanding. Hubble found that the light from distant galaxies was shifted toward lower frequencies, to the red end of the spectrum. This is known as a red Doppler shift, or a red-shift. If the galaxies were moving toward Hubble telescope, the light would have been blue-shifted.

Section 1

Doppler Effects in sound :

Definition: Doppler effect in sound is the change in the frequency of the sound observed by an observer due to the velocity of the observer and the source of sound.which are relative to the medium in which the sound passes though.

Theory: velocity of the observer and of the source are relative to the medium in which the waves go through are important.

Use Of Medium: For waves that propagate in a medium, such as sound waves, the velocity of the observer and of the source are relative to the medium in which the waves are transmitted. The total Doppler effect may therefore result from motion of the source, motion of the observer, or motion of the medium.

Example: Acoustics Doppler Current Profiler,siren medical applications such as echocardiograms the Leslie speaker etc.

Doppler Effects in light:

Definition: Doppler effect in light is the apparent change in the frequency of the light observed by an observer due to the relative motiuon between the observer and the source of light.

Theory: Only the relative difference in velocity between the observer and the source are important.

Use Of Medium: The resulting Doppler shift in detected frequency occurs for any form of wave. Light requires no medium, and the Doppler shift for light traveling in vacuum depends only on the relative speed of the observer and source.

Example: Astronomical applications,satellite etc.

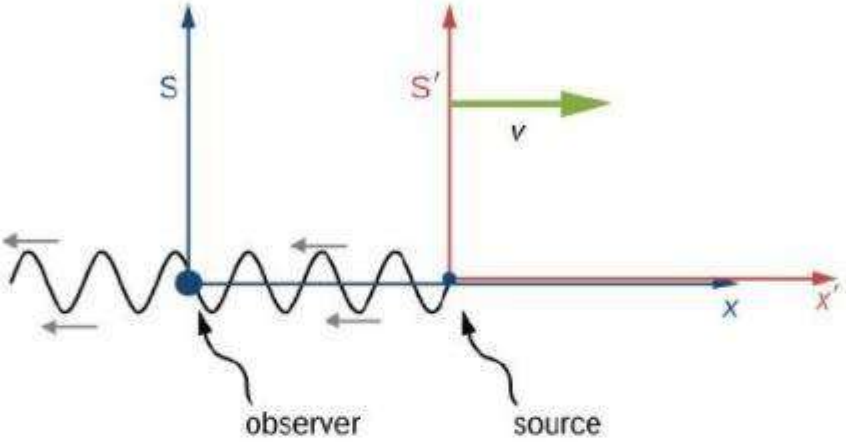
Sound waves propagate through a medium, while light does not require a medium to pass through. Therefore, for the doppler effect in sound, the velocity of the observer and the source are relative to the medium in which the waves go through are important, whereas for the doppler effect in light, only the relative difference in velocity between the observer and the source are important. Thus, this is the key difference between doppler effect in sound and light.

SECTION 2

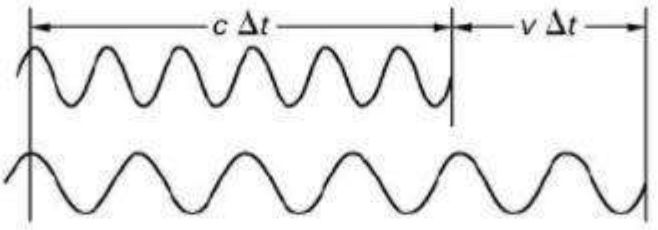
Relativistic Doppler Effcect: If a source of sound and a listener are moving farther apart, the listener encounters fewer cycles of a wave in each second, and therefore lower frequency, than if their separation remains constant. For the same reason, the listener detects a higher frequency if the source and listener are getting closer. The resulting Doppler shift in detected frequency occurs for any form of wave. For sound waves, however, the equations for the Doppler shift differ markedly depending on whether it is the source, the observer, or the air, which is moving. Light requires no medium, and the Doppler shift for light traveling in vacuum depends only on the relative speed of the observer and source.

Suppose an observer in S sees light from a source in S′ moving away at velocity v. The wavelength of the light could be measured within S′ — for example, by using a mirror to set up standing waves and measuring the distance between nodes. These distances are proper lengths with S′ as their rest frame, and change by a factor $\sqrt{1 - (\frac{v}{c})^2}$

when measured in the observer’s frame S, where the ruler measuring the wavelength in S′ is seen as moving.



(a)



(b)

(a) When a light wave is emitted by a source fixed in the moving inertial frame S', the observer in S sees the wavelength measured in S'. to be shorter by a factor . (b) Because the observer sees the source moving away within S, the wave pattern reaching the observer in S is also stretched by the factor .

If the source were stationary in S, the observer would see a length cΔt of the wave pattern in time Δt. But because of the motion of S' relative to S, considered solely within S, the observer sees the wave pattern, and therefore the wavelength, stretched out by a factor of

$$\frac{c\Delta t_{period} + v\Delta t_{period}}{c\Delta t_{period}} = 1 + \frac{v}{c}$$

as illustrated in (b) of Figure. The overall increase from both effects gives

$$\begin{aligned}\lambda_{obs} &= \lambda_{src}\left(1+\frac{v}{c}\right)\sqrt{\frac{1}{1-\frac{v^2}{c^2}}}\\ &= \lambda_{src}\left(1+\frac{v}{c}\right)\sqrt{\frac{1}{\left(1+\frac{v}{c}\right)\left(1-\frac{v}{c}\right)}}\\ &= \lambda_{src}\sqrt{\frac{\left(1+\frac{v}{c}\right)}{\left(1-\frac{v}{c}\right)}}\end{aligned}$$

where λsrc is the wavelength of the light seen by the source in S' and λobs is the wavelength that the observer detects within **S**.

Red Shifts and Blue Shifts:

The observed wavelength of electromagnetic radiation is longer (called a “red shift”) than that emitted by the source when the source moves away from the observer. Similarly, the wavelength is shorter (called a “blue shift”) when the source moves toward the observer. The amount of change is determined by

$$\lambda_{obs} = \lambda_s\sqrt{\frac{\left(1+\frac{v}{c}\right)}{\left(1-\frac{v}{c}\right)}}$$

where λS is the wavelength in the frame of reference of the source, and V is the relative velocity of the two frames S and S'. The velocity V is positive for motion away from an observer and negative for motion toward an observer. In terms of source frequency and observed frequency, this equation can be written as

$$f_{obs} = f_s \sqrt{\frac{\left(1 - \frac{v}{c}\right)}{\left(1 + \frac{v}{c}\right)}}$$

Notice that the signs are different from those of the wavelength equation.

The Non Relativistic Doppler Effect:

As we saw in our discussion of the properties of electromagnetic radiation , relative motion of a source creates a shift in the frequency of light (or any other wave) called the Doppler effect. The adjacent right figure illustrates, as does this animation. The Doppler effect for light leads to a displacement in the position of spectral lines. Motion of the source away from the observer causes a shift to longer wavelengths that is termed a red shift. Motion of the source toward the observer causes a shift to shorter wavelengths that is termed a blue shift.

The Doppler effect is particularly important in astronomy because it allows a rather simple determination of the radial velocity (velocity along the line of sight) for a source emitting electromagnetic waves. Since binary stars are in orbit around their center of mass, the measurement of Doppler shifts in the light coming from binary systems provides an important way to learn about this motion.

The Doppler Shift Formula-

In nonrelativistic approximation, the radial velocity and the Doppler shift of spectral lines are related by the formulas given in the adjacent left figure. The nonrelativistic approximation is valid when the velocities v are much less than the speed of light c. If v is comparable to c, a slightly more complex formula must be used but the effect is still to allow a velocity to be computed from a Doppler shift. The radial velocities in all normal binary star systems are much smaller than the speed of light.

SECTION 3

APPLICATIONS OF DOPPLER EFFECT-

1. Scientists use the Doppler effect to observe distant stars

The Doppler effect is a very useful tool for astronomers. Stars are constantly emitting electromagnetic waves in all directions that we can observe from here on Earth.As the star rotates around its center of mass and moves in space, the wavelengths of its EM radiation shift accordingly relative to our position on Earth. We observe this as very subtle changes in the EM spectrum, notably the visible light portion of it. When the star moves towards us its EM emission wavelengths get compressed and becomes slightly bluer (blueshifts). When the star moves away from us, its emitted light becomes ever so slightly redder, or redshifts. To observe this effect, astronomers use something called a spectrograph (a prism-like apparatus) that separates out incoming light waves into different colors.In the star's outer layer, atoms absorb light at specific wavelengths. These can be observed as "missing" by appearing as dark lines in different colors of the sun's emitted spectrum.These are useful as markers to measure the size of the Doppler shift. If the star is on its own (no planets or other nearby stars) this pattern should remain relatively constant over time.If there is a companion star around, the gravitational pull of this unseen body will affect the other star's movement at certain points of its orbit. This will produce a noticeable change in the overall pattern of the Doppler shift over time.

2. The Doppler effect is used to find exoplanets

Just like companion stars, the Doppler effect can be used to find, or at least surmise, their presence around a distant star. As these planets are so small, relatively speaking, it is very hard to observe them directly using conventional telescopes. Even if we could, they are often obscured from view by the overwhelming glare of their parent star. Any star that has exoplanets will "wobble" ever so slightly about its axis. We can use the Doppler effect to find candidate star systems. However, it should be noted we can only find larger planets akin to Jupiter or bigger using this approach. The effect will be more subtle than a companion star, but it is useful to determine the planet's orbital period (aka the length of a "year") and the likely shape of its orbit, and also its likely minimum mass. For smaller exoplanets, like another Earth-sized planet, other methods are required. Specialist apparatus like NASA's Kepler spacecraft, look for drops in a parent sun's emitted radiation as planets move across the surface of their sun. Called a "transit method", astronomers can calculate the relative drop in brightness of a star and use that data to calculate the size of the body that transited past the sun. We can even work out how far the exoplanet is and infer information about its likely atmospheric composition. The Doppler effect, if the observing apparatus is sensitive enough, can even be used to observe the likely atmospheric condition of the planet. According to MIT, in 2010 one of their postdoc graduates, Simon Albrecht, was able to discover that color shifts in the light absorbed by the planet indicated that strong winds were likely present within its atmosphere. To date, over 4,000 exoplanets (as of September 3rd, 2020 NASA announced that we have confirmed 4,276) have been discovered using things like the Doppler effect. There are also thousands of "candidate" exoplanets that are yet to be officially confirmed. Amazingly, the first exoplanet was discovered over three decades ago during the 1990s. Since then the number has grown exponentially. As our observing apparatus gets more complex and sensitive over time, who knows what will be able to discover about these distant worlds.

CONCLUSION

If observer is moving in a direction opposite to that of the sound waves (source), its frequency will appear to observer to be higher.

Similarly, if sound waves (source is) are moving towards you (observer), its frequency will appear to observer to be higher than the original frequency.

On the other hand, if observer is moving away from the sound waves (source) or if observer is moving in the same direction of sound waves, its frequency will appear to observer to be lower than the original frequency.

This report helped me to give me a deep knowledge of the of Doppler Effect. It further helped me to realize to know the relativistic and non-relativistic Doppler effect etc.

BIBLIOGRAPHY

For the completion of this project and giving it a final shape, I have consulted the following books and websites.

Sites-

https://www.physics-and-radio-electronics.com/physics/dopplereffect.html

https://en.wikipedia.org/wiki/Doppler_effect

Books-

1. The doppler effect in other stories by Allene steele
2. Vibration and waves by A.P. French

ACKNOWELEDGEMENT

Primarily I would like to express my gratitude to Ramakrishna Mission Vivekananda Centenary College, Rahara for including the project as part of our academic curriculum. This has provided me an opportunity to learn something beyond the text book. I wish to express my deep gratitude and sincere thanks to revered Principal Maharaj Swami Kamalasthananda, Ramakrishna Mission Vivekananda Centenary College, Rahara, for his encouragement and the facilities provided for this project work.

I extend my sincere thanks to Dr. Rajen Kundu, Professor, Ramakrishna Mission Vivekananda Centenary College, Rahara and my mentor the project by who suggested me the topic up this project as an introduction to the subject.

I take this opportunity to express my deep sense of gratitude for his valuable support and constructive comments.

HISTORICAL DEVELOPMENT OF KEPLER'S LAW

The project submitted, in partial fulfilment of the requirement for the assignments in ***PHSA CC-XI, PHSA CC-XII, PHSA DSE-I, PHSA DSE-II*** *Paper (* ***Semester V*** *) in the Department of Physics*

Submitted by

Ankan Maity

Registration No: A01-1112-111-046-2019

Supervisor Teacher: Prof. Bhaskar Haldar

RAMAKRISHNA MISSION VIVEKANANDA CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

HISTORICAL DEVELOPMENT OF KEPLER'S LAW

▪ Introduction

In Astronomy, **Kepler's laws of planetary motion** are three scientific laws describing the motion of planets around the Sun.

Most planetary orbits are nearly circular, and careful observation and calculation are required in order to establish that they are not perfectly circular. Calculations of the orbit of Mars indicated an elliptical orbit. From this, **Johannes Kepler** inferred that other bodies in the Solar System, including those farther away from the Sun, also have elliptical orbits.

Kepler's law, published between **1609** and **1619**, improved the **Heliocentric Theory** of **Nicolaus Copernicus**, explaining how the planets' speeds varied, and using elliptical orbits rather than circular orbits with epicycles.

Isaac Newton showed in **1687** that relationships like Kepler's would apply in the Solar System to a good approximation, as a consequence of his own laws of motion and law of universal gravitation**.**

▪ History

Johannes Kepler published his first two laws about planetary motion in **1609**, having found them by analyzing the astronomical observations of **Tycho Brahe**. Kepler's third law was published in 1619. Kepler had believed in the **Copernican model** of the solar system, which called for circular orbits, but he could not reconcile **Brahe's** highly precise observations with a circular fit to Mars' orbit – Mars coincidentally having the highest eccentricity of all planets except Mercury. His first law reflected this discovery.

Kepler in **1621** and **Godefroy Wendelin** in **1643** noted that Kepler's third law applies to the four brightest moons of Jupiter. The second law, in the **"Area law"** form, was contested by **Nicolaus Mercator** in a book from **1664**, but by **1670** his **Philosophical Transactions** were in its favour. As the century proceeded it became more widely accepted. The reception in Germany changed noticeably between **1688**, the year in which **Newton's *Principia*** was published and was taken to be basically **Copernican**, and **1690**, by which time work of **Gottfried Leibniz** on Kepler had been published.

Newton was credited with understanding that the second law is not special to the inverse square law of gravitation, being a consequence just of the radial nature of that law; while the other laws do depend on the inverse square form of the attraction. **Carl Runge** and **Wilhelm Lenz** much later identified a symmetry principle in the phase space of planetary motion which accounts for the first and third laws in the case of Newtonian gravitation, as conservation of angular momentum does via rotational symmetry for the second law.

Comparison to Copernicus

Kepler's laws improved the model of Copernicus. If the eccentricities of the planetary orbits are taken as zero, then Kepler basically agreed with Copernicus:

1. The planetary orbit is a circle.
2. The Sun is at the center of the orbit.
3. The speed of the planet in the orbit is constant.

The eccentricities of the orbits of those planets known to Copernicus and Kepler are small, so the foregoing rules give fair approximations of planetary motion, but Kepler's laws fit the observations better than does the model proposed by Copernicus.

Kepler's corrections are not at all obvious:

1. The planetary orbit is not a circle, but an ellipse.
2. The Sun is not at the center but at a focal point of the elliptical orbit.
3. Neither the linear speed nor the angular speed of the planet in the orbit is constant, but the areal speed (closely linked historically with the concept of angular momentum) is constant.

Formulary

The mathematical model of the kinematics of a planet subject to the laws allows a large range of further calculations.

First law of Kepler

The orbit of every planet is an ellipse with the Sun at one of the two foci.

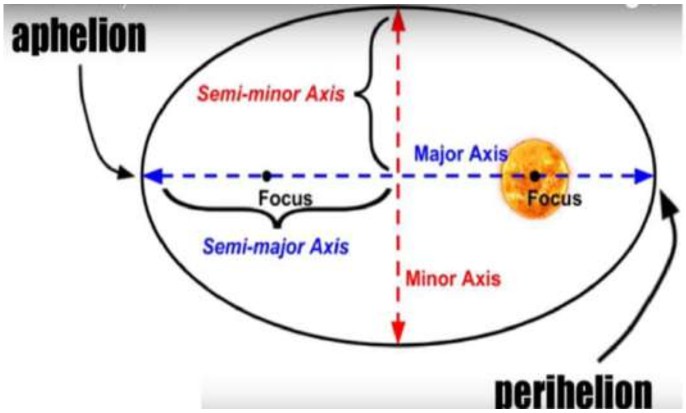


Figure: Kepler's first law placing the Sun at the focus of an elliptical orbit.

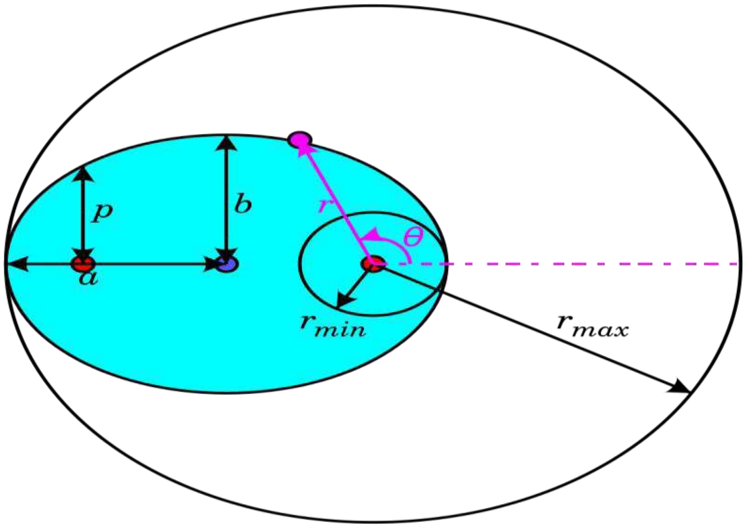


Figure: Heliocentric coordinate system (r, θ) for ellipse.semi-major axis *a*, semi-minor axis *b* and semi-latus rectum p.

Mathematically, an ellipse can be represented by the formula:

$$r = \frac{p}{1+\varepsilon\cos\theta}$$

where p is the semi-latus rectum, ε is the eccentricity of the ellipse, r is the distance from the Sun to the planet, and θ is the angle to the planet's current position from its closest approach, as seen from the Sun. So (r, θ) are polar coordinates.

For an ellipse $0 < \varepsilon < 1$; in the limiting case $\varepsilon = 0$, the orbit is a circle with the Sun at the centre (i.e. where there is zero eccentricity).

At θ = 0°, perihelion, the distance is minimum

$$r_{min} = \frac{p}{1+\varepsilon}$$

At θ = 90° and at θ = 270° the distance is equal to p.

At θ = 180°, aphelion, the distance is maximum (by definition, aphelion is – invariably – perihelion plus 180°)

$$r_{max} = \frac{p}{1-\varepsilon}$$

The semi-major axis *a* is the arithmetic mean between r_{min} and r_{max} :

$$\mathrm{r_{max} - a = a - r_{min}}$$

$$a = \frac{p}{1-\varepsilon^2}$$

The semi-minor axis b is the geometric mean between r_{min} and r_{max} :

$$\frac{\mathrm{r_{max}}}{b} = \frac{b}{\mathrm{r_{min}}}$$

$$b = \frac{p}{\sqrt{1-\varepsilon^2}}$$

The semi-latus rectum p is the harmonic mean between r_{min} and r_{max} :

$$\frac{1}{\mathrm{r_{min}}} - \frac{1}{p} = \frac{1}{p} - \frac{1}{\mathrm{r_{max}}}$$

$$pa = \mathrm{r_{max}} \mathrm{r_{min}} = \mathrm{b}^2$$

The eccentricity ε is the coefficient of variation between $\mathrm{r_{min}}$ and $\mathrm{r_{max}}$:

$$\varepsilon = \frac{\mathrm{r_{max}} - \mathrm{r_{min}}}{\mathrm{r_{max}} + \mathrm{r_{min}}}$$

The area of the ellipse is

$$A = \pi ab$$

The special case of a circle is $\varepsilon = 0$, resulting in $r = p = \mathrm{r_{min}} = \mathrm{r_{max}} = \mathrm{a} = \mathrm{b}$ and $A = \pi r^2$

➢ Second law of kepler

A line joining a planet and the Sun sweeps out equal areas during equal intervals of time.

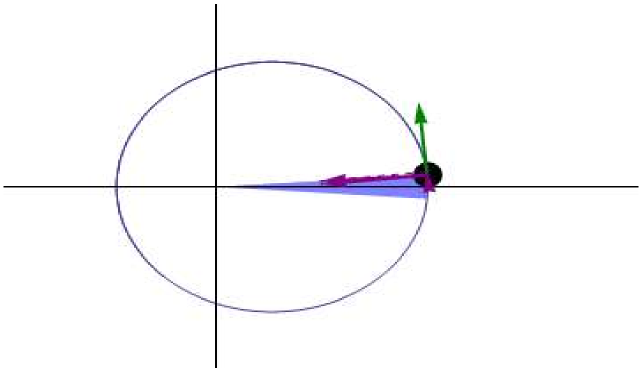


Figure: The same (blue) area is swept out in a fixed time period. The green arrow is velocity. The purple arrow directed towards the Sun is the acceleration. The other two purple arrows are acceleration components parallel and perpendicular to the velocity.

The orbital radius and angular velocity of the planet in the elliptical orbit will vary. This is shown in the animation: the planet travels faster when closer to the Sun, then slower when farther from the Sun. Kepler's second law states that the blue sector has constant area.

In a small time dt the planet sweeps out a small triangle having base line r and height $r\ d\theta$ and area $dA = \frac{1}{2} \cdot r \cdot r\ d\theta$ and so the constant areal velocity is $\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt}$

The area enclosed by the elliptical orbit is πab. So the period P satisfies

$$P \cdot \frac{1}{2} r^2 \frac{d\theta}{dt} = \pi ab$$

and the mean motion of the planet around the Sun $n = \frac{2\pi}{P}$ satisfies $r^2 d\theta = abn\ dt$

And so,

$$\frac{dA}{dt} = \frac{abn}{2} = \frac{\pi ab}{P}$$

➢ Third law of Kepler

The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit.

Using Newton's Law of gravitation published in 1687, this relation can be found in the case of a circular orbit by setting the centripetal force equal to the gravitational force:

$$mr\omega^2 = G\frac{mM}{r^2}$$

Then, expressing the angular velocity in terms of the orbital period and then rearranging, we find Kepler's Third Law:

$$mr\left(\frac{2\pi}{T}\right)^2 = G\frac{mM}{r^2}$$

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

$$T^2 \propto r^3$$

A more detailed derivation can be done with general elliptical orbits, instead of circles, as well as orbiting the center of mass, instead of just the large mass. This results in replacing a circular radius r, with the elliptical semi-major axis a, as well as replacing the large M mass with $M + m$. However, with planet masses being so much smaller than the Sun, this correction is often ignored. The full corresponding formula is:

$$\frac{a^3}{T^2} = \frac{G(M+m)}{4\pi^2} \approx \frac{GM}{4\pi^2} \approx 7.496 \cdot 10^{-6} \left(\frac{AU^3}{days^2}\right) \text{ is constant}$$

Where M is the mass of the Sun, m is the mass of the planet, and G is the gravitational constant, T is the orbital period and a is the elliptical semi-major axis.

- ## Planetary acceleration

According to Kepler's first and second law the acceleration of a planet is i.e.

1. The *direction* of the acceleration is towards the Sun.
2. The *magnitude* of the acceleration is inversely proportional to the square of the planet's distance from the Sun (the *inverse square law*).

This implies that the Sun may be the physical cause of the acceleration of planets.

Newton defined the force acting on a planet to be the product of its mass and the acceleration (Newton's laws of motion). So:

1. Every planet is attracted towards the Sun.
2. The force acting on a planet is directly proportional to the mass of the planet and is inversely proportional to the square of its distance from the Sun.

The Sun plays an unsymmetrical part, which is unjustified. So he assumed, in Newton's law of universal gravitation:

1. All bodies in the Solar System attract one another.
2. The force between two bodies is in direct proportion to the product of their masses and in inverse proportion to the square of the distance between them.

As the planets have small masses compared to that of the Sun, the orbits conform approximately to Kepler's laws. Newton's model improves upon Kepler's model, and fits actual observations more accurately.

Acceleration vector

From the heliocentric point of view consider the vector to the planet $\boldsymbol{r} = r\hat{\boldsymbol{r}}$ where r is the distance to the planet and $\hat{\boldsymbol{r}}$ is a unit vector pointing towards the planet.

$$\frac{d\hat{\boldsymbol{r}}}{dt} = \dot{\hat{\boldsymbol{r}}} = \dot{\theta}\hat{\boldsymbol{\theta}}, \qquad \frac{d\hat{\boldsymbol{\theta}}}{dt} = \dot{\hat{\boldsymbol{\theta}}} = -\dot{\theta}\hat{\boldsymbol{r}}$$

Where, $\hat{\boldsymbol{\theta}}$ is the unit vector whose direction is 90 degrees counterclockwise of $\hat{\boldsymbol{r}}$, and $\boldsymbol{\theta}$ is the polar angle.

Now, velocity vector $\dot{\boldsymbol{r}} = \dot{r}\hat{\boldsymbol{r}} + r\dot{\theta}\hat{\boldsymbol{\theta}}$ and

the acceleration vector $\ddot{\boldsymbol{r}} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{\boldsymbol{r}} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{\boldsymbol{\theta}}$

So, $\ddot{\boldsymbol{r}} = a_r\hat{\boldsymbol{r}} + a_\theta\hat{\boldsymbol{\theta}}$

where the **radial acceleration** $a_r = \left(\ddot{r} - r\dot{\theta}^2\right)$ and the **transversal acceleration** $a_\theta = \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)$

Inverse square law

Kepler second law says that, $r^2\dot{\theta} = nab$ is constant.

The transversal acceleration is zero:

$$\frac{d\left(r^2\dot{\theta}\right)}{dt} = r\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right) = ra_\theta = 0$$

So the acceleration of a planet obeying Kepler's second law is directed towards the Sun.

The radial acceleration is $a_r = \ddot{r} - \frac{n^2a^2b^2}{r^3}$

Kepler's first law states that the orbit is described by the equation:

$$\frac{p}{r} = 1 + \varepsilon\cos\theta$$

Using the following equations we get:

$$pa_r = -\frac{n^2a^2b^2}{r^2}$$

The relation $b^2 = pa$ gives the simple final result $a_r = -\frac{n^2 a^3}{r^2}$

This means that the acceleration vector $\ddot{\boldsymbol{r}}$ of any planet obeying Kepler's first and second law satisfies the **inverse square law** $\ddot{\boldsymbol{r}} = -\frac{\boldsymbol{\alpha}}{r^2}\hat{\boldsymbol{r}}$ where $\alpha = n^2 a^3$ is constant.

α has the same value for all the planets. So the inverse square law for planetary accelerations applies throughout the entire Solar System.

The inverse square law is a differential equation whose solutions include the Keplerian motions, but they also include motions where the orbit is a hyperbola or parabola or a straight line.

Newton's law of gravitation

By Newton's second law, the acceleration of solar system body number i due to the gravitational force is:

$$\ddot{\boldsymbol{r}}_{\boldsymbol{i}} = G \sum_{i \neq j} m_j \, r_{ij}{}^{-2} \hat{\boldsymbol{r}}_{ij}$$

Where G is the gravitational constant, m_j is the mass of body j, r_{ij} is the distance between body *i* and body *j*, $\hat{\boldsymbol{r}}_{ij}$is the unit vector from body *i* towards body j.

In the three-body case the accelerations are:

$$\ddot{\boldsymbol{r}}_{Sun} = Gm_{Earth} r_{Sun,Earth}{}^{-2} \hat{\boldsymbol{r}}_{Sun,Earth} + Gm_{Moon} r_{Sun,Moon}{}^{-2} \hat{\boldsymbol{r}}_{Sun,Moon}$$

$$\ddot{\boldsymbol{r}}_{Earth} = Gm_{Sun} r_{Earth,Sun}{}^{-2} \hat{\boldsymbol{r}}_{Earth,Sun} + Gm_{Moon} r_{Earth,Moon}{}^{-2} \hat{\boldsymbol{r}}_{Earth,Moon}$$

$$\ddot{\boldsymbol{r}}_{Moon} = Gm_{Sun} r_{Moon,Sun}{}^{-2} \hat{\boldsymbol{r}}_{Moon,Sun} + Gm_{Earth} r_{Moon,Earth}{}^{-2} \hat{\boldsymbol{r}}_{Moon,Earth}$$

Conclusion

In conclusion, kepler’s laws are still vaild today and have an important place in the history of science, astronomy and cosmology. They are the key step in the revolution which moved from Earth-centred model to the heliocentric model, and they led the discovery of Newton’s laws. As the modern scientists are still discover more about the universe and working in the light of discoveries made by earlier fellow scientists, thus, it justified Newton’s statement that, “if I have been able to see any further, it was only because I stood on the shoulders of giants”.

References

"Johannes Kepler: His Life, His Laws and Times." Kepler Mission. 4 Aug. 2009.
Web.http://kepler.nasa.gov/johannes/.

"Newton's Three Laws of Motion." Astronomy 161 The Solar System.
Department of Physics & Astronomy University of Tennessee, 10 Aug. 2000.
Web.<http://csep10.phys.utk.edu/astr161/lect/history/newton3laws.html>.

Stern, David P. "Kepler's Three Laws of Planetary Motion".
NASA Polar, Wind, and Geotail Projects.7 Feb. 2008.
Web.http://www-istp.gsfc.nasa.gov/stargaze/Kep3laws.html.

Weir, Maurice D. "Planetary Motion and Satellites."
Thomas' calculus. 11th ed. Boston: Pearson Addison Wesley, 2005. 950-56. Print.

Wikipedia, https://en.wikipedia.org/wiki/Kepler%27s_laws_of_planetary_motion

Acknowledgement

In the accomplishment of this project successfully, many people have given me their support and guidance. Now I am utilizing this time to thank all the people who have been concerned with this project .

Primarily I would thank my project supervisor **Prof. Bhaskar Haldar** for his guidance and support which help me to complete this project in time , Besides him I would like to thank my friends for their help which also help me to complete this project work.

TITLE OF THE PROJECT :

THERMOELECTRIC GENERATOR

The project submitted, in partial fulfilment of the requirement for the assignments in ***CC-XI, CC-XII, DSE-I, DSE-II*** *Paper (Semester* ***V*** *) in the Department of Physics.*

Submitted by

RUDRANARAYAN GIRI

Registration No : A01-1112-111-047-2019

Supervisor Teacher : Prof. KALYAN BRATA CHATTERJEE

RAMAKRISHNA MISSION VIVEKANANDA CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

Introduction:

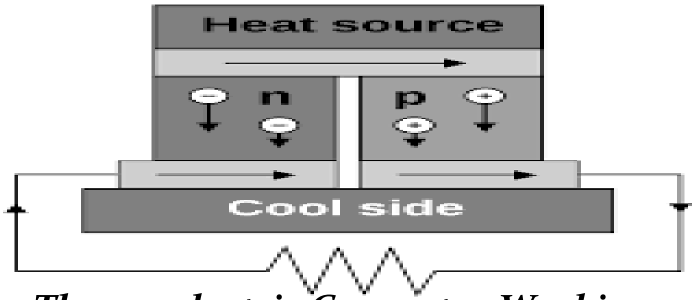
In our daily life we come across a number of devices based on effect of electric current. The current may be produced from heating, magnetic, chemical and lighting etc. Heat is produced when a current is passing through a conductor. This phenomenon is called joule effect. It is also possible to convert heat energy into electrical energy. Current can be generated by maintaining a temperature difference either between different parts of same conductor or different conductors join at their ends. The branch of electricity that deals with the conversion of heat energy into electrical energy is called thermoelectricity. Through this phenomenon called the seebeck effect (a form of thermoelectric effect) thermoelectric generator, a solid state device converts heat flux directly into electric energy. It is one of the alternate and renewable sources for the utilization of decay warmth present in huge amount in the surrounding.

History:

In 1821,Thomas Johann Seebeck discovered that a thermal gradient formed between two dissimilar conductors can produce electricity. The thermoelectric effect is the fact that a temperature gradient in a conducting material results in heat flow; this results in the diffusion of charge carriers. The flow of charge carriers between the hot and cold regions in turn creates a voltage difference. In 1834, Jean Charles Athanase Peltier discovered the reverse effect , that running an electric current through the junction of two dissimilar conductors could ,depending on the direction of the current, cause it to act as a heater or cooler. It was not until 1855 that William Thomson drew the connection between the seebeck and peltier effects, which was the first significant contribution to the understanding of thermoelectric phenomena. In 1910 Edmund Altenkirch, a German scientist, satisfactorily calculated the potential efficiency of thermoelectric generator.

Construction:

Thermoelectric generators are the devices that are solid-state heat components constructed of two essential junctions which are p-type and n-type. The P-type junction has an increased concentration of *+ve* charge and the n-type junction has an increased concentration of *-ve* charged elements. The p-type components are doped in the condition to have more positive charged carriers or holes thus providing a positive Seebeck coefficient. In a similar way, n-type components are doped to have more negative charged carriers thus providing a negative type of Seebeck coefficient.



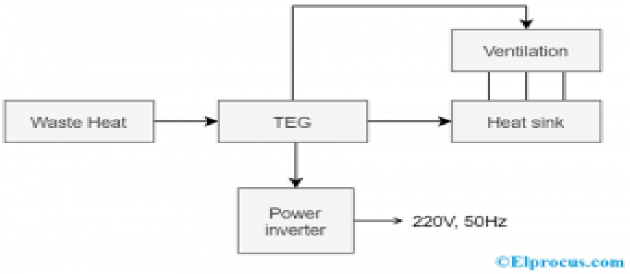
Thermoelectric Generator Working

With the passage of the electrical connection between the two junctions, every positively charged carrier moves to the n-junction, and similarly negatively charged carrier moves to the p-junction. In the thermoelectric generator construction, the most implemented element is lead telluride.

It is the component that is constructed of tellurium and lead which have minimal amounts of either sodium or bismuth. In addition to this, the other elements that are used in this device construction are bismuth sulphide, tin telluride, bismuth telluride, indium arsenide, germanium telluride, and many others. With these materials, thermoelectric generator design can be done.

Thermoelectric Generator Working Principle:

The thermoelectric generator working is dependent on the Seebeck effect. In this effect, a loop that is formed in between the two various metals generates an emf when the metal junctions are maintained at various temperature levels. As because of this scenario, these are also termed as Seebeck power generators. The thermoelectric generator block diagram is shown as:



A thermoelectric generator is generally included with a heat source that is maintained at high values of temperature and a heat sink is also included. Here, the heat sink temperature has to be less than that of the heat source. The change in temperature values for the heat source and heat sink allows the flowing current across the load section.

In this kind of energy transformation, there exist no transitional energy conversions dissimilar to the other types of energy conversion. As because of this, it is termed as direct energy transformation. The generated power because of this Seebeck effect is of single-phase DC type and is represented as I^2R_L where R_L corresponds to resistance value at load. The output voltage and power values can be increased in two ways. One is by increasing the temperature variation that rises in between hot and cold edges and the other is to form a series connection with thermoelectric power generators. The voltage of this TEG device is given by $V = \alpha\Delta\, T$,Where ‘α’ corresponds to the Seebeck coefficient and ‘Δ’ is the temperature variation in between the two junctions. With this, the current flow is given by

$\boldsymbol{I = \frac{V}{R+R_L}}$. From this, the current equation is

$$\boldsymbol{I = {}^{\alpha\Delta T}\!/_{R + R_L}}$$

From this, the power flow across the load section is

$$\text{P at load} = \boldsymbol{R_L} \cdot (\frac{\boldsymbol{\alpha\Delta T}}{\boldsymbol{R+R_L}})^{\mathbf{2}}$$

The power rating is more when R reaches to RL, then

P_{max} = $(\alpha\Delta\text{T})^2/4\text{R}$. There will be current flow till the time when there is heat supply to the hot edge and removal of heat from the cold edge. And the developed current is in DC form and it can be transformed into AC type through inverters. The voltage values can be more augmented through the implementation of transformers.

This kind of energy conversion can also be reversible where the energy flow path can be changed back. When both the DC power and load are removed from the edges, then heat can be simply withdrawn from thermoelectric generators. So, this is the thermoelectric generator theory behind working.

Thermoelectric Generator Efficiency Equation :

The efficiency of this device is represented as the proportion of generated power at the resistor at load section to the heat flow across the load resistor. This ratio is represented as

$$\text{Efficiency} = \frac{(\textbf{Genareted power at } \boldsymbol{R_L}).}{(\textbf{Heat flow 'Q'})} = (\boldsymbol{I^2 R_L})/\mathbf{Q}$$

$$\text{Efficiency} = \{\boldsymbol{R_L}\left(\frac{\boldsymbol{\alpha\Delta T}}{\boldsymbol{R+R_L}}\right)^{\mathbf{2}}\}/\mathbf{Q}$$

This is how the efficiency of the thermoelectric generator can be calculated.

Thermoelectric Generator Types:

Based on the TEG device size, kind of heat source and source for heat sink, power ability and application purpose, TEG' s are mainly classified as three types and those are:

- Fossil fuel generators
- Nuclear fueled generators
- Solar source generators

Advantages and Disadvantages of TEG:

The advantages of the thermoelectric generator are:

- As all the components used in this TEG device are solid-state, they have enhanced reliability
- The extreme range of fuel sources

- TEG devices are constructed to deliver power ranging not minimal to that of mW and greater than KW which means they have huge scalability
- These are direct energy transformation devices
- Silently operated
- Minimal size
- These can function even at extreme and zero range of gravitational forces

The disadvantages of thermoelectric generator are:

- These are a bit expensive when compared to other kinds of generators
- These have minimal efficiency
- Minimal thermal properties
- These devices need more output resistance.

Thermoelectric Generator Applications :

- For enhancing the fuel performance of cars, the TEG device is mostly employed. These generators make use of heat that is generated at the time of vehicle operation
- Seebeck Power Generation is utilized to provide power for the spacecraft.
- Thermoelectric generators are implemented to provide power for the remote stations such as weather systems, relay networks, and others.

Conclusion :

Let us summaries what I have done in this project. Upto here, I discussed what thermoelectricity is, how it works, what a thermoelectric generator is. I have discussed about the working principle of TEG. Besides this, I saw that we can construct a TEG. I told about the advantages and disadvantages of it and also discussed about the applications of TEG.

Acknowledgement :

I would like to thank our principal, Swami Kamalasthananda Maharaj and our physics departmental professor Dr. Kalyan Brata Chatterjee, whose valuable guidance has ones that helped me to complete this. Their instructions and suggestions help me very much. Besides this I would like to thank my parents and friends who helped me with their suggestions.

BIBLIOGRAPHY :

- www.google.com
- www.wikipedia.com
- www.scribd.com
- www.researchgate.net
- www.elprocus.com
- www.allprojectreports.com

TITLE OF THE PROJECT :

SIMULATION OF BRIDGE RECTIFIER CIRCUIT

The project submitted, in partial fulfilment of the requirement for the assignments in ***CC-XI, CC-XII, DSE-I, DSE-II*** *Paper (Semester* ***V*** *) in the Department of Physics.*

Submitted by

SUBHAM MONDAL

Registration No : ***A01-1112-111-048-2019***

Supervisor Teacher : ***Prof. ANJAN KUMAR CHANDRA***

RAMAKRISHNA MISSION VIVEKANANDA
CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

INTRODUCTION :

*A **rectifier** is an electrical device which converts the Alternating current (AC, which periodically reverses its direction) to Direct current (DC, which flows only in one direction). The process is called **Rectification**.*

Now a days we use many electrical machines in our house. Many electrical parts of them are controlled by DC current. So we need to convert the supplied current (which is AC current) to DC current where we actually needed. But now there arises a question that why AC current is supplied instead of DC ? We are supplied AC because ....(i) AC is simpler to produce than DC. (ii) AC is also less expensive than DC to generate. (iii) The waste of power is negligible for AC during transmission. And so on ....

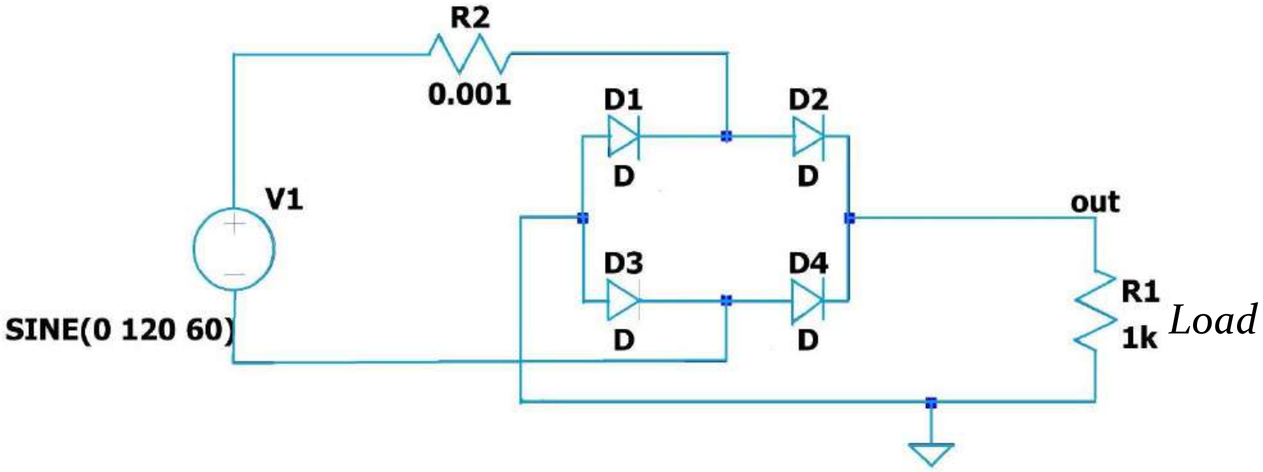
So according to our need we have to convert AC to DC. There are many rectifier devices which do this job. Such as vacuum tube diodes, mercury-arc valves, copper and selenium oxide rectifiers, semiconductor diodes, silicon-controlled rectifiers and other silicon-based semiconductor switches.

Rectifier circuits may be single-phase or multi-phase. Most low power rectifiers for domestic equipment are single phase, but three phase rectification is very important for industrial applications and for the transmission of energy as DC. In our daily life we use many electrical equipment where this bridge rectifier is used, such as rechargeable lithium-ion battery torch, mobile charger and so many things. Even the total TV circuit is controlled by DC current. Besides it has many useful applications which we will discuss accordingly.

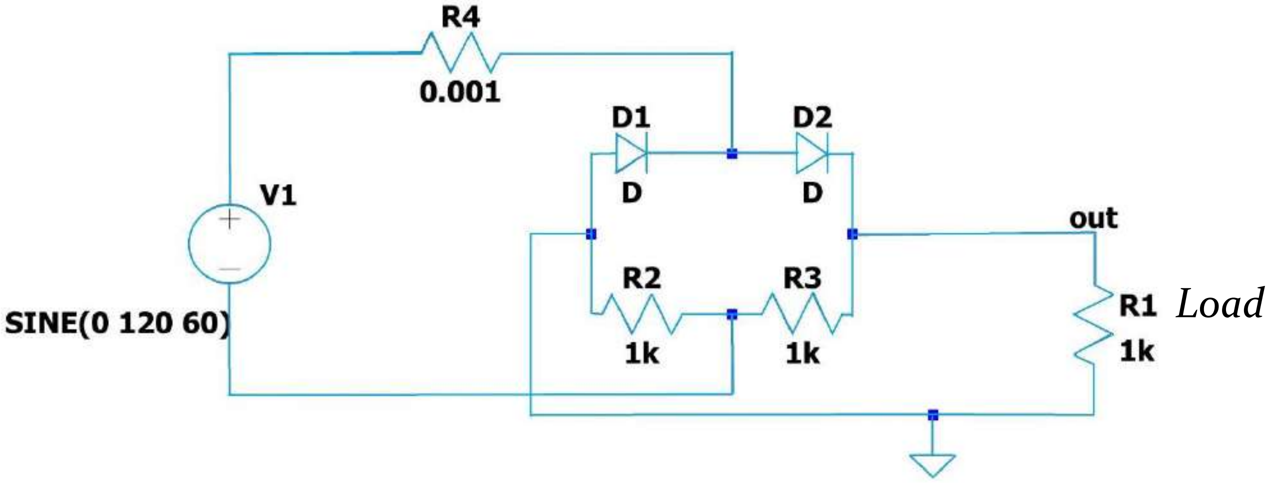
*Here we will discuss only the **Bridge Rectifier** circuit using the semiconductor diode. Bridge rectifier circuit conventionally uses 4 diodes. This is called **4 diode bridge rectifier circuit or simply Bridge rectifier**.*

Besides we can construct bridge rectifier circuit using 2 diodes instead of 4 diodes. This is very special. Now come to the point that why we use the 2 diode bridge rectifier ? Because in the case of an ideal diode, the 4 diode bridge rectifier give the output peak voltage exactly same as input AC peak voltage. But in case of 2 diode rectifier we can control the output peak voltage by controlling the resistance value. We will describe both the bridge rectifier circuits.

CIRCUIT DIAGRAM :



FOUR DIODE BRIDGE RECTIFIER CIRCUIT



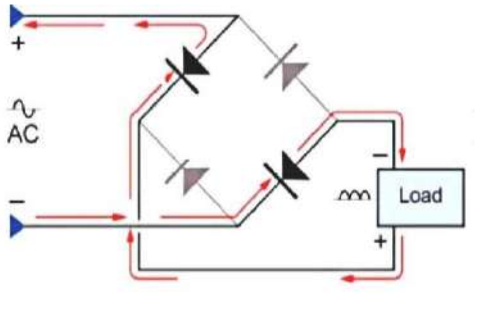
TWO DIODE BRIDGE RECTIFIER CIRCUIT

WORKING PRINCIPLE :

4 diode bridge rectifier :

Firstly, let's discuss the working principle of 4 diode bridge rectifier circuit. In this circuit the four diodes labelled D1, D2, D3, D4 are arranged in "series pairs" with only two diodes conduct current during each "half cycle" of AC input voltage. The working of Bridge rectifier circuit during each half cycle is discussed below as "positive half cycle" and "negative half cycle".

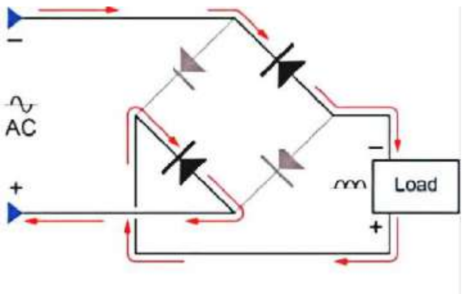
Positive Half Cycle :



Positive half cycle

During the positive half cycle of the AC supply (shown in the figure below), the diode D2 and D3 will go in conducting mode while the other two diodes D1 and D4 will become reverse biased. So, they conduct no current. Then the current flows through the diode D2 and the load resistance and the diode D3. The direction of electron flow is shown in this figure.

Negative Half Cycle :



Negative half cycle

During the negative half cycle of the AC supply (shown in the figure below), the diode D1 and D4 will go in conducting mode while the other two diodes D2 and D3 will become reverse biased. So, they conduct no current. Then the current flows through the diode D4 and the load resistance and the diode D1. The direction of electron flow is shown in this figure.

In the subsequent half cycles of the AC current the above process are repeated. In both the half cycles it is clear, that current flows.

through the load resistor in only one direction. Although, the current in the overall circuit changes its direction periodically, yet the current inside the load resistor is unidirectional.

===

2 diode bridge rectifier :

Now, let's discuss the working principle of 2 diode bridge rectifier circuit in brief. The circuit diagram of 2 diode bridge rectifier is same as the 4 diode bridge rectifier except the two diodes D3 and D4 are replaced by two resistors R2 and R3 respectively in the 2 diode bridge rectifier.

In this rectifier circuit also there are positive half cycle and negative half cycle. In the positive half cycle the diode D2 is in conducting mode and the diode D1 is in non-conducting mode (i.e, reverse biased). So the current inside the circuit flows through the diode D2 and the load resistor and the resistors R2 and R3.

In the negative half cycle the diode D1 is in conducting mode and the diode D2 is in non-conducting mode (i.e, reverse biased). So the current inside the circuit flows through the resistors R2 and R3 the load resistor and the diode D1.

Here also the current flows through the load resistor in only one direction. The specialty of this circuit is that, we can control peak value of the output voltage by changing the value of the resistors R2 and R3. That is, if the input voltage has the peak value 120 volt (as an example) then the peak value of the output voltage can be anything. By suitable choice of R2 and R3 we can get the output voltage as we needed.

METHODOLOGY :

*To check the rectification phenomena we have used simulation technique and the software that has been used is "**LTspice**". It is a high performance SPICE simulation software and waveform viewer for facilitating the simulation of analog circuits. We have studied the input and output waveforms of 4 diode bridge rectifier circuit and 2 diode bridge rectifier circuit. In 4 diode bridge rectifier circuit we have used a sinusoidal input voltage with peak value 120 volt. We have measured the output voltage across the load which is basically a 1kΩ resistor. In the filter circuit we have used a 1000 μF capacitor.*

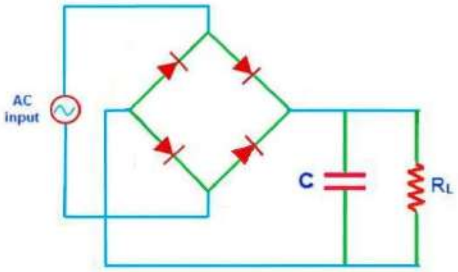
In 2 diode bridge rectifier circuit we have used two resistor R2 and R3. In this case also we have a sinusoidal input waveform of peak voltage 120 volt. Here we have studied the output waveforms of a 2 diode bridge rectifier by taking two different set of R2 and R3 (R2 = R3 = 1kΩ and R2 = R3 = 2kΩ). For this two different set of values of R2 and R3 we have studied the change in the peak voltage of output waveform. So using 2 diode bridge rectifier we can produce the output voltage as we required. This is most interesting fact about 2 diode bridge rectifier circuit. This tells us the usefulness of 2 diode bridge rectifier circuit.

RESULTS :

In the Bridge Rectifier circuit the input AC waveform is basically a sinusoidal waveform which periodically changes it direction.

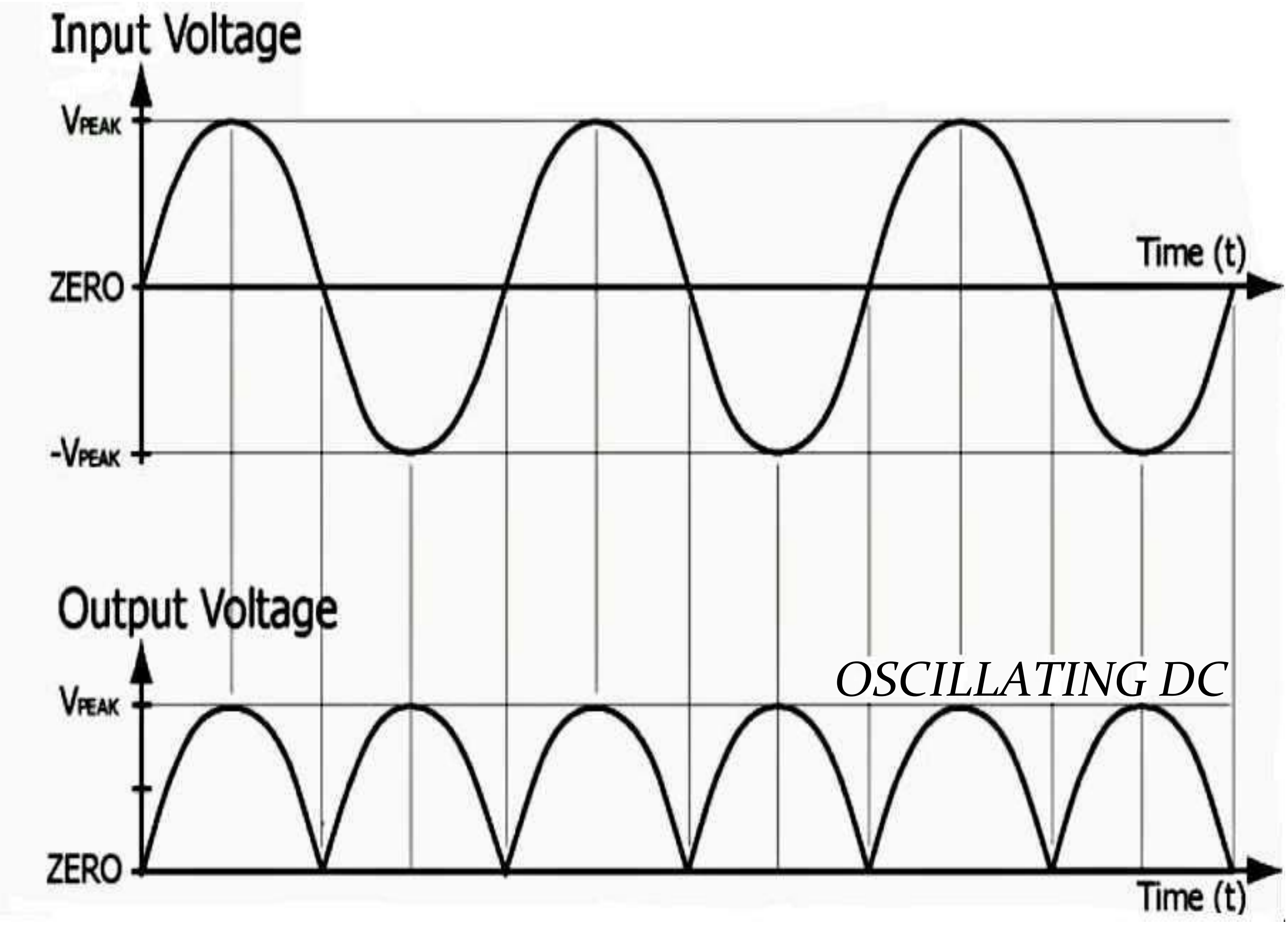
Now if we measure the output waveform across the load using voltmeter, we will find that the output waveform will also oscillate periodically with the time period half of the input waveform. But now the output waveform does not change its direction like AC input. So, this output waveform is called ***Oscillating DC****.*

But mostly we need smoother DC not oscillating DC. So, to get smoother DC we have to connect a capacitor parallel to the load resistor. This is called ***Filter Circuit****. The filter circuit is shown in the right.*

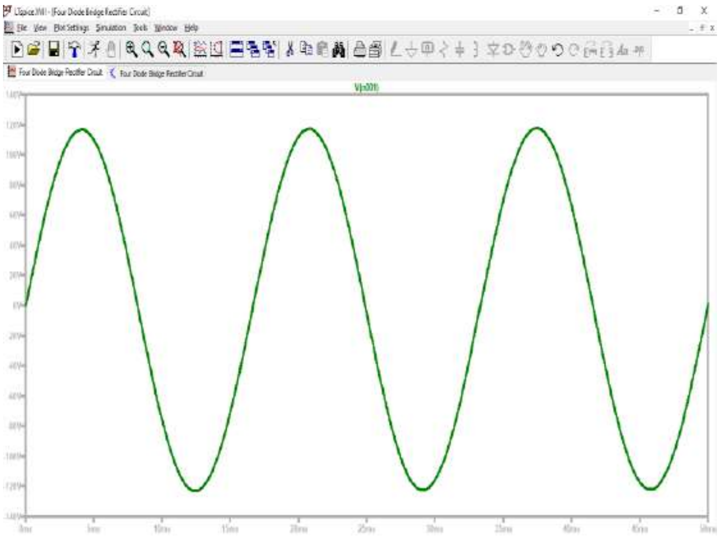


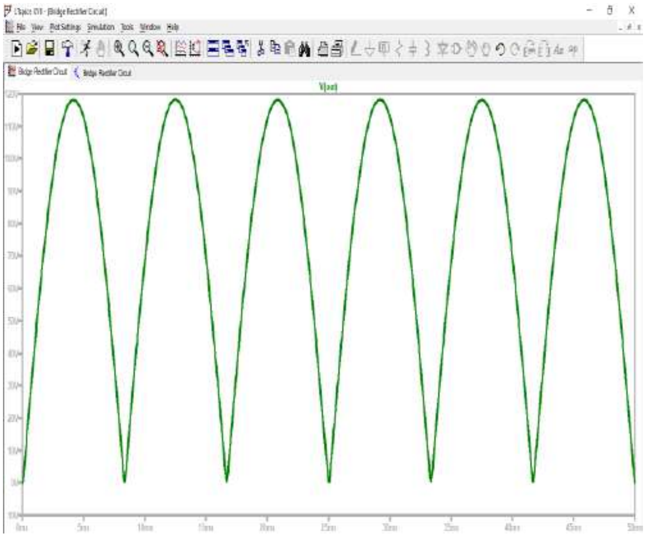
The input and the output waveforms are shown below……….

INPUT AND OUTPUT WAVEFORM :



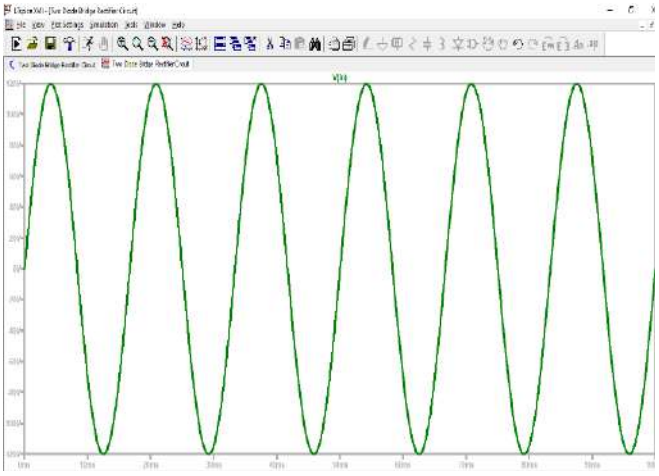
Four diode bridge rectifier simulation :



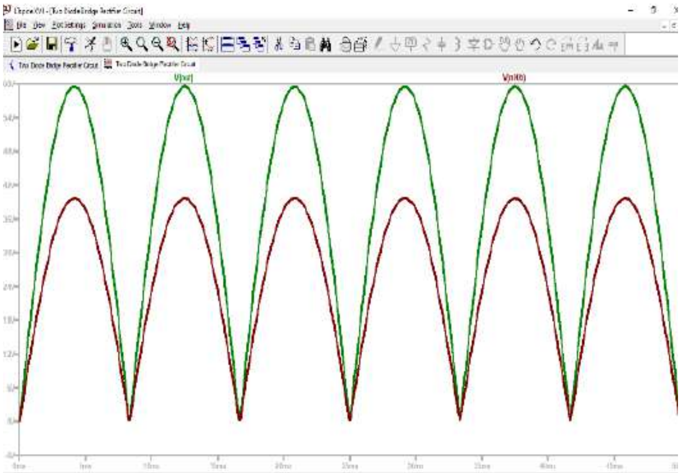


Input *Output*

Two diode bridge rectifier simulation :

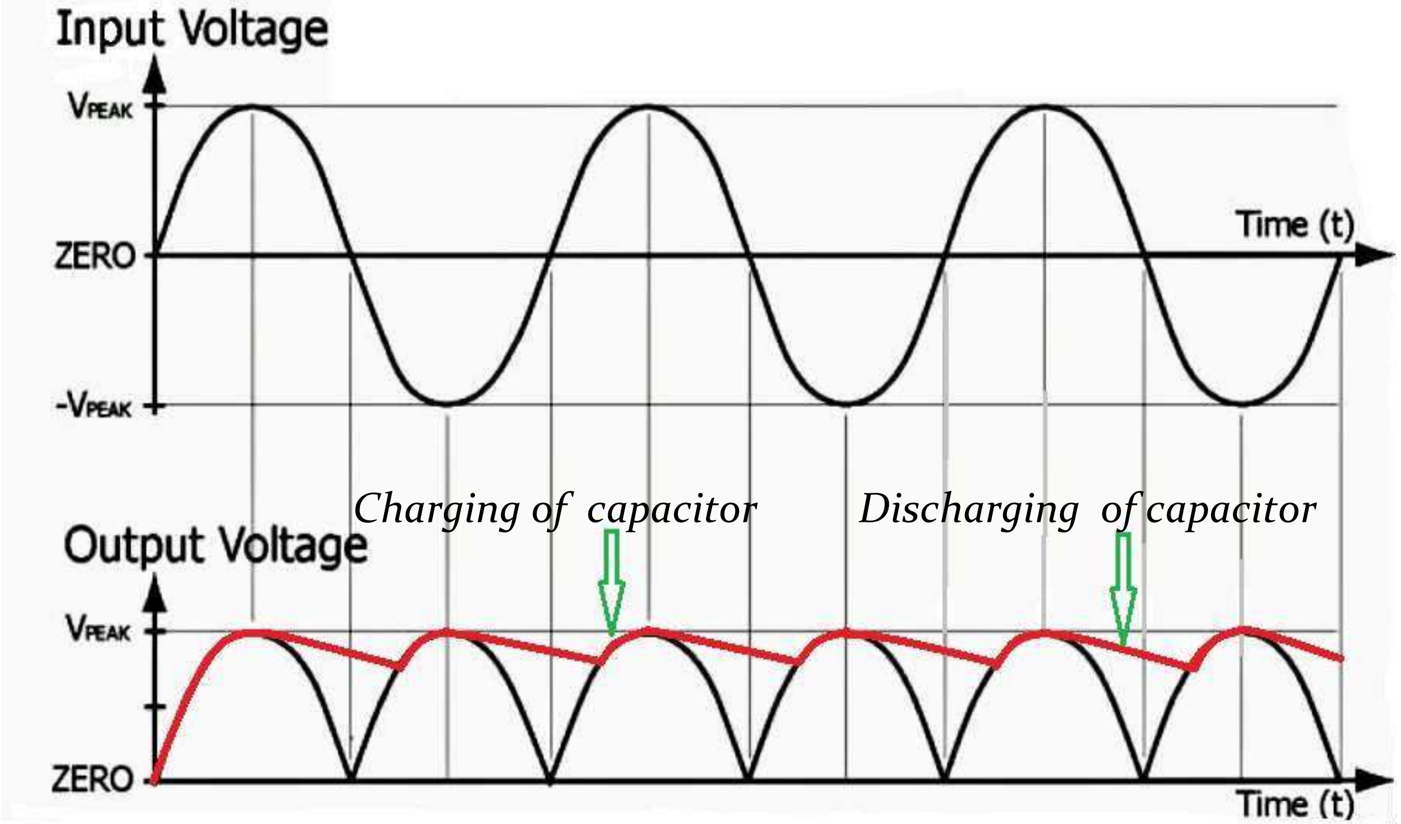


Input



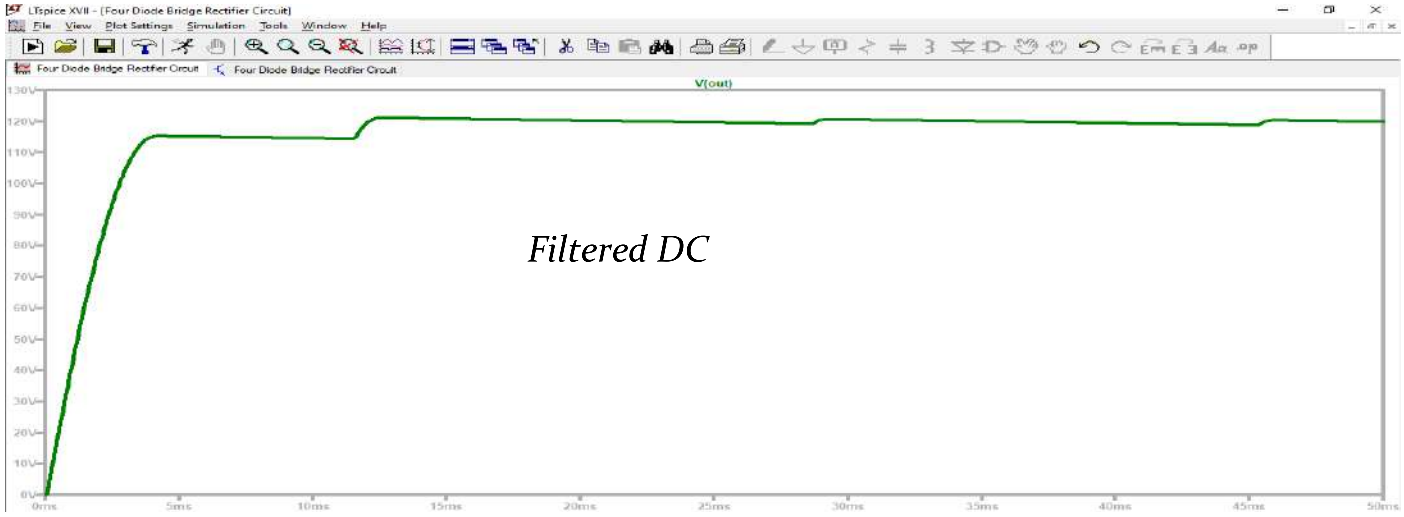
Output(using 2 different values of R2 & R3)

OUTPUT WAVEFORM OF FILTER CIRCUIT :



Charging of capacitor

Discharging of capacitor



Filtered DC

APPLICATIONS :

Regulated DC power supply is often required for many electronic applications. One of the most reliable and convenient ways is to convert the available AC mains power supply into DC supply. This conversion of AC signal to DC signal is done using a rectifier. Here we will discuss some applications of bridge rectifier circuit………….

- *Because of the low cost (when compared with centre-tapped), these are widely used in power supply circuits.*
- *In the modulating radio signals to detect its amplitude concept of the bridge rectifier is essential.*
- *For electric wielding it requires a steady supply of DC in a polarized manner, this is possible by the full-wave rectifying circuit.*
- *Because of the efficient nature of the bridge rectifier, it is preferred as the part of the various appliances power supply unit.*
- *The high AC voltage can be converted into the low DC value with the capable nature of the bridge rectifier.*
- *For the powering up of the devices it may be LED or DC motor this type of rectifiers are preferably used.*
- *Bridge rectifier is used in any charging device such as any cell phone charger, laptop, tablet and power bank charger and in any Bluetooth device charger.*
- *It also used in TV circuit and in laptop circuit and in computers etc.*

Besides, due to low cost and very simple circuitry and high efficiency Bridge rectifier is widely used than any other rectifier circuit.

CONCLUSION :

Let us summarize what we have done in this project. We have discussed what a rectifier is, how it works and what is a Bridge Rectifier. We have also shown how a bridge rectifier circuit look like. We have explained about the working principle of a bridge rectifier. We have seen that generally a bridge rectifier is made of 4 diodes. Besides we have shown that we can construct a bridge rectifier circuit using 2 diodes only. We have discussed about the importance of the filter circuit inside a bridge rectifier circuit. We have also discussed why we need the bridge rectifier, what are the applications of bridge rectifier.

*Now let's end our discussion keeping in mind some precautions when we work some sort of electric circuit. (i) We have to keep ourselves safe from high voltage. (ii) We have to solder the wires very safely. (iii) While soldering we should not touch the soldering tip. We have to keep in mind that "**safety first**".*

ACKNOWLEDGEMENT :

In the accomplishment of this project successfully, many people have given me their heart pledged support. I would like to thank all the people who have helped me in this project.

I would like to thank our Principal, Swami KAMALASTHANANDA MAHARAJ. I would like to thank our Physics teacher Dr. ANJAN KUMAR CHANDRA, whose valuable guidance has helped me to complete this project successfully. His instructions and suggestions helped me very much.

Besides I would like to thank my parents and friends who have helped me with their valuable suggestions and guidance. These all helped me in completion of this project.

BIBLIOGRAPHY

- *www.google.com*
- *www.scribd.com*
- *www.slideshare.net*
- *www.wikipedia.com*

Title of the Project

The project submitted, in partial fulfilment of the requirement for the assignments in ………. Paper (Semester…….) in the Department of Physics

Submitted by: Atanu Maity

(**Name of the student**)

Registration No: A01-1112-111-049-2019

Supervisor Teacher: Prof. Palash Nath

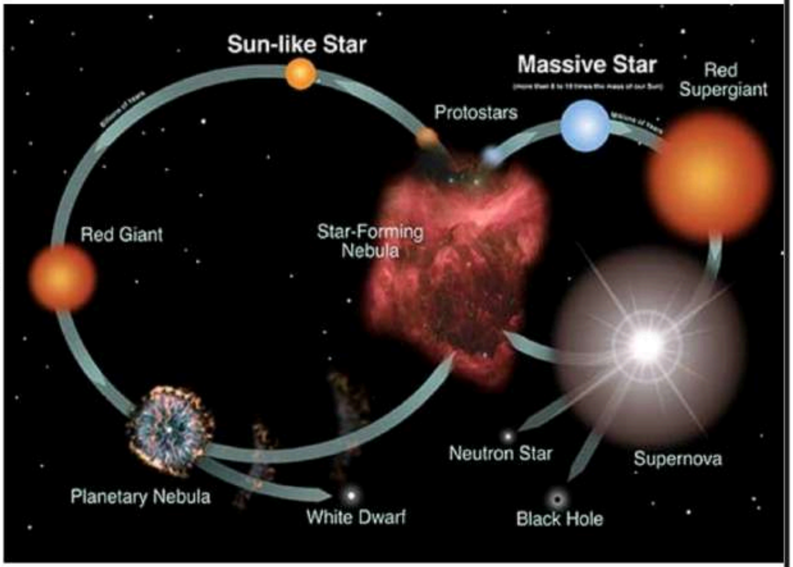
RAMAKRISHNA MISSION VIVEKANANDA CENTENARY COLLEGE

P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

Introduction to the Birth of star:

There are countless suns and countless earths all rotating round their suns in exactly the same way as the planets of our system. We see only the suns because they are the largest bodies and are luminous, but their planets remain invisible to us because they are smaller and non-luminous. The unnumbered worlds in the universe are all similar in form and rank and subject to the same forces and the same laws.

A star's nuclear evolution begins when hydrogen is fused into helium, but that can only occur when the core temperature exceeds 10 to 12 million K. Since stars form from cold interstellar material, we must understand how they collapse and eventually reach this "ignition temperature" to explain the birth of stars. Star formation is a continuous process, from the birth of our Galaxy right up to today. We estimate that every year in our Galaxy, on average, three solar masses of interstellar matter are converted into stars. This may sound like a small amount of mass for an object as large as a galaxy, but only three new stars (out of billions in the *G*alaxy) are formed each year.



Characterization of stars:

Brightness:

Two characteristics define brightness: luminosity and magnitude. Luminosity is the amount of light that a star radiates. The size of the star and its surface temperature determine its luminosity. Apparent magnitude of a star is its perceived brightness, factoring in size and distance, while absolute magnitude is its true brightness irrespective of its distance from earth.

Color:

A star's color depends on its surface temperature. Cooler stars tend to be redder in color, while hotter stars have a bluer appearance. Stars in the mid ranges are white or yellow, such as our sun. Stars can also blend colors, such as red-orange stars or blue-white stars.

Surface Temperature:

Astronomers measure a star's temperature on the Kelvin scale. Zero degrees on the Kelvin scale is theoretically absolute and is equal to -273.15 degrees Celsius. The coolest, reddest stars are approximately 2,500 K, while the hottest stars can reach 50,000 K. Our sun is about 5,500 K.

Size:

Astronomers measure the size of a given star in terms of our own sun's radius. Thus, a star that measure 1 solar radii would be the same size as our sun. The star Rigel, which is much larger than our sun, measures 78 solar radii. A star's size, along with its surface temperature, will determine its luminosity.

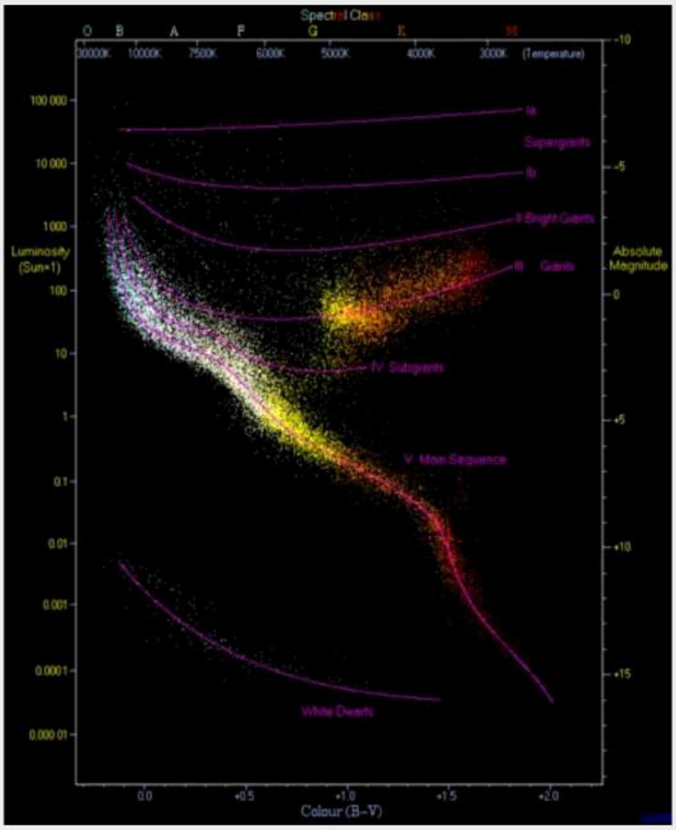
Mass:

A star's mass is also measured in terms of our own sun, with 1 equal to the size of our sun. For instance, Rigel, which is much larger than our sun, has a mass of 3.5 solar masses. Two stars of a similar size may not necessarily have the same mass, as stars can vary greatly in density.

The Hertzsprung–Russell diagram:

The Hertzsprung–Russell diagram, abbreviated as H–R diagram, HR diagram or HRD, is a scatter plot of stars showing the relationship between the stars' absolute magnitudes or luminosities versus their classifications or effective temperatures. The diagram was created independently around 1910 by Ejnar Hertzsprung and Henry Norris Russell, and represented a major step towards an understanding of stellar evolution.

An observational Hertzsprung–Russell diagram with 22,000 stars plotted from the Hipparcos Catalogue and 1,000 from the Gliese Catalogue of nearby stars. Stars tend to fall only into certain regions of the diagram. The most prominent is the diagonal, going from the upper-left (hot and bright) to the lower-right (cooler and less bright), called the main sequence. In the lower-left is where white dwarfs are found, and above the main sequence are the subgiants, giants and supergiants. The Sun is found on the main sequence at luminosity 1 (absolute magnitude 4.8) and B−V color index 0.66 (temperature 5780 K, spectral type G2V).



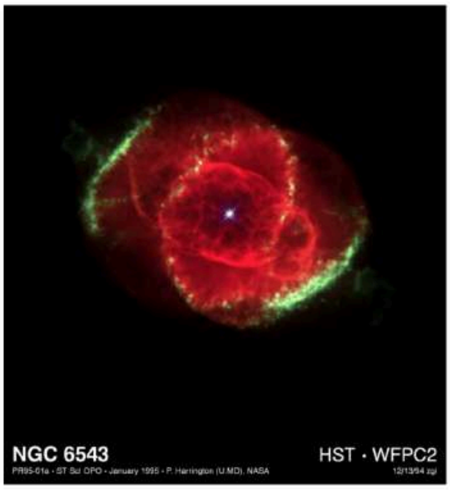
Birth Of Star: Most stars are born inside great clouds of gas and dust called nebulas. The process begins when a nebula starts to shrink, then divides into smaller, swirling clumps. Each clump becomes ball-shaped, and as it continues to shrink the material in it gets hotter and hotter.

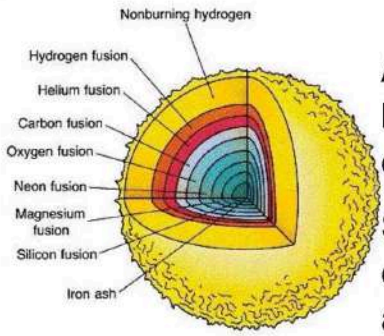
Source Of energy: The Sun is Earth's major source of energy, yet the planet only receives a small portion of its energy and the Sun is just an ordinary star. Many stars produce much more energy than the Sun. The energy source for all stars is nuclear fusion.Stars are made mostly of hydrogen and helium, which are packed so densely in a star that in the star's center the pressure is great enough to initiate nuclear fusion reactions. In a nuclear fusion reaction, the nuclei of two atoms combine to create a new atom. Most commonly, in the core of a star, two hydrogen atoms fuse to become a helium atom. Although nuclear fusion reactions require a lot of energy to get started, once they are going they produce enormous amounts of energy.In a star, the energy from fusion reactions in the core pushes outward to balance the inward pull of gravity. This energy moves outward through the layers of the star until it finally reaches the star's outer surface. The outer layer of the star glows brightly, sending the energy out into space as electromagnetic radiation, including visible light, heat, ultraviolet light, and radio waves.

In particle accelerators, subatomic particles are propelled until they have attained almost the same amount of energy as found in the core of a star. When these particles collide head-on, new particles are created. This process simulates the nuclear fusion that takes place in the cores of stars. The process also simulates the conditions that allowed for the first helium atom to be produced from the collision of two hydrogen atoms in the first few minutes of the universe.

Death of a Star: For stars less than about 25 solar masses the end of their lives is to evolve to white dwarfs after substantial mass loss. Due to atomic structure limits, all white dwarfs must mass less than the Chandrasekhar limit. If their initial mass is more than the Chandrasekhar limit, then they must lose their envelopes during their planetary nebula phase till they are below this mass limit. An example of this is the Cat's Eye Nebula shown below:

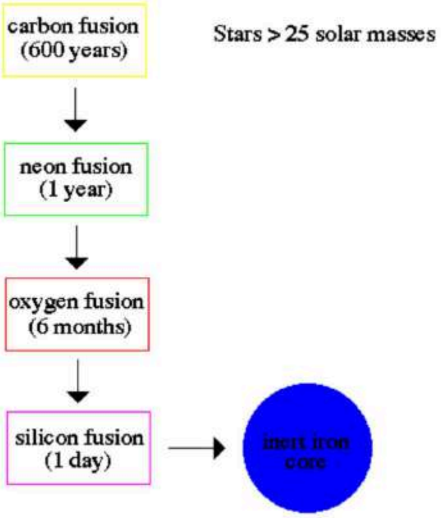
At what stage a star leaves the AGB (Asymptotic Giant Branch) and becomes a white dwarf depends on how fast it runs out of fuel in its core. Higher mass stars will switch from helium to carbon burning and extend their lifetimes. Even higher mass stars will burn neon after carbon is used up. However, once iron is reached, fusion is halted since iron is so tightly bound that no energy can be extracted by fusion. Iron can fuse, but it absorbs energy in the process and the core temperature drops.





After evolving to white dwarfs, stars with original masses less than 25 solar masses slowly cool to become black dwarfs and suffer heat death.

Stars greater than 25 solar masses undergo a more violent end to their lives. Carbon core burning lasts for 600 years for a star of this size. Neon burning for 1 year, oxygen burning about 6 months (i.e. very fast on astronomical timescales). At 3 billion degrees, the core can fuse silicon nuclei into iron and the entire core supply is used up in one day.



An inert iron core builds up at this time where successive layers above the core consume the remaining fuel of lighter nuclei in the core. The core is about the size of the Earth, compressed to extreme densities and near the Chandrasekhar limit. The outer regions of the star have expanded to fill a volume as large as Jupiter's orbit from the Sun. Since iron does not act as a fuel, the burning stops.

The sudden stoppage of energy generation causes the core to collapse and the outer layers of the star to fall onto the core. The infalling layers collapse so fast that they `bounce' off the iron core at close to the speed of light. The rebound causes the star to explode as a supernova.

The energy released during this explosion is so immense that the star will out shine an entire galaxy for a few days. Supernova can be seen in nearby galaxies, about one every 100 years (therefore, if you survey 100 galaxies per year you expect to see at least one supernova a year). One such supernova (1991T) is shown below in the galaxy M51.

Conclusion:

Stars do not live forever, just like people. Stars are born, live their lives, changing or *evolving* as they age, and eventually they die. Often stars do this in a much more spectacular way than humans do!
Scientists speak of stellar evolution when talking about the birth, life and death of stars. The lifetime of individual stars is way too long for humans to observe the evolution of a single star, so how do scientists study stellar evolution? This is possible as there are so many stars in our galaxy, so we can see lots of them at different stages of their lives. In this way, astronomers can build up an overall picture of the process of stellar evolution. In this chapter you will discover how stars are born, how they evolve, and how they die.

Acknowledgement :

Primarily I would thank God for being able to complete this project with success. Then I would like to thank our principal, Swami Kamalasthananda Maharaj and our physics departmental professor Dr. Palash Nath, whose valuable guidance has ones that helped me to complete this. Their instructions and suggestions help me very much. Besides this I would like to thank my parents and friends who helped me with their suggestions.

References:

www.wikipedia.org

www.lumenlearning.com

abyss.uoregon.edu

GEOTHERMAL ENERGY

The project submitted, in partial fulfilment of the requirement for the assignments in ***CC-XI, CC-XII, DSE-I, DSE-II*** *Paper (* ***Semester-V*** *) in the Department of* ***Physics***

Submitted by

Arkadeb Ghosh

Registration No: A01-1112-111-008-2018

Supervisor Teacher: Bhaskar Halder

RAMAKRISHNA MISSION VIVEKANANDA CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

❖ Geothermal Energy

➢ Introduction:

Geothermal energy is the thermal energy in the Earth's crust which originates from the formation of the planet and from radioactive decay of materials in currently uncertain but possibly roughly equal proportions. The high temperature and pressure in Earth's interior cause some rock to melt and solid mantle to behave plastically, resulting in parts of the mantle convecting upward since it is lighter than the surrounding rock and temperatures at the core–mantle boundary can reach over 4000 °C (7200 °F).

Geothermal heating, for example using water from hot springs has been used for bathing since Paleolithic times and for space heating since ancient Roman times, however more recently geothermal power, the term used for generation of electricity from geothermal energy, has gained in importance. It is estimated that the earth's geothermal resources are theoretically more than adequate to supply humanity's energy needs, although only a very small fraction is currently being profitably exploited, often in areas near tectonic plate boundaries.

➢ History:

Hot springs have been used for bathing at least since Paleolithic times. The oldest known spa is a stone pool on China's Lisan mountain built in the Qin Dynasty in the 3rd century BCE, at the same site where the Huaqing Chi palace was later built. In the first century CE, Romans conquered Aquae Sulis, now Bath, Somerset, England, and used the hot springs there to feed public baths and underfloor heating. The admission fees for these baths probably represent the first commercial use of geothermal power. The world's oldest geothermal district heating system in Chaudes-Aigues, France, has been operating since the 15th century. The earliest industrial exploitation began in

1827 with the use of geyser steam to extract boric acid from volcanic mud in Larderello, Italy.

In 1892, America's first district heating system in Boise, Idaho was powered directly by geothermal energy, and was copied in Klamath Falls, Oregon in 1900. The first known building in the world to utilize geothermal energy as its primary heat source was the Hot Lake Hotel in Union County, Oregon, whose construction was completed in 1907. A deep geothermal well was used to heat greenhouses in Boise in 1926, and geysers were used to heat greenhouses in Iceland and Tuscany at about the same time. Charlie Lieb developed the first downhole heat exchanger in 1930 to heat his house. Steam and hot water from geysers began heating homes in Iceland starting in 1943.

In the 20th century, demand for electricity led to the consideration of geothermal power as a generating source. Prince Piero Ginori Conti tested the first geothermal power generator on 4 July 1904, at the same Larderello dry steam field where geothermal acid extraction began. It successfully lit four light bulbs. Later, in 1911, the world's first commercial geothermal power plant was built there. It was the world's only industrial producer of geothermal electricity until New Zealand built a plant in 1958. In 2012, it produced some 594 megawatts.

In 1960, Pacific Gas and Electric began operation of the first successful geothermal electric power plant in the United States at The Geysers in California. The original turbine lasted for more than 30 years and produced 11 MW net power.

The binary cycle power plant was first demonstrated in 1967 in the USSR and later introduced to the US in 1981. This technology allows the generation of electricity from much lower temperature resources than previously. In 2006, a binary cycle plant in Chena Hot Springs, Alaska, came on-line, producing electricity from a record low fluid temperature of 57 °C (135 °F).

➢ Resource:

The Earth's internal thermal energy flows to the surface by conduction at a rate of 44.2 terawatts (TW), and is replenished by radioactive decay of minerals at a rate of 30 TW. These power rates are more than double humanity's current energy consumption from all primary sources, but most of this energy flow is not recoverable. In addition to the internal heat flows, the top layer of the surface to a depth of 10 m (33 ft) is heated by solar energy during the summer, and releases that energy and cools during the winter.

Outside of the seasonal variations, the geothermal gradient of temperatures through the crust is 25–30 °C (77–86 °F) per km of depth in most of the world. The conductive heat flux averages 0.1 MW/km2. These values are much higher near tectonic plate boundaries where the crust is thinner. They may be further augmented by fluid circulation, either through magma conduits, hot springs, hydrothermal circulation or a combination of these.

The thermal efficiency and profitability of electricity generation is particularly sensitive to temperature. The most demanding applications receive the greatest benefit from a high natural heat flux, ideally from using a hot spring. The next best option is to drill a well into a hot aquifer. If no adequate aquifer is available, an artificial one may be built by injecting water to hydraulically fracture the bedrock. This last approach is called hot dry rock geothermal energy in Europe, or enhanced geothermal systems in North America. Much greater potential may be available from this approach than from conventional tapping of natural aquifers.

Estimates of the potential for electricity generation from geothermal energy vary sixfold, from 0.035to2TW depending on the scale of investments. Upper estimates of geothermal resources assume enhanced geothermal wells as deep as 10 kilometres (6 mi), whereas existing geothermal wells are rarely more than 3 kilometres (2 mi) deep. Wells of this depth are now common in the petroleum industry. The deepest research well in the world, the Kola superdeep borehole, is 12 kilometres (7 mi) deep.

Geothermal power is electrical power generated from geothermal energy. Technologies in use include dry steam power stations, flash steam power stations and binary cycle power stations. Geothermal electricity generation is currently used in 26 countries, while geothermal heating is in use in 70 countries.

As of 2019, worldwide geothermal power capacity amounts to 15.4 gigawatts (GW), of which 23.86 percent or 3.68 GW are installed in the United States. International markets grew at an average annual rate of 5 percent over the three years to 2015, and global geothermal power capacity is expected to reach 14.5–17.6 GW by 2020. Based on current geologic knowledge and technology the GEA publicly discloses, the Geothermal Energy Association (GEA) estimates that only 6.9 percent of total global potential has been tapped so far, while the IPCC reported geothermal power potential to be in the range of 35 GW to 2 TW. Countries generating more than 15 percent of their electricity from geothermal sources include El Salvador, Kenya, the Philippines, Iceland, New Zealand, and Costa Rica.

➢Geothermal Power:

Geothermal power is considered to be a sustainable, renewable source of energy because the heat extraction is small compared with the Earth's heat content. The greenhouse gas emissions of geothermal electric stations are on average 45 grams of carbon dioxide per kilowatt-hour of electricity, or less than 5 percent of that of conventional coal-fired plants.

As a source of renewable energy for both power and heating, geothermal has the potential to meet 3-5% of global demand by

2050. With economic incentives, it is estimated that by 2100 it will be possible to meet 10% of global demand.

Geothermal electric plants were traditionally built exclusively on the edges of tectonic plates where high-temperature geothermal resources are available near the surface. The development of binary cycle power plants and improvements in drilling and extraction technology enable enhanced geothermal systems over a much greater geographical range. Demonstration projects are operational in Landau-Pfalz, Germany, and Soultz-sous-Forêts, France, while an earlier effort in Basel, Switzerland, was shut down after it triggered earthquakes. Other demonstration projects are under construction in Australia, the United Kingdom, and the United States of America. In Myanmar over 39 locations capable of geothermal power production and some of these hydrothermal reservoirs lie quite close to Yangon which is a significant underutilized resource.

➢Geothermal heating:

Geothermal heating is the direct use of geothermal energy for some heating applications. Humans have taken advantage of geothermal heat this way since the Paleolithic era. Approximately seventy countries made direct use of a total of 270 PJ of geothermal heating in 2004. As of 2007, 28 GW of geothermal heating capacity is installed around the world, satisfying 0.07% of global primary energy consumption. Thermal efficiency is high since no energy conversion is needed, but capacity factors tend to be low (around 20%) since the heat is mostly needed in the winter.

Geothermal energy originates from the heat retained within the Earth since the original formation of the planet, from radioactive decay of minerals, and from solar energy absorbed at the surface. Most high temperature geothermal heat is harvested in regions close to tectonic plate boundaries where volcanic activity rises close to the surface of the Earth. In these areas, ground and groundwater can be found with temperatures higher than the target temperature of the application. However, even cold ground contains heat, below 6 metres (20 ft) the undisturbed ground temperature is consistently at the Mean Annual Air Temperature and it may be extracted with a ground source heat pump.

Geothermal energy comes in either vapor-dominated or liquid-dominated forms. Larderello and The Geysers are vapor-dominated. Vapor-dominated sites offer temperatures from 240 to 300 °C that produce superheated steam.

➢Liquid dominated plant :

Liquid-dominated reservoirs (LDRs) are more common with temperatures greater than 200 °C (392 °F) and are found near young volcanoes surrounding the Pacific Ocean and in rift zones and hot spots. Flash plants are the common way to generate electricity from these sources. Pumps are generally not required, powered instead when the water turns to steam. Most wells generate 2–10 MW of electricity. Steam is separated from a liquid via cyclone separators, while the liquid is returned to the reservoir for reheating/reuse. As of 2013, the largest liquid system is Cerro Prieto in Mexico, which generates 750 MW of electricity from temperatures reaching 350 °C (662 °F). The Salton Sea field in Southern California offers the potential of generating 2000 MW of electricity

Lower-temperature LDRs (120–200 °C) require pumping. They are common in extensional terrains, where heating takes place via deep circulation along faults, such as in the Western US and Turkey. Water passes through a heat exchanger in a Rankine cycle binary plant. The water vaporizes an organic working fluid that drives a turbine. These binary plants originated in the Soviet Union in the late 1960s and predominate in new US plants. Binary plants have no emissions.

➢ Renewability and Sustainability :

Geothermal power is considered to be renewable because any projected heat extraction is small compared to the Earth's heat content. The Earth has an internal heat content of 1031 joules (3·1015 TWh), approximately 100 billion times the 2010 worldwide annual energy consumption. About 20% of this is residual heat from planetary accretion; the remainder is attributed to past and current radioactive decay of naturally occurring isotopes. For example, a 5275 m deep borehole in United Downs Deep Geothermal Power Project in Cornwall, England, found granite with very high thorium content, whose radioactive decay is believed to power the high temperature of the rock.

Natural heat flows are not in equilibrium, and the planet is slowly cooling down on geologic timescales. Human extraction taps a minute fraction of the natural outflow, often without accelerating it. According to most official descriptions of geothermal energy use, it is currently called renewable and sustainable because it returns an equal volume of water to the area that the heat extraction takes place, but at a somewhat lower temperature. For instance, the water leaving the ground is 300 degrees, and the water returning is 200 degrees, the energy obtained is the difference in heat that is extracted. Current research estimates of impact on the heat loss from the Earth's core are based on a studies done up through 2012. However, if household and industrial uses of this energy source were to expand dramatically over coming years, based on a diminishing fossil fuel supply and a growing world population that is rapidly industrializing requiring additional energy sources, then the estimates on the impact on the Earth's cooling rate would need to be re-evaluated.

Geothermal power is also considered to be sustainable thanks to its power to sustain the Earth's intricate ecosystems. By using geothermal sources of energy present generations of humans will not endanger the capability of future generations to use their own resources to the same amount that those energy sources are presently used. Further, due to its low emissions geothermal energy is considered to have excellent potential for mitigation of global warming.

Even though geothermal power is globally sustainable, extraction must still be monitored to avoid local depletion. Over the course of decades, individual wells draw down local temperatures and water levels until a new equilibrium is reached with natural flows. The three oldest sites, at Larderello, Wairakei, and the Geysers have experienced reduced output because of local depletion. Heat and water, in uncertain proportions, were extracted faster than they were replenished. If production is reduced and water is reinjected, these wells could theoretically recover their full potential. Such mitigation strategies have already been implemented at some sites. The long-term sustainability of geothermal energy has been demonstrated at the Lardarello field in Italy since 1913, at the Wairakei field in New Zealand since 1958, and at The Geysers field in California since 1960.

Falling electricity production may be boosted through drilling additional supply boreholes, as at Poihipi and Ohaaki. The Wairakei power station has been running much longer, with its first unit commissioned in November 1958, and it attained its peak generation of 173 MW in 1965, but already the supply of high-pressure steam was faltering, in 1982 being derated to intermediate pressure and the station managing 157 MW. Around the start of the 21st century it was managing about 150 MW, then in 2005 two 8 MW isopentane systems were added, boosting the station's output by about 14 MW. Detailed data are unavailable, being lost due to re-organisations. One such re-organisation in 1996 causes the absence of early data for Poihipi (started 1996), and the gap in 1996/7 for Wairakei and Ohaaki; half-hourly data for Ohaaki's first few months of operation are also missing, as well as for most of Wairakei's history.

➢ Environmental effects:

Fluids drawn from the deep Earth carry a mixture of gases, notably carbon dioxide (CO_2), Hydrogen Sulfide (H_2S), methane (CH_4) and ammonia (NH_3). These pollutants contribute to global warming, acid rain, and noxious smells if released. Existing geothermal electric plants emit an average of 122 kilograms (269 lb) of CO_2 per megawatt-hour (MW·h) of electricity, a small fraction of the emission intensity of conventional fossil fuel plants. But a few plants emit more than gas-fired power, at least in the first few years, such as some geothermal power in Turkey. Plants that experience high levels of acids and volatile chemicals are usually equipped with emission-control systems to reduce the exhaust.

In addition to dissolved gases, hot water from geothermal sources may hold in solution trace amounts of toxic elements such as mercury, arsenic, boron, and antimony. These chemicals precipitate as the water cools, and can cause environmental damage if released. The modern practice of injecting cooled geothermal fluids back into the Earth to stimulate production has the side benefit of reducing this environmental risk.

Plant construction can adversely affect land stability. Subsidence has occurred in the Wairakei field in New Zealand. In Staufen im Breisgau, Germany, tectonic uplift occurred instead, due to a previously isolated anhydrite layer coming in contact with water and turning into gypsum, doubling its volume. Enhanced geothermal systems can trigger earthquakes as part of hydraulic fracturing. The project in Basel, Switzerland was suspended because more than 10,000 seismic events measuring up to 3.4 on the Richter Scale occurred over the first 6 days of water injection.

Geothermal has minimal land and freshwater requirements. Geothermal plants use 3.5 square kilometres per gigawatt of electrical production (not capacity) versus 32 square kilometres and 12 square kilometres for coal facilities and wind farms respectively. They use 20 litres of freshwater per MW·h versus over 1,000 litres per MW·h for nuclear, coal, or oil.

➢Conclusion:

Geothermal energy is renewable and sustainable. Geothermal energy can be used in a much more significant way in future. Due to its eco-friendly nature, geothermal energy can be a crucial source of renewable source of energy in future.

➢Reference:

- The Geothermal Collection by the University of Hawaii at Manoa
- Geothermal Resources Council
- Energy Efficiency and Renewable Energy – Geothermal Technologies Program
- Geothermal Energy Association
- International Energy Agency Geothermal Energy Homepage

➢Acknowledgement:

In the accomplishment of this project successfully, many people have given me their support and guidance. Now I am utilizing this time to thank all the people who have been concerned with this project.

Primarily I would thank my project supervisor Prof. Bhaskar Haldar for his guidance and support which help me to complete this project in time , Besides him I would like to thank my friends for their help which also help me to complete this project work.

Metamaterials and Negative Refractive Index

The project submitted, in partial fulfilment of the requirement for the assignments in ***PHSA CC-XI, PHSA CC-XII, PHSA DSE-I, PHSA DSE-II*** *Papers (****Semester V****) in the Department of* ***Physics*** *(session:* ***2021-2022****)*

Submitted by

Aniket Misra

Registration No: A01-1112-111-015-2018

Supervisor Teacher: ***Dr. Anjan Chandra***

RAMAKRISHNA MISSION VIVEKANANDA CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

OBJECTIVES:

Here we'll discuss distinctive properties of metamaterial, prominently negative index materials (NIM). We'll come across recent advances in metamaterials research and discuss the potential that these materials may hold for realizing new and seemingly exotic electromagnetic phenomena. The engineered response of metamaterials has had a dramatic impact on the physics, optics, and engineering communities. The advent of metamaterials has yielded new opportunities to realize physical phenomena that were previously only theoretical exercises.

Keywords: Metamaterials; negative-index of refraction materials; split-ring Resonator; permeability; permittivity.

INTRODUCTION :

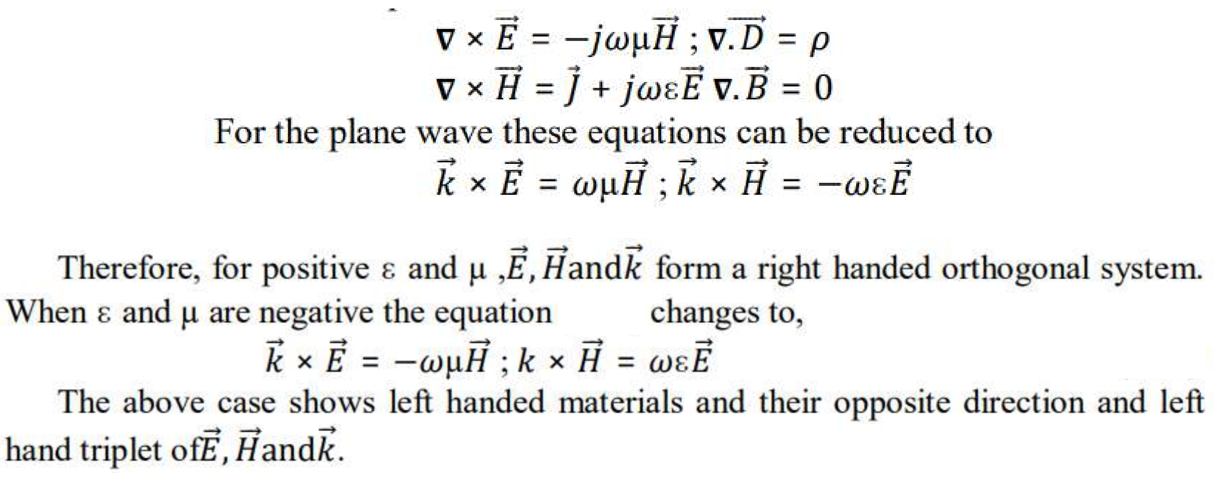
Let us consider light passing through a plate of glass. As visible light has a wavelength that is hundreds of times larger than the atoms of which the glass is composed, the atomic details lose importance in describing how the glass interacts with light. In practice, we can average over the atomic scale, conceptually replacing the otherwise inhomogeneous medium by a homogeneous material characterized by just two macroscopic electromagnetic parameters: the electric permittivity, ε, and the magnetic permeability, μ. From the electromagnetic point of view, the wavelength, λ, determines whether a collection of atoms or other objects can be considered a material. The electromagnetic parameters ε and μ need not arise strictly from the response of atoms or molecules. Any collection of objects whose size and spacing are much smaller than λ can be described by an ε and μ. Here, the values of ε and μ are determined by the scattering properties of the structured objects. Although such an inhomogeneous collection may not satisfy our intuitive definition of a material, an electromagnetic wave passing through the structure cannot explain the difference. From the electromagnetic point of view, we have created an artificial material or metamaterial.

EARLIER ADVANCES :

In 1898, Sir J.C. Bose showed the possibility of existence of artificial material by conducting microwave experiment on twisted structure. Later, the physicist Victor Veselgo (1968) presented theoretical investigation and Pendry *et. al.* in 1996 used an artificial wired medium whose permittivity is negative to realize artificial electric plasma. Followed by this, in 1999 magnetic plasma was realized whose permeability is negative using split-ring resonators (SRR). Smith *et. al.* (2004) had realized gradient refractive index medium to bend electromagnetic waves. The word "metamaterial" was first coined by Rodger M. Walser (2001) who gave the following definition "Metamaterials are defined as macroscopic composites having a man-made, three-dimensional, periodic cellular architecture designed to produce an optimized combination, not available in nature.''

❖ Theoretical Aspects:

Metamaterial can be characterized by using Maxwell equations. Transformation of Maxwell equations have a prominent role in describing Metamaterial which is given below as Maxwell equation in time domain:



$$\nabla \times \vec{E} = -j\omega\mu\vec{H} \, ; \nabla . \overrightarrow{D} = \rho$$

$$\nabla \times \vec{H} = \vec{J} + j\omega\varepsilon\vec{E} \; \nabla . \vec{B} = 0$$

For the plane wave these equations can be reduced to

$$\vec{k} \times \vec{E} = \omega\mu\vec{H} \, ; \vec{k} \times \vec{H} = -\omega\varepsilon\vec{E}$$

Therefore, for positive ε and μ ,$\vec{E}, \vec{H}$and$\vec{k}$ form a right handed orthogonal system. When ε and μ are negative the equation changes to,

$$\vec{k} \times \vec{E} = -\omega\mu\vec{H} \, ; k \times \vec{H} = \omega\varepsilon\vec{E}$$

The above case shows left handed materials and their opposite direction and left hand triplet of$\vec{E}, \vec{H}$and$\vec{k}$.

❖ Metamaterials and Resonant Response:

Why does a set of conductors shaped into Swiss rolls behave like a magnetic material ? In this structure, the coiled copper sheets have a self-capacitance and self-inductance that create a resonance. The currents that flow when this resonance is activated couple strongly to an applied magnetic field, yielding an effective permeability that can reach quite high values. This field transference was demonstrated by arranging an antenna in the shape of the letter M as the source and mapping the transmitted magnetic field distribution. On resonance, the Swiss roll structure uniformly transmitted the incident field pattern across the slab (Fig. 1B), and the resolution matched that predicted by theory. The signal from that spin pattern had to be conveyed faithfully back to the receiver. This experiment demonstrated that a high-performance metamaterial could act as a magnetic face plate and convey information from one side to the other without loss of spatial information.

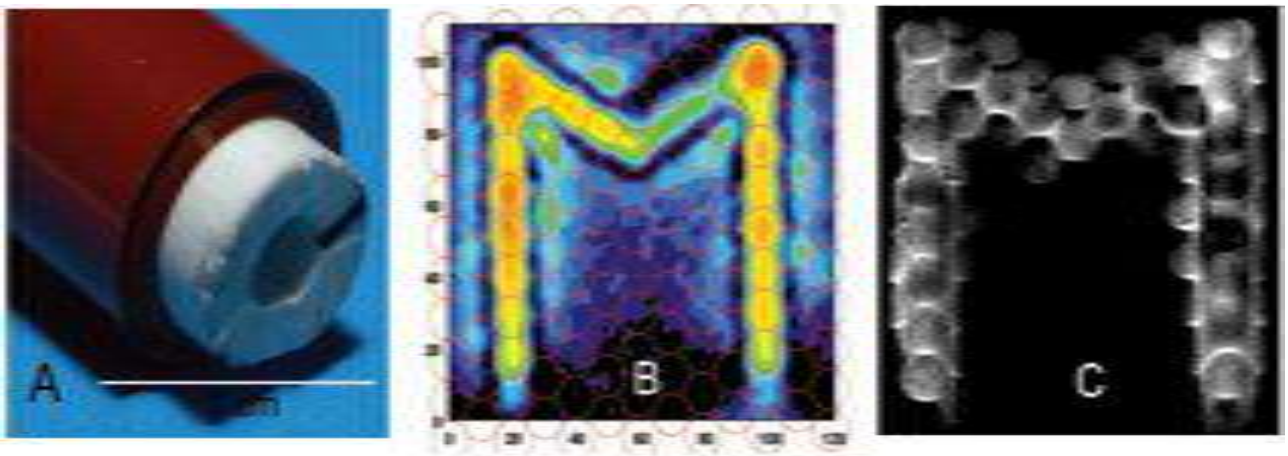


Fig. 1. **(A) A single element of Swiss roll metamaterial. (B) An array of these elements is assembled into a slab and the RF magnetic field from an M-shaped antenna below the slab, is reproduced on the upper surface. The red circles show the location of the rolls (1 cm in diameter). (C) The resulting image taken in an MRI machine, showing that the field pattern is transmitted back and forth through the slab.**

❖ Artificial Magnetism:

Artificial magnetic metamaterials have unique properties; at lower frequencies magnetism is also exhibited by existing conventional materials. At higher frequencies, nearly all materials have electronic resonances that result from lattice vibration. A frequency range of particular interest occurs between 1 and 3 THz, a region that represents a natural breakpoint between magnetic and electric response in conventional materials. Metamaterials can be constructed to provide this response. At higher frequencies, the split ring resonator (SRR), another conducting structure, can be conveniently used to achieve a magnetic response. The SRR consists of a planar set of concentric rings, each ring with a gap. Because the SRR is planar, it is easily fabricated by lithographic methods at scales appropriate for low frequencies to optical frequencies. Both the Swiss roll metamaterial and the THz SRR metamaterial illustrate the advantage of developing artificial magnetic response.

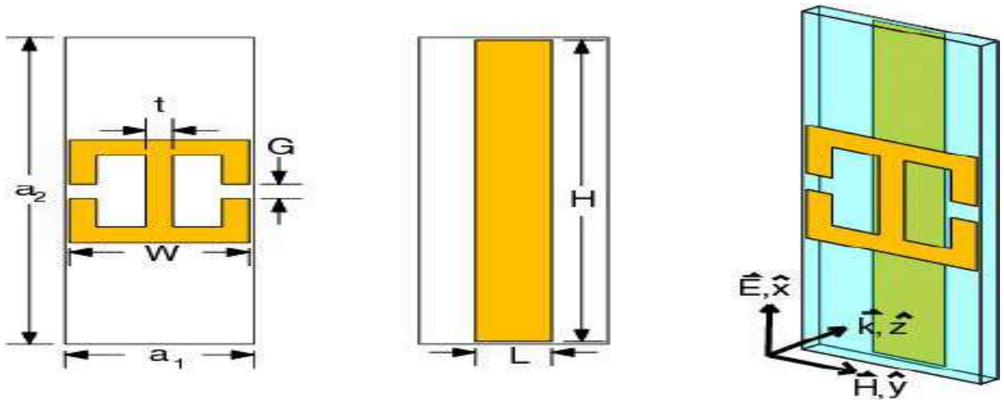


Fig.2. **First perfect electromagnetic metamaterial absorber structure**

Negative Material Response:

A material having either (but not both) ε or μ negative is opaque to electromagnetic radiation. In conventional materials, the group and phase velocities are parallel. By contrast, the group and phase velocities point in opposite directions when ε<0 and μ<0 (Fig.3). The reversal of phase and group velocity in a material implies that sign of the refractive index, ***n***, must be taken as negative. In 2000, a composite structure based on SRRs was introduced for which the negative μ occurred at frequencies above the resonant frequency and negative ε occurred below a cut-off frequency; by choosing the parameters of the wire lattice such that the cut-off frequency was significantly above the SRR resonant frequency, the composite was made to have an overlapping region where both ε and μ were negative.

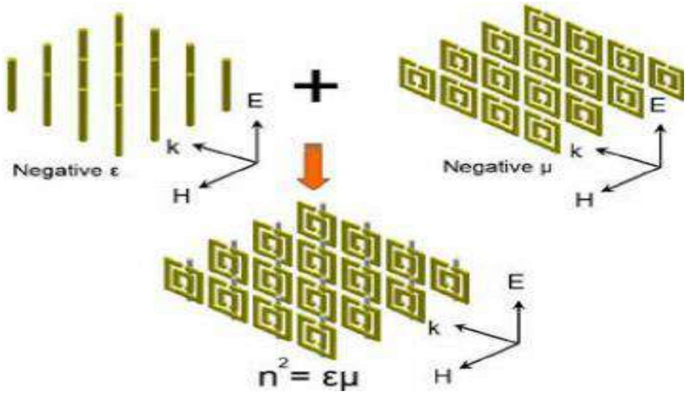


Fig.3. **Negative refractive index materials with a periodic array of SRRs**

Negative Refraction and Subwavelength Resolution:

The quantitative statement of refraction is embodied in Snell's law, which relates the exit angle of a beam, q_2, as measured with respect to a line drawn perpendicular to the interface of the material, to the angle of incidence, q_1, by the formula **sin (q_1) = n sin (q_2).** The refractive index determines the amount by which the beam is deflected. If the index is positive, the exiting beam is deflected to the opposite side of the surface normal, whereas if the index is negative, the exiting beam is deflected the same side of the normal (Fig. 4).

In 2001, a Snell's law experiment was performed on a wedge-shaped metamaterial designed to have a negative index of refraction at microwave frequencies. In this experiment, a beam of microwaves was directed onto the flat portion of the wedge sample, passing through the sample undeflected, and then refracting at the second interface. The angular dependence of the refracted power was then measured around the circumference, establishing the angle of refraction.

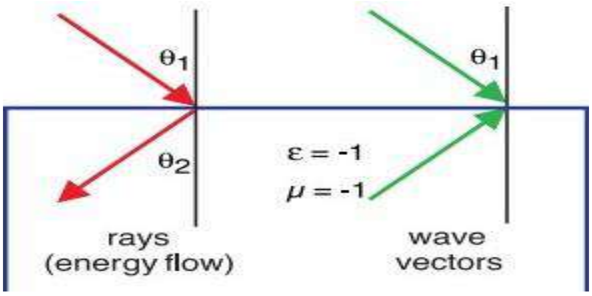


Fig.4. **Negative refraction: A ray enters a negatively refracting medium and is bent the wrong way relative to the surface normal, forming a V-shape at the interface (left). The wave vector and group velocity point in opposite directions (right).**

The result of the experiment (Fig. 5) indicated quite clearly that the wedge sample refracted the microwave beam in a manner consistent with Snell's law. Figure **5.B** shows the detected power as a function of angle for a Teflon wedge (n = 1.5, blue curve) compared to that of the NIM wedge (red curve). The location of the peak corresponding to the negative index material (NIM) wedge implies an index of –2.7.

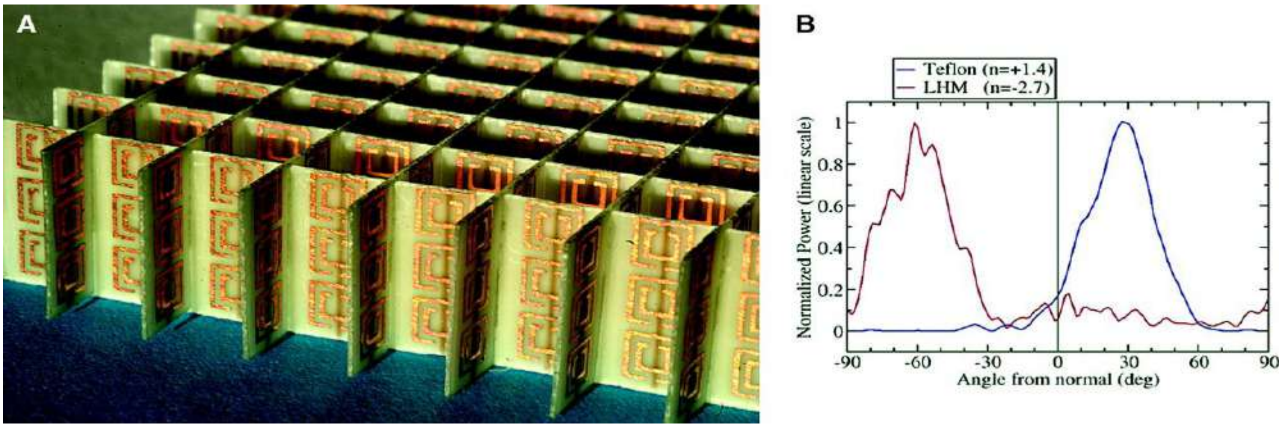


Fig. 5.(A) **A negative index metamaterial formed by SRRs. The height of the structure is 1 cm. (B) The power detected as a function of angle in a Snell's law experiment performed on a Teflon sample (blue curve) and a negative index sample (red curve).**

❖ Applications:

1. ***Metamaterial as antenna:*** Metamaterial coatings have been used to enhance the radiation and matching properties of electrically small electric and magnetic dipole antennas. Metamaterial step up the radiated power.

The newest Metamaterial antenna radiate 95% of input radio signal at 350 MHz's. Experimental metamaterial antenna is as small as one fiftieth of a wavelength.

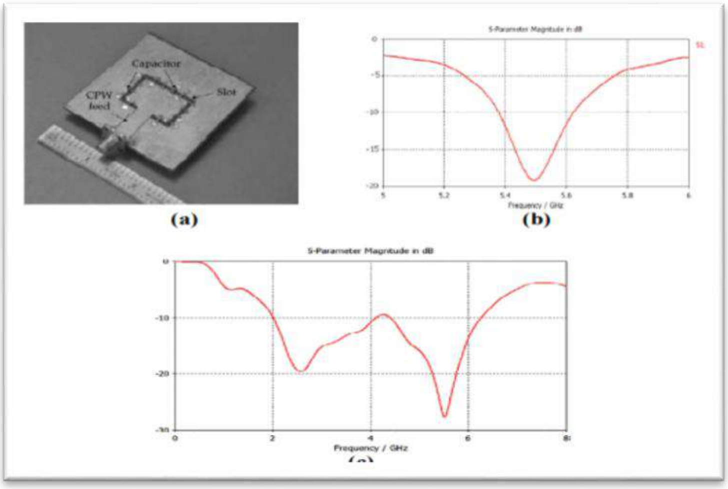


Fig.6. **(a)** **3D image of CPW feeding of an antenna** **(b)** **result of simple patch antenna** **(c)** **result of Metamaterial antenna.**

2. ***Metamaterial as Absorber:*** The first Metamaterial based absorber by Landy (2008) utilizes three layers, two metallic layers and dielectric and shows a simulated absorptivity of 99% at 11.48 GHz. Experimentally, Landy was able to achieve an absorptivity of 88%. The difference between simulated and measured results were due to fabrication errors.

3. ***Metamaterial as superlens:*** Superlens uses metamaterials to go beyond the diffraction limit. Ramakrishna (2005) showed, it has resolution capabilities that go beyond ordinary microscopes. Conventional optical materials suffer a diffraction limit because only the propagating components are transmitted from a light source. The non-propagating components, the evanescent waves, are not transmitted.

One way to improve the resolution is to increase the refractive index but it is limited by the availability of high-index materials. The road to the super lens is

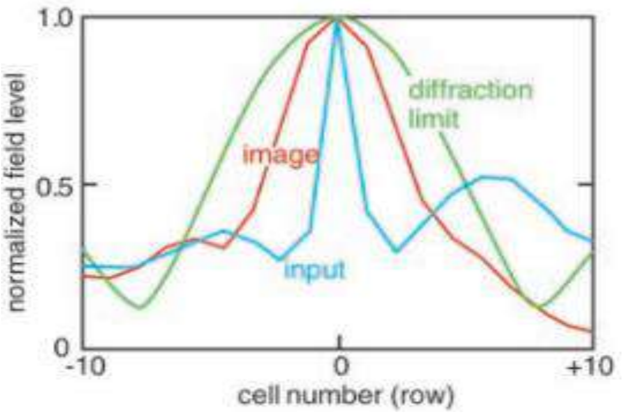


Fig.7. **Resolution of measured data (red) are less than perfect results (blue) due to losses, but better than the diffraction limit (green).**

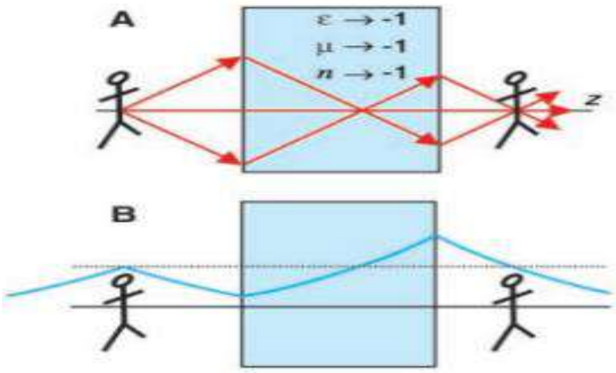


Fig.8. **Perfect Lensing: A slab of negative material effectively removes an equal thickness of space for (A) the far field and (B) the near field, translating the object into a perfect image.**

its aptitude to significantly enhance and recover the evanescent waves that carry information at very small scales. No lens is yet able to completely reconstitute all the evanescent waves emitted by an object. So, the future challenge is to design a superlens which can constitute all evanescent waves to get perfect image.

4. ***Networking:*** Signals can travel further with higher gain using holographic beamforming. In the case of 5G networks, that means that the mm Waves in the higher frequency bands will not be blocked as easily by buildings and could travel farther than a few kilometres. As a result, holographic beamforming antennas boost throughput for cellular networks and possibly replace fibre optic cables as backhaul links between base stations.

5. ***Navigation (RADAR & GPS):*** Metamaterial electronically scanned array (MESA) radar can be reduced from vehicle-mounted systems to a paperback book's size with a comparable reduction in cost. Today's conventional electronically-scanned array (ESA) radars are too large and too expensive. These cost/performance attributes open up applications for MESAs in drones and other unmanned aerial vehicles, autonomous ground vehicles (including automobiles), portable security systems, and for countering threats from unmanned air systems (UAS), including complex environments with drones, cruise missiles, aircraft, and ballistic missiles.

6. ***Metamaterial as cloaks:*** Cloaking can be achieved by cancellation of the electric and magnetic field generated by an object or by guiding the electromagnet wave around the object. Guiding the wave means transforming the coordinate system in such a way that inside the hollow cloak

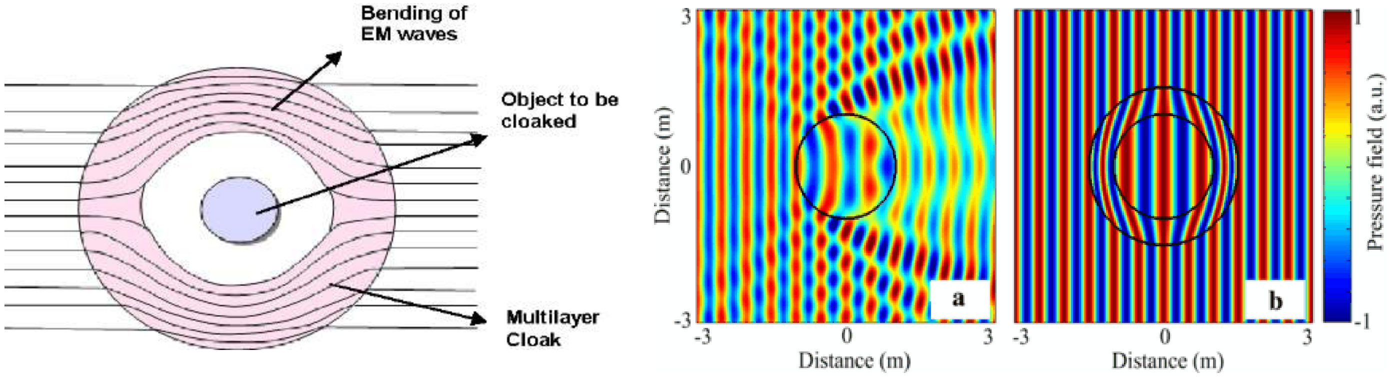


Fig.9. **Demonstration of metamaterial cloaking: (a) Impingement of waves in a solid cylinder, (b) in a solid cylinder surrounded by invisibility cloak.**

electromagnetic field will be zero this makes the region inside the shell disappear. The basic idea consists of covering the object with an invisibility cloak that guides the waves around such object and returns them to their original shape once they have crossed the cloak. Since these waves recover their original shape as if there were no object, this mechanism confers the object with the property of being undetectable by any sensor placed in front of or behind it.

The principles of the cloaking phenomenon also apply to **sound waves**, which led to the introduction of acoustic cloaks. Apart from the possible military applications, acoustic cloaking can be applied to develop structures for diverting impinging noise or new types of walls improving the acoustics in concert halls. Biomedical devices like "transparent" probes for ultrasonography and acoustic sensors that do not distort the field they are measuring are also potential applications of acoustic cloaks.

7. ***Metamaterial as sensor:*** Metamaterial opens a door for designing sensor with specified sensitivity. Metamaterials provide tools to significantly enhance the sensitivity and resolution of sensors. Metamaterial sensors are used in agriculture, biomedical etc. In agriculture the sensors are based on resonant material and employ SRR to gain better sensitivity, in bio medical wireless strain sensors are widely used, nested SRR based strain sensors have been developed to enhance the sensitivity.

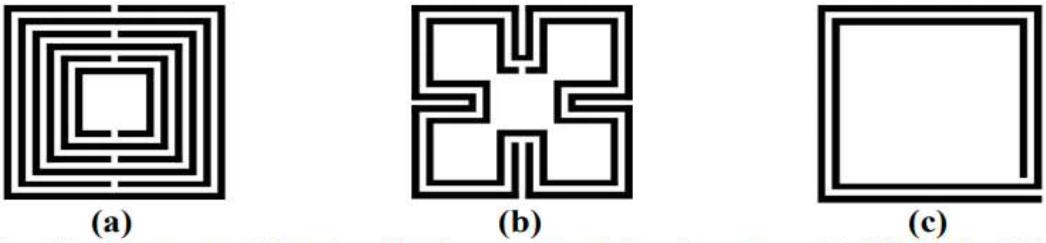

(a) (b) (c)

Fig. 10: Metamaterial unit cells that are used for the sensor (a) Multiple SRR (b)Sierpinski SRR(c) Spiral Resonator

8. ***Charging purposes & MRI:*** Researchers at Mitsubishi Electric Research Laboratories have built a prototype wireless power transfer system incorporating a NIM lens. One potential application for NIM lenses is in charging electric vehicles as well as mobile broadband in vehicles.

❖ Acknowledgements:

First of all, I would like to pay my gratitude to hon'ble professor ***Dr. Anjan Chandra***, our project-guide for guiding me in choosing the topic of the project as well as render clarity to all my confusions.

I'll also thank ***Dr. Ashok K. Pal***, hon'ble HOD of Physics Department for giving us the opportunity of pursuing this project work alongside our studies.

I'm thankful to my classmates for always being there to solve any issues.

At last but not the least, I'm indebted to all the professors of the ***Department of Physics, RKMVCC*** for teaching us all the concepts that directly or indirectly helped me in completing the project.

❖ Conclusion:

Metamaterial are expected to have an impact across the entire range of technologies where electromagnetic radiation are used and will provide a flexible platform for technological advancement. The most interesting application is as an absorber and also as sensors for humidity, soil moisture measurement etc. From the progress and interest in this field it is clear that the future of metamaterials lies in the field of optics and medical. This is closely linked to advancements in nanotechnology.

References :

1. D.R. Smith, J.B. Pendry, M.C.K. Wiltshire, Review in Science vol-305 (Aug.,2004)

2. T. J. Yen et al., Science 303, 1494 (2004).

3. W. L. Barnes, A. Dereux, T. W. Ebbesen, Nature 424, 824 (2003).

4. Adnan Noor (2010), Metamaterial Electromagnetic Absorbers and Plasmonic Structures, pp.42.

5. A.Sihvola (2007), Metamaterials in electromagnetics, Metamaterials1, 2-11.

Stellar Evolution The project submitted, in partial fulfilment of the requirement for the assignments in CC-11, CC-12, DSE-1, DSE-2 Papers (Semester V) in the Department of Physics

Submitted by Kushal Ghosh

Registration No: A01-1112-111-047-2018

Supervisor Teacher: Dr. Palash Nath

STAR FORMATION and EVOLUTION

Objects that appear as specks of lights in the night sky carry with them the most astonishing and profound theories which make the most fertile minds to dive into speculation to unravel the mysteries of their birth and evolution.

With advances in the field of astronomy and with the help of technology developed over years of research scientists and astronomers have been able to put forward several theories which can explain the underlying behaviour of interstellar objects one of them being stars.

What is a star?

A star is a celestial body which consists of a sphere of luminous plasma held together by it's own gravitational force of attraction.

Stars can generate light of their own. In other words they create their own sustained energy which suggests that they inevitably need a source of fuel to generate such enormous amounts of energy which keeps them glowing for thousands of years. Stars are born out of diffuse molecular clouds of gas, in regions of space known as 'stellar nurseries'. These gas clouds contain the hydrogen and heavier elements that provide the fuel source for the star's lifetime.

The birth of a star is a violent and chaotic event with gas flowing in and being ejected outwards at speeds up to hundreds of kilometres per second. Understanding the violent and chaotic processes that characterise the birth of new stars is one of the biggest challenges in contemporary astrophysics. While advances in infrared detection technologies have made it possible to observe aspects of the star formation process directly, there are still many mysteries surrounding the exact physics involved.

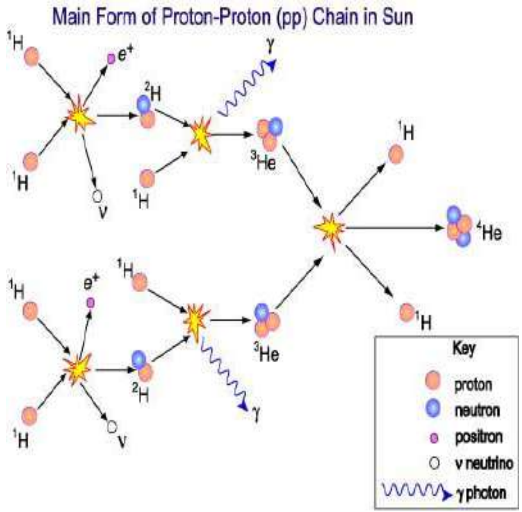
Now, what might be the possible source of that amount of energy sustained for such a long time?

Although there are different possibilities but they are not all feasible. There are certain constraints which poses restriction on the processes…………..Like we might have

a. Chemical combustion: this process is inefficient and cannot sustain the lifetime of the star for a very long time.

b. Radioactivity: Radioactive elements have not been detected inside the sun(our nearest star) or any other star. If found then in traces.

c. Nuclear fusion: This might be a feasible option. The presence of light elements like Hydrogen(H) and Helium(He) also serve as a strong evidence to suggest that thi migt be process involved in providing energy to a star.



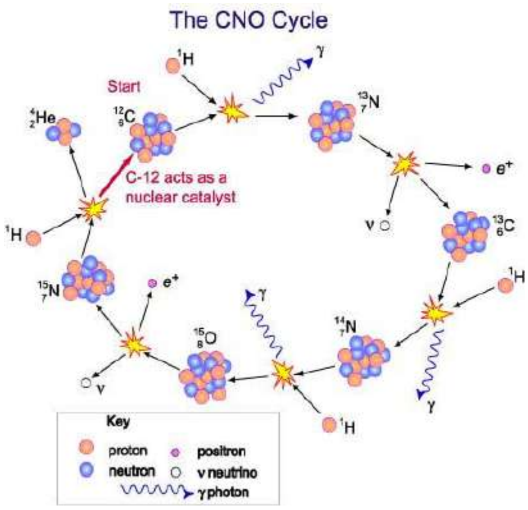
The proton-proton fusion is the mechanism which provides energy to the sun --------

$H^1 + H^1 \longrightarrow H^2 + e^+$(positron) + v(neutrino) + energy(γ photon)

$H^1 + H^2 \longrightarrow He^3$ + energy(γ photon)

$He^3 + He^3 \longrightarrow He^4 + H^1 + H^1$

${}^1H({}^1H, e^+ \nu_e)\ {}^2D({}^1H,\gamma)\ {}^3He({}^3He, 2\,{}^1H)\ {}^4He$



The Carbon-Nitrogen-Oxygen cycle is the mechanism which provides energy to bigger stars------

$^{12}C + p \longrightarrow {}^{13}N + \gamma(\text{energy})$

$^{13}N \longrightarrow {}^{13}C + e^+(\text{positron}) + \nu_e$

$^{13}C + p \longrightarrow {}^{14}N + \gamma(\text{energy})$

$^{14}N + p \longrightarrow {}^{15}O + \gamma(\text{energy})$

$^{15}O \longrightarrow {}^{15}N + e^+(\text{positron}) + \nu_e$

$^{15}N + p \longrightarrow {}^{12}C + {}^{4}He$

${}^{12}C({}^1H,\gamma)\ {}^{13}N(e^+\nu_e)\ {}^{13}C({}^1H,\gamma)\ {}^{14}N({}^1H,\gamma)\ {}^{15}O(e^+\nu_e)\ {}^{15}N({}^1H, {}^4He)\ {}^{12}C$

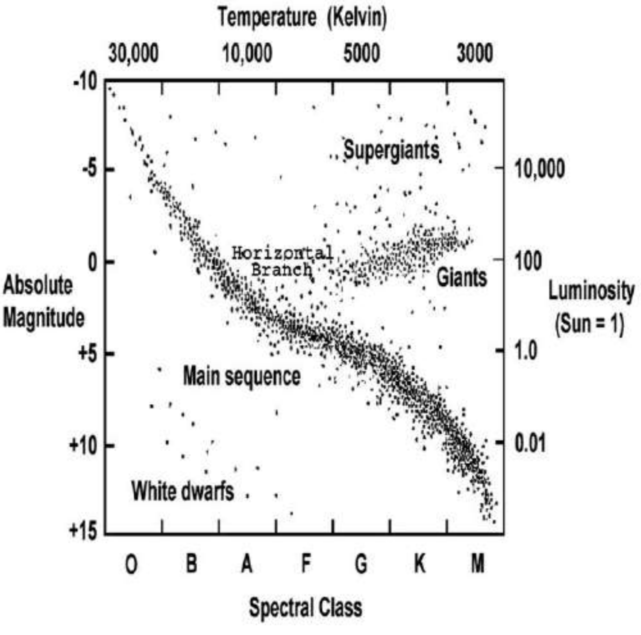
So far we have seen that what fuels a star after it's formation. These reaction guide us and show how they generate energy and in such a sustained and efficient manner. Now there might be a question that how a star might be born and how does it not collapse into itself.

How is a Star Born?

Stars are born within the clouds of dust and scattered throughout most galaxies in the form of a nebula. Turbulence deep within these clouds gives rise to knots with sufficient mass that the gas and dust can begin to collapse under its own gravitational attraction. As the cloud collapses, the material at the centre begins to heat up. Known as a protostar, it is this hot core at the heart of the collapsing cloud that will one day become a star.

Studies help to predict that the spinning clouds of collapsing gas and dust may break up into two or three blobs; this would explain why the majority the stars in the Milky Way are paired or in groups of multiple stars. As the cloud collapses, a dense, hot core forms and begins gathering dust and gas. Not all of this material ends up as part of a star — the remaining dust can become planets, asteroids, or comets or may remain as dust.

There exists an order in the birth of the stars. This order or rule guides into the formation and maturation of a star as it appears to us. The 'Hertzsprung-Russel' diagram plots the power(brightness) as a function of surface temperature (spectral class). This diagram in itself helps to determine the fate and formation of a star.



Clouds of dust and gases(Nebula)

↓ Perturbation

Irregularities in density causes gas to collapse due to net gravitational pull

↓ Collapse continues

Temperature increases.

↓ The cloud breaks to blobs

The core of the cloud collapses faster to conserve angular momentum

↓

H_2 gas breaks to H atoms

Onset of fusion reaction. Marks birth of star.

As shown in the Hertzsprung-Russell diagram, Main Sequence stars span a wide range of luminosities and colours, and can be classified according to those characteristics. The smallest stars, known as red dwarfs, may contain as little as 10% the mass of the Sun and emit only 0.01% as much energy, glowing feebly at temperatures between 3000-4000K. Despite their diminutive nature, red dwarfs are by far the most numerous stars in the Universe and have lifespans of tens of billions of years.

On the other hand, the most massive stars, known as hypergiants, may be 100 or more times more massive than the Sun, and have surface temperatures of more than 30,000 K. Hypergiants emit hundreds of thousands of times more energy than the Sun, but have lifetimes of only a few million years. Although extreme stars such as these are believed to have been common in the early universe, today they are extremely rare - the entire Milky Way galaxy contains only a handful of hypergiants.

Now after this discussion we might have a little talk about the fate of stars corresponding to the diagram.

Fate of stars corresponding to Hertzsprung-Russel diagram-------

The dots in the diagram each represent a stellar body having a certain energy respective to a temperature. The clusters of these dots are found to be the most dense in the 'Main Sequence' with another region having considerable no. of dots which represent 'Giants'. There are some erratic dots at both farther ends which represent the less probable or have a short lived lifespan.

Coming to this we must know how does the life span of an star vary------

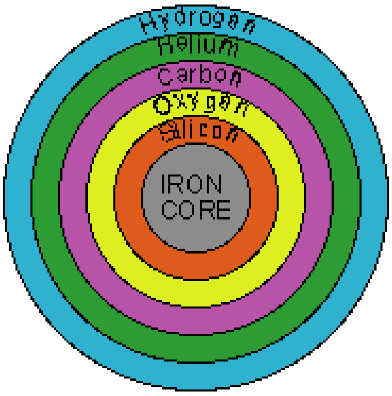
A note on the lifespan of stars : The duration of lifespan of a star primarily depends on the abundance of fuel within the star and how fast the star consumes that fuel.

i. Stars that are the size of our sun or smaller are quite abundant and have less power. So despite having lesser fuel the have longer life span due to lesser fuel consumption.
ii. Stars that are very large are less probable and it is obvious as well since due to their large size they have huge amounts of energy causing very quick consumption of their fuel(despite having larger fuel content).
iii. Very small stars are the most abundant due their high density and quiescent power they tend to outlive the universe itself as they exhaust their fuel extremely slowly.

In general the main sequence stars are the most abundant and spans over a wide range of luminosities and colours. They are fuelled by the hydrogen present within the core of the star that undergoes nuclear fusion to emanate enormous amount of energy and helium. The star does not collapse into itself due to outflow of the humongous energy and it's radiation pressure. the smaller of the stars named as 'Red Dwarfs' are by far the most abundant in the universe. They may contain as little as 10% the mass of the Sun and emit only 0.01% as much energy, glowing feebly at temperatures between 3000-4000K. They have such a large life span that some are predicted to even outlive our universe.

On the other hand, the most massive stars, known as hypergiants, may be 100 or more times more massive than the Sun, and have surface temperatures of more than 30,000 K. Hypergiants emit hundreds of thousands of times more energy than the Sun, but have lifetimes of only a few million years. Although extreme stars such as these are believed to have been common in the early Universe, today they are extremely rare - the entire Milky Way galaxy contains only a handful of hypergiants.

A star that is deprived of the energy production needed to support it, the core begins to collapse into itself and becomes much hotter. Hydrogen is still available outside the core, so hydrogen fusion continues in a shell surrounding the core. The increasingly hot core also pushes the outer layers of the star outward, causing them to expand and cool, transforming the star into a red giant.



If the star is sufficiently massive, the collapsing core may become hot enough to support more exotic nuclear reactions that consume helium and produce a variety of heavier elements up to iron. However, such reactions offer only a temporary reprieve. Gradually, the star's internal nuclear fires become increasingly unstable - sometimes burning furiously, other times dying down. These variations cause the star to pulsate and throw off its outer layers, enshrouding itself in a cocoon of gas and dust. What happens next depends on the size of the core.

The Main Sequence stars are the most numerous and are commonly observed and studied. As already said that supergiants are rare and were more common to the early universe so we are less prior to them.

The average star : For average stars like the Sun, the process of ejecting its outer layers continues until the stellar core is exposed. This dead, but still ferociously hot stellar cinder is called a White Dwarf. The question is that why didn’t the white dwarf collapse further? The answer lies within the high density of the core where the entire mass of the star is concentrated. The huge pressure exerted by the out flow of the electrons from within the core of the star leads to balancing pressure which keeps the star from collapsing further. But it does not end there!

Dynamical equilibrium Inside a star there are two forces at play, the outward force due to nuclear fusion and the inward force due to gravitational attraction. For a stable star to form these two forces must balance each other. If the nuclear force becomes greater than the gravitation then the star will shed off its mass, and if the gravitation dominates the nuclear force the star might collapse upon itself. If the mass of the sun is greater than 0.08 time the mass of the sun, then the energy generated by fusion provides thermal pressure to stop the collapse, however if the mass of the star is less than 0.08 solar mass then it is the degeneracy pressure which stops the gravitational collapse, this prevents the core from being hot enough to initiate hydrogen fusion and the star becomes a brown dwarf. After a certain time the hydrogen in the core is used up and the star is left with an inert helium core. The core shrinks in size along with its inner layers. This results in the hydrogen shell surrounding the core to get hot enough to begin fusion. This H-burning shell produces more energy than main sequence phase, as a result the star expands. This is called the red giant phase. H-burning shell produces more helium which is accumulated in the inert core, thus the core contracts further due to added mass, increasing the pressure. This phenomenon accelerates with time. The star grows massively in size during this. During this stage and after the outer surface of the star is not very tightly bound and is prone to loss due to stellar winds. Eventually after the red giant phase the core becomes hot enough to initiate helium fusion and the star starts to fuse helium to form carbon with a surrounding hydrogen burning shell. In this phase the energy generation is steady. The star becomes smaller and hotter. Afterwards we can see a repetition of previous phenomenon as an inert core of carbon forms surrounded by shells of helium and hydrogen. (a) (b) Fig 4(a): equilibrium between pressure and gravity. Fig 4(b): expansion of photosphere Death and decay Low mass stars such as our sun never reach the temperature required for fusion beyond carbon to take place and no additional heat and gas pressure is generated. Thus the core begins to collapse again. This shrinking is stopped by electron degeneracy pressure before it gets hot enough to fuse carbon and oxygen into magnesium and silicon. The outer layer of the star begins to fall apart, producing a planetary nebula. The planetary nebula’s material slowly begins to drift away, leaving a dense hot core of the star. This leftover remains of the star, entirely supported by electron degeneracy pressure is called a white dwarf star. Newly formed, it is quite hot and looks white oar blue, however with time it cools down enough and becomes red. For high mass stars, whose mass are in the range 2−3 Msunthe nuclear reactions proceed until the core becomes mainly iron, beyond this the density becomes so high that the neutron degeneracy pressure comes into play and stops further shrinking. The neutrons star die in a truly magnificent way. The gravitational forces crushes its core, releasing extreme gravitational energy which leads to the explosion of the star as a supernova. If the iron core of the collapsing star has mass larger than 3 Msunit becomes a black hole. (a) (b) Fig 5(a): planetary nebula. Fig 5(b): supernova explosion Conclusion We see that the journey of a star from its birth to death is a marvellous phenomenon of nature. This journey of a star is not very much unlike that of a living being, it starts its life, matures and dies, albeit in a time scale which is humongous when compared to that of any living thing we know of. Studying such matter one cannot help but come face to face with the realisation that compared to these how small and insignificant our lives are. Yet, as grand as these cosmic entities be, they too do not exist forever. So in a sense these grand objects are as mortal as we are, only that they exist for a timescale which is much larger than ours. However, the inevitability of death harbours the promise of creation. When large star dies in a supernova explosion or a small star sheds its planetary nebula, it spreads throughout the space the materials of creation, without which the universe would remain barren. These materials form other cosmic objects such as planets and other stars. This grand dance of cosmic birth and death enables beings from the third planet of a medium sized star to direct its gaze towards the sky and marvel at the cosmic objects, because without this dance, neither of these would have existed. Acknowledgement I would like to thank Dr. Palash Nath for guiding me throughout this assignment. I am thankful to Dr. Asok kumar Pal, Head of the department of physics. I am also indebted to Swami Kamalasthananda, Principal of Ramakrishna Mission Vivekananda Centenary College, Rahara. References Figure 1: Lecture notes on Stellar Structure and Evolution, Jørgen Christensen-Dalsgaard (Institut for Fysik og Astronomi, Aarhus Universitet) Figure 2, 3, 4(a), 4(b): The life cycle of stars, Shadia Habbal (Institute for Astronomy, University of Hawaii) Figure 5(a): Hubble telescope, NASA Figure 5(b): supernova 1987 A, Anglo-Australian observatory Lecture notes on Stellar Structure and Evolution, Jørgen Christensen-Dalsgaard (Institut for Fysik og Astronomi, Aarhus Universitet) The life cycle of stars, Shadia Habbal (Institute for Astronomy, University of Hawaii) Stellar Evolution: Birth, Life and Death of Stars, John R. Percy(International Astronomical Union, University of Toronto, Canada) Astronomy a beginner’s guide to the universe, Chaisson, Mcmillan Encyclopedia Britannica

PREDATOR-PREY MODEL(LOTKA–VOLTERRA MODEL)

The project submitted, in partial fulfilment of the requirement for the assignments in **PHSA CC Xl , PHSA CC Xll , PHSA DSE-l , PHSA DSE-ll**

Paper (Semester 5th) in the Department of Physics

Submitted by

Spandan Bhattacharjee

Registration No: A01-1112-111-001-2018

Supervisor Teacher: Prof. Kalyanbrata Chatterjee

RAMAKRISHNA MISSION VIVEKANANDA CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

Predator-Prey Model (**Lotka-Volterra Model**)

Predator-prey models are arguably the building blocks of the bio- and ecosystems as biomasses are **grown** out of their resource masses. Species compete, evolve and disperse simply for the purpose of seeking resources to sustain their struggle for their very existence. Depending on their specific settings of applications, they can take the forms of resource-consumer, plant-herbivore, parasite-host, tumor cells (virus)-immune system, susceptible-infectious interactions, etc. They deal with the general loss-win interactions and hence may have applications outside of ecosystems. When seemingly competitive interactions are carefully examined, they are often in fact some forms of predator-prey interaction in disguise.

So in this model we are going to study the change of populations the predator and the prey and the dependence of their population number on one another which is known as the predator-prey dynamics .

For example , lets take rabbits (prey) and foxes(predator)

So let u = no. of prey (rabbits)
v = no. of predators (foxes)
And u and v are functions of time .

In general , if we try to explain the predator – prey dynamics in words without any equations , the explanation goes like this -> When prey increases in number , the predators have more food , hence they grow and prey decreases . After a certain time , there is shortage in prey and hence predators have less food available and they start decreasing and the prey starts increasing and this cycle continues . This is what creates the nice predator – prey dynamics .

We want to figure out the change in population of prey and predator over time –

$$\frac{du}{dt} = \dot{u} = u - uv \quad \text{----------} \ (1)$$

Where the first term (u) represents rabbits growing themselves exponentially without predator (reproduction term) and the second term (uv) represents rabbits being eaten by foxes (predation term)

We write the same thing for foxes –

$\frac{dv}{dt} = \dot{v} = \alpha uv - \alpha v$ ---------- (2)

Where α = proportionality constant

Where the first term (αuv) represents that the foxes will grow when there is more food available and also for their own reproduction and the second term (αv) the no . of foxes decreases when there are too many foxes and small prey(more competition in catching and eating rabbits) where as rabbits don't face that problem because we are assuming that there is always enough food available for the rabbits .

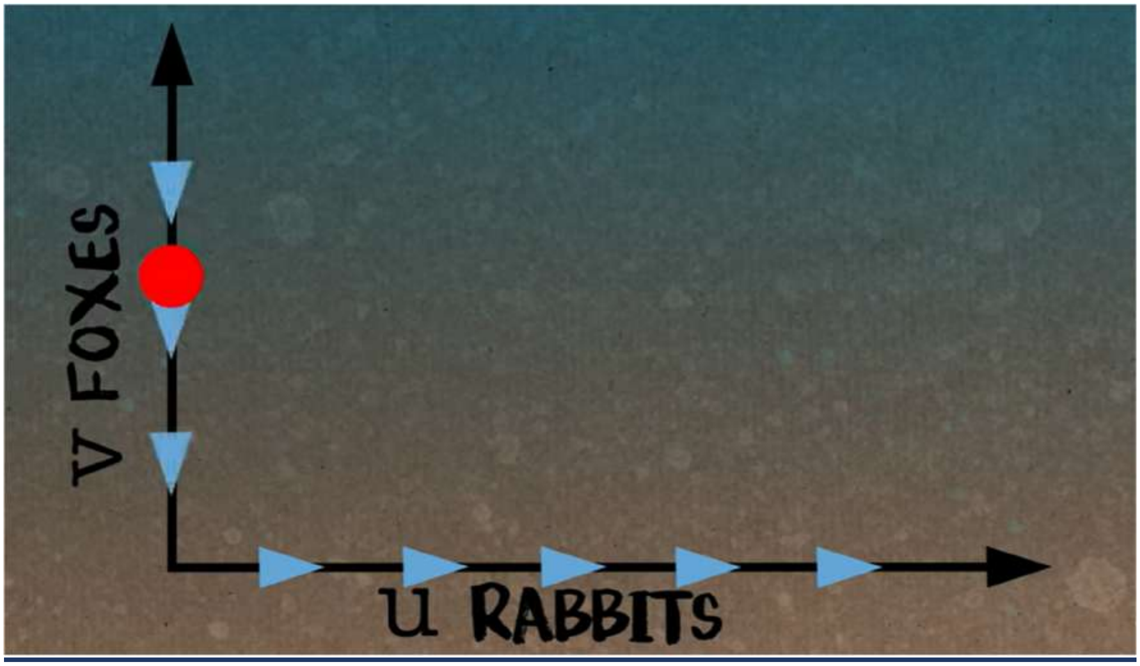
So our model so far –

For rabbits – 1) They grow with increase in their own population
2) Decrease with increase in fox population

For foxes – 1) Increase with population of rabbits
2) Decrease with their own population

Eq (1) and Eq (2) are coupled differential equations . This is because of the complex dynamics between the predator and prey as they are so interdependent on one another

Plotting The Phase Space



Lets see what happens on the two axes first :

In u – axis → v = 0 (zero predators)

In v – axis → u = 0 (zero prey)

So putting v = 0 in eq(1) (no predators) , we get

$\dot{u}$ = u => The rate of change of 'u' (rabbits / prey) is equal to value of u (rabbit / prey population)

That is the rabbit / prey population keeps on increasing exponentially when v = 0 (zero foxes / predators)

Putting u = 0 in eq(2) (no prey) , we get

$\dot{v}$ = – αv => The rate of change of 'v' (foxes / predator) is equal to value of – αv (– α times fox / predator population)

That is suppose there are some no . of foxes initially with zero prey . Now surely they rae going to decrease as there is no prey (no food available)

<u>The Steady States</u> : It is a particular population number where both the predator and the prey are happy . It sticks to a constant number . There is no change in their population number with time.

Hence , $\dot{u} = \dot{v} = 0$

$\Rightarrow (u\text{-}uv) = 0 \quad \Rightarrow \; u(1\text{-}v) = 0 \; \Rightarrow \; u = 0$ or $v = 1$

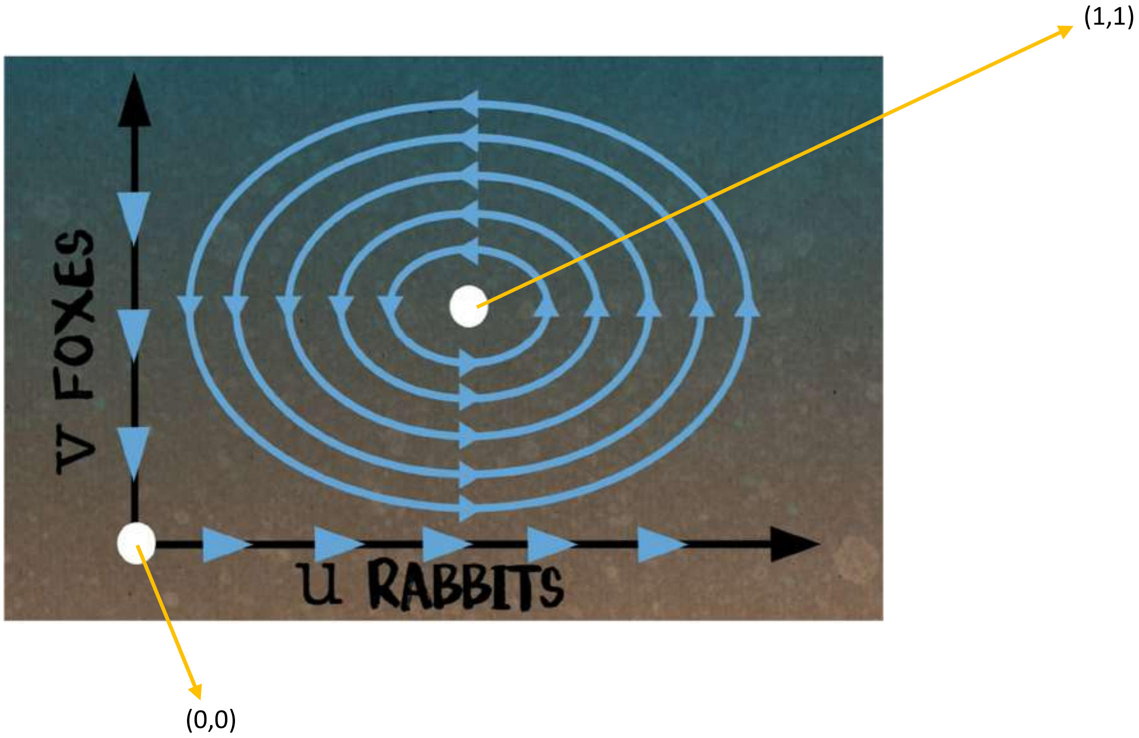
And $\alpha uv - \alpha v = 0 \; \Rightarrow \; \alpha v(u\text{-}1) = 0 \; \Rightarrow \; v = 0$ or $u = 1$

So we have two steady states :- $\boxed{\begin{array}{l} u = 0 \\ v = 0 \end{array}}$ and $\boxed{\begin{array}{l} u = 1 \\ v = 1 \end{array}}$

No predator
No prey

One prey per
one predator

After doing stability analysis , we get the point (u = 1,v = 1) as the center and near this point of the population , the population pathways look like an ellipse as shown in figure below and the direction of the curve will be as per the rules of phase plane (and that is the trajectories can never cross and all the directions follow)



Plotting population vs time :

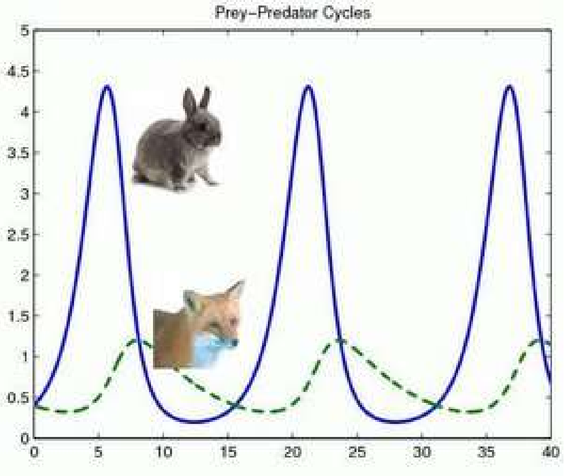
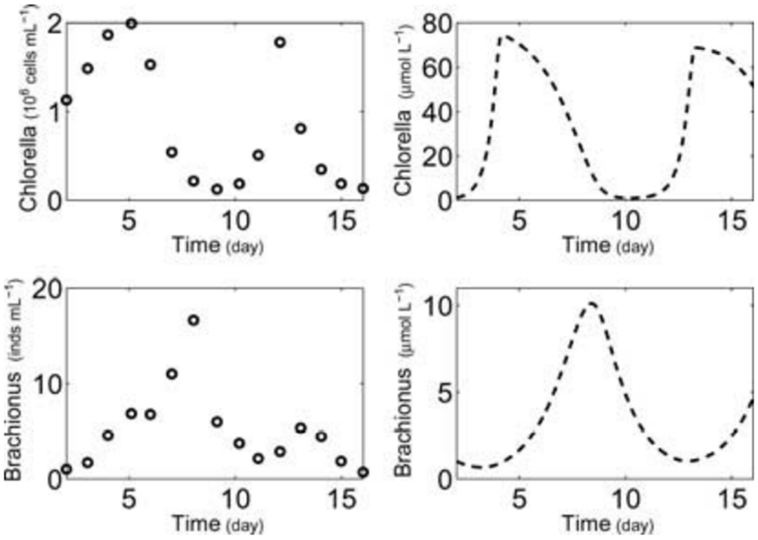


Figure 1: Periodic activity generated by the Predator-Prey model.

Conclusion :

Though we have considered the simplest model , still amazingly the results and graphs obtained from the model very closely resembles the actual data and graphs. Some real life data and graphs are shown below :

Example 1 :



Example of experimental predator–prey dynamics. The two graphs on the left show the experimental observations for Chlorella (algal prey) and Brachionus (rotifer predator) at nitrogen concentration (inflow) N∗ = 80 µmol/L and dilution rate δ = 0.68/day (from Yoshida et al. [2003]). The two graphs on the right show the ODE solutions of system (1) with Fussmann et al.'s (2000) parameter values, using the first observations as the initial values. Note the quantitative differences between observed and predicted time series and the necessity to estimate a scaling factor between observed concentrations (in numbers of organisms) and model predictions (all state variables in units of nitrogen).

Example 2 :

The experimental data set for this research was collected by Bruce Tomkins and Peter Ridland at the Plant Research Institute, Burnley, Victorian Department of Agriculture in 1981. The experiment was carried out on carnation with predatory mite P. persimilis as predator and twospotted mite T. urticac as prey. The study was conducted in two identical glasshouses. In each glasshouse, 50 carnation plants were grown in polystyrene boxes with two rows of five plants per box. Plant stems were supported by layers of wire mesh which were added as plants grew. The fluctuations in mite numbers in each glasshouse were followed by taking plant samples weekly. For each species the number of adult females, adult males, juveniles and eggs present were counted as well as plant percentage infestation. Complete data set and the detailed information on how the data was collected in the glasshouse experiment are given in [9]. Here is presented only a part of the data called "16E Total". These experimental data used for the model simulations are plotted in Figure 1, where the time t unit used is in weeks, and prey and predator populations u and v are in units of the plant percentage infestation.

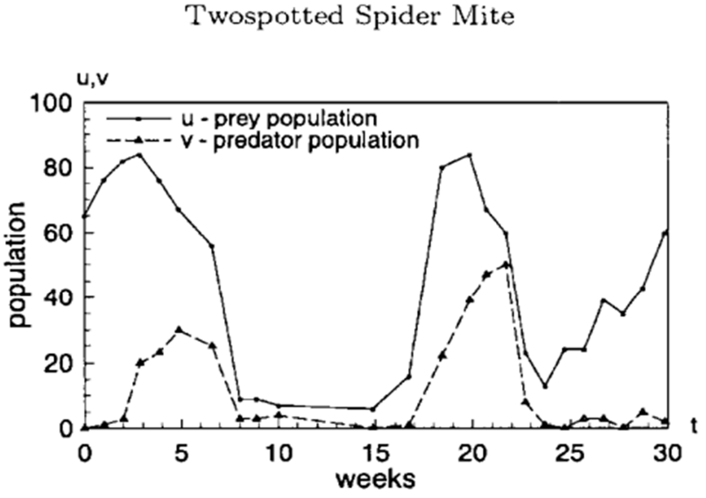

Twospotted Spider Mite

Figure 1. Predator-prey graph — observed data values "16Etotal".

Python Program For Predator – Prey Model

```
pred = float(input("Enter the predator population : ")) # use pred and prey here
prey = float(input("Enter the prey population :"))

periods = int(input("Enter the number of periods: "))
```

```
A=float(input("Enter the value .1: "))
B=float(input("Enter the value .01 : "))
C=float(input("Enter the value .01 : "))
D=float(input("Enter the value .00002: "))

for i in range(periods):

  # update both pred and prey at once (so no temp vars are needed)
  # also, lots of unneeded parentheses were removed

  prey, pred = prey*(1 + pred), pred*(-α + α*prey)

  print("After period {} there are {:.0f} predators, and {:.0f} prey"
      .format(i, pred, prey))
```

References :

1) A) Video links :

   https://youtu.be/M0nRWcF1WJw

   https://youtu.be/c2LPt9z4RJY

   B) Website links:

   Twospotted spider mite predator-prey model - ScienceDirect

   biom_942.tex (sfu.ca)

   Predator-prey model - Scholarpedia

2) Books :
   - M.L Boas
   - Arfken
   - Piskunov

3) Co-operation of departmental professors (Special thanks to KC sir)

Fractals

The project submitted, in partial fulfilment of the requirement for the assignments in ***CC-XI, CC-XII, DSE-I*** *and* ***DSE-II*** *Paper (Semester V) in the Department of Physics*

Submitted by

Atanu Ghosh

Registration No: A01-1112-111-020-2018

Supervisor Teacher:

Sankhasubhra Nag

RAMAKRISHNA MISSION VIVEKANANDA
CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

Introduction:

This is a project that regarding fractals on the basic level. I have written this report based on my understanding of fractals hoping that it would give a simpler introduction to fractals. Basically, here we have covered,

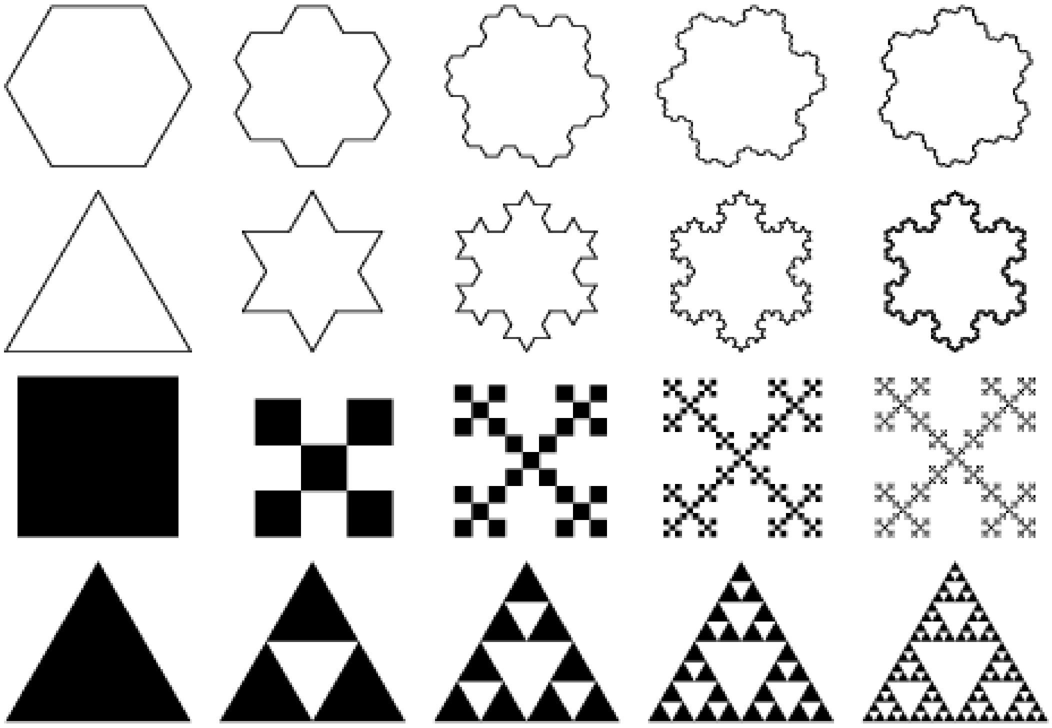
- The basic of fractals
- The various simple fractal shapes
- Relation to real life fractals
- Determining the number of fractals in a shape
  and,
- Properties of fractals.

These are important for gaining a basic understanding of fractals. As fractals are important for they change the most basic ways we analyze and understand an experimental data.

What are fractals:

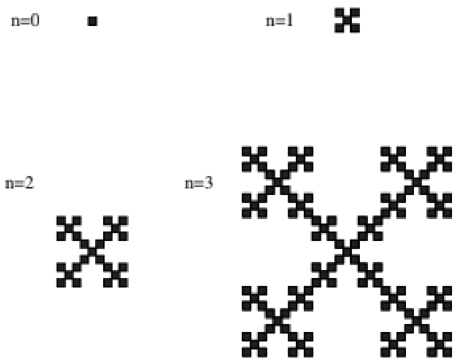
In a simple word a fractal is a never-ending pattern. Fractals are complex pattern that are self similar across different scales. They are patters that repeat over and over to form several shapes.

Some forms of fractals are as follows:



Inserted from Feldman D.P

Self-similarity: The fractals are all self-similar geometrical objects. Imagine a snowflake, if we break an arm of a snowflake, it will come out identical to the snowflake albeit in a smaller size.



In the above figure, we see at n = 0 we start with a small square. We can think of it as a seed playing the function of the initial condition. To ger into n = 1 we must take 4 similar squares at each corner. Then for n = 2 similarly we must lace four n = 1 shapes at four corners and thus repeating it over we get the snowflakes.

Real life fractals: Some real-life fractals are given below

treehugger.com

fractalsaco.weebly.com

mathigon.org

These are some of the fractals we encounter in real life. Though they can't be said to be ideal fractals as mathematically and in real life fractals are quite different. For example, a fern we can zoom in on the fern, and we will see smaller copies of the fern. But eventually this stop, and the fern no longer looks like little ferns. Instead, we start to see individual cells. Thus, real life and mathematical fractals are quite different. There are no perfect fractals in real life.

Dimensions: Now we will characterize fractals on basis of dimensions. By defining dimension in terms of the scaling properties of a shape, we will come up with a quantitative way of describing fractals.

How many little things fit inside a big thing: Suppose we have a line segment and then magnify it by a factor of 3. The line segment is now three times as long as before. Hence, three of the small line segments fit inside the new, larger line segment. Now, try the same thing with a square. We start with a small square, and then magnify it by a factor of 3. This means that it is now three times as long and three times as tall as it was before. We can see 9 copies of the small square fit inside the bigger square. Finally, let us imagine the same experiment with a cube. We start with a small cube and magnify it by a factor of 3. The cube is now three times as wide, three times as tall, and three times as deep. In this bigger cube, one can fit 27 of the smaller cubes.

Now in all instances, the magnification factor is 3. All lengths in the shape are three times as long as they were previously. Now from here we an see the objects **dimensions** determines how many of similar shapes can fit inside. The relation of the dimension and the number of small copies is

$$\textit{number of small copies} = (\textit{magnification factor})^{D}$$

D = Dimension

Now from this formula we can get dimension in fractional quantity too. Let it be any dimension like 1.465

So, what does dimension 1.465 mean?

There are several ways to think about this. First, a dimension between 1 and 2 means that the shape has some qualities of two-dimensional objects and some of one-dimensional objects.

Another way of giving meaning to a dimension like 1.465 is as follows. The essential feature of a fractal is that it is self-similar; it is made up of small parts that each resemble the whole, and those small parts are made up of smaller parts that resemble the whole, and so on. For a fractal it is not always meaningful to speak of the average or typical size of a component of a fractal. Rather, we want to capture something about what stays the same as we examine the fractal at different length scales.

Example:

1. **Dimensions of cantor set:**

This is a cantor set. The step of construction is given in the figure. The magnification factor is 3, as it was for the snowflake; each line segment must be stretched to 3 times its length to be as long as the line segments at the previous step. And the number of small copies is 2. At each step there are two small Cantor sets which can be scaled up to reproduce the original. Thus, the dimension equation for the Cantor set is:

$$3^D = 2.$$

To solve for D, we take the logarithm of both sides:

$$\log(3D) = \log (2).$$

Simplifying and solving for D, we obtain:

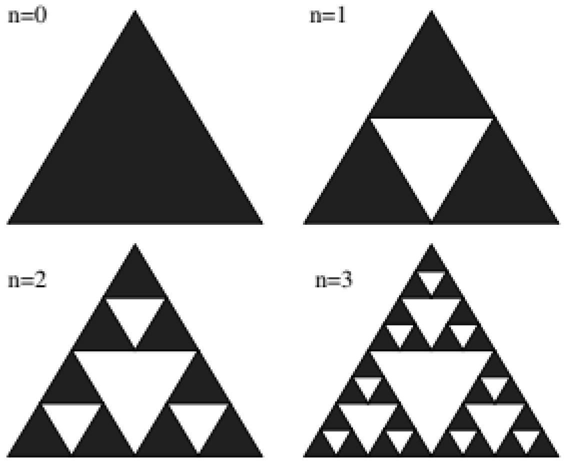
$D \log(3) = \log(2),$

$D = \log(2)$

$\log(3) \approx 0.6309$

The dimension between 1 and 0 indicates that the Cantor set is in some regards line-like (one-dimensional) and in some regards point-like (zero-dimensional).

2. Dimension of Sierpinski triangle:



Feldman D.P

As a final example, let us determine the self-similarity dimension of the Sierpinski triangle, a fractal given in the above figure. Looking at the n = 1 stage, we can see that there are 3 small copies of the triangle inside the full shape. How big is each small copy compared to the large one? The magnification factor for this fractal is 2. The easiest way to see this is to look at one of the sides of one of the small triangles. We can see that this side of the small triangle is exactly half the length of the side of the full triangle. So, we would need to magnify it by a factor of 2 for it to be as large as the big triangle. Thus, our dimension equation is:

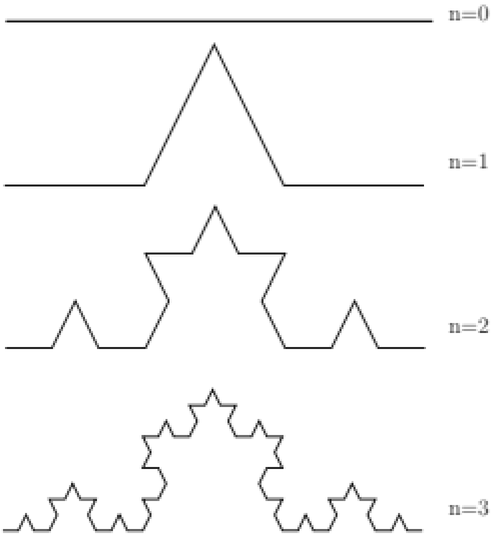
$$2^D = 3$$

Solving for D, we obtain

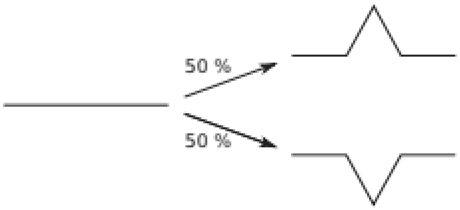
D = log (3)
log (2) ≈ 1.585.

Random fractals: Let us talk about random fractals. Theses are fractals produced due to randomness or irregularity. Such as

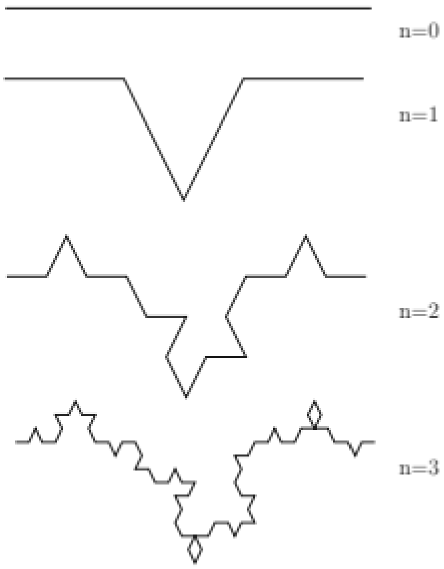
1. **The random Koch curve:**



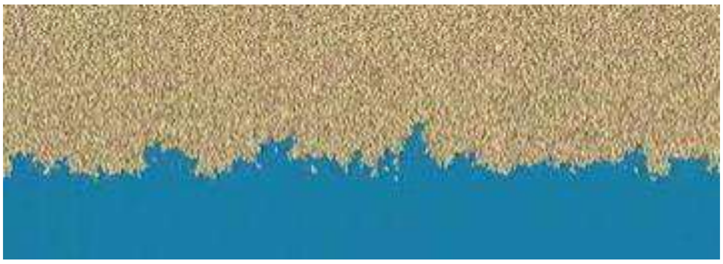
Here the Koch curve is generated by a simple line segment. In the first step of iteration, the segment is divided into four smaller segments. Another way to think of this is that the initial line segment gets bent and stretched upward so it has a triangle in the middle. This process is then iterated. In the next step, moving from n = 1 to n= 2, again each line segment is replaced by four smaller line segments. This procedure is repeated as one goes from n = 2 to n = 3 in the figure. Now, each time we replace a line segment with a bent segment, half of the time we bend the line up, and half of the time we bend it down.



Repeated application of this iteration rule produces a shape known as a random Koch curve. Now due to the probability of bending up and down by ½ the fractals can or can't be symmetric as given in the next figure



This fractal is basically a representation of real-life fractals found in coastline



Fractalfoundation.org

So let us conclude here with the basic summary about fractals being:

(1) They exhibit self-similarity across a range of scales. This self-

similarity can be exact or approximate.

(2) They are not well described by usual geometric forms like circles, cones, and lines.

(3) The self-similarity dimension, or some similar dimension, is larger than the topological dimension.

Conclusion: Idea of fractals are important because fractals are a nice way of storing information, particularly when you are describing the behaviour of a group of objects that all act similarly to each other. It can also be used to explain a lot of phenomena in our daily life.

References: Book- Chaos and fractals An Elementary introduction by David P. Feldman, Wikipedia, Sites- www.worldscientific.com, mathigon.org etc.

Acknowledgement:

In the accomplishment of this project successfully, many people have given me their support and guidance. Now I am utilizing this time to thank all the people who have been concerned with this project. Primarily I would thank my project supervisor Prof. Sankhasubhra Nag Sir for his guidance and support which help me to complete this project in time, besides him I would like to thank my friends for their help which also help me to complete this project work. Our physics department and college had also given considerable support and help in doing the project.

Asymmetric Vibration and Combination Tone

The project submitted, in partial fulfilment of the requirement for the assignments in PHSA-CC-XI, PHSA-CC-XII, PHSA-DSE-I, PHSA-DSE-II Paper (Semester-V) in the Department of Physics

Submitted by :

(**Krishnakeshab Nayak**)

Registration No: **A01-1112-111-033-2018**

Supervisor Teacher: Prof. Atisdipankar Chakraborty

RAMAKRISHNA MISSION VIVEKANANDA
CENTENARY COLLEGE
P.O. RAHARA, KOLKATA-700118
WEST BENGAL, INDIA

Introduction：

This is a project that regarding Asymmetric vibration and Combination tone on the basic level. I have written this report based on my understanding of Asymmetric vibration and Combination tone hoping that it would give a simpler introduction to Asymmetric vibration and Combination tone. Basically, here I have covered,

- **Asymmetric Vibration**
- **Combination tones**
    1. Defination
    2. Derivation
    3. Example
    4. Conclusion
- **References**

Asymmetric Vibration

We will assume that there is a the squared term of displacement in restoring force - the direction of this term will not change with the direction of displacement so it will sometimes decrease and sometimes increase the original restoring force.The form of the equation of motion will be:

$$\ddot{x} + p^2x + ax^2 = 0 \tag{1.1}$$

Or,
$$\ddot{x} = -\ p^2x - ax^2 \tag{1.2}$$

So, that is the value of the force on unit mass. The first term is restoring i.e on the opposite side of the displacement for all values of displacement, but the second term is not like that - if displacement is negative then it tries to increase velocity towards the direction of displacement. If potential energy is 'V' then

$$\int_0^x (p^2x + ax^2)\, dx\ = \frac{p^2x^2}{2} + \frac{ax^3}{3} \tag{1.3}$$

the graph of 'V' will be

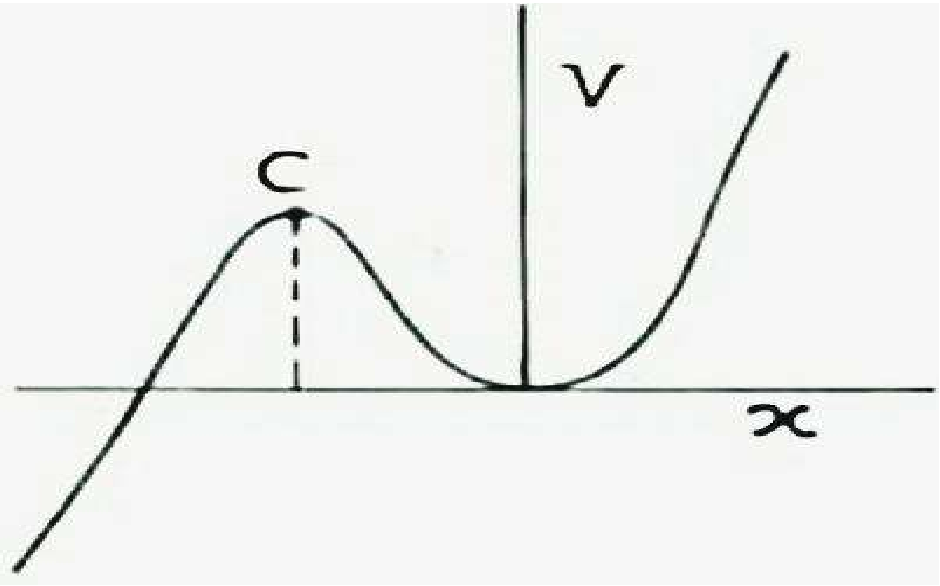


Figure : 1.1

In potential there are 1st and 2nd powered term of displacement. So, here are two minima.
There is a maxima of 'V' at the point C on the left side of the figure.For mild vibrations,maxima state of 'V' is a miserable state and the total force to the left will be congruent with the displacement and the particle will never return. However, at present we assume that the amplitude is so small that the term 'ax^2' is almost negligible compared to 'p^2x'.
Now putting $x = x_0$ sin pt in the equation (1.1) we can get,

$$\ddot{\mathbf{x}} + \mathbf{p}^2\mathbf{x} + \mathbf{a}\mathbf{x}_0^2 \sin^2 \mathbf{pt} = 0 \tag{1.4}$$

Or, $$\ddot{x} + p^2x + \frac{ax_0^2}{2} = \frac{ax_0^2}{2}\cos 2pt \qquad (1.4)$$

If we put $x + \frac{ax_0^2}{2p^2} = y$ then above equation will be

$$\ddot{y} + p^2y = \frac{ax_0^2}{2}\cos 2pt \qquad (1.5)$$

Solution of above differential equation (1.5) will be

$$y = -\frac{ax_0^2}{6p^2}\cos 2pt \qquad (1.6)$$

Now, putting x in equation (1.6) in terms of y we can get, $$x = -\frac{ax_0^2}{6p^2}\cos 2pt - \frac{ax_0^2}{2p^2} + x_0 \sin pt \qquad (1.7)$$

That means, the normal frequency p/2π remains unchanged but the midpoint of oscillation has shifted somewhat.The reason for this shift, however, is that the term ax^2 is the source of an asymmetric ball relative to the origin x = 0. Again, this effect has given rise to the oscillation of its double frequency without the original normal frequency.

Combination tone

Definition : When two pure, strong tones of frequencies q_1 and q_2 act on the same system (such as a membrane or an air cavity), a series of new notes may be heard having frequencies (q_1-q_2), (q_1+q_2) and in general $(aq_1 \pm bq_2)$ where a and b are small integers. They have been termed combination tones by Helmholtz. The first two of these form the first order combination tones; (q_1-q_2), is called the difference tone, and (q_1+q_2) the summation tone. The rest are known as higher order tones. They may have frequencies $q_1- 2q_2$, $2q_1 -q_2$, q_1+2q_2, etc. Of these, the difference tone generally the most and easily perceived. The intensities of the various combination tones depend on the intensities of the fundamental (Fig : 1.2).

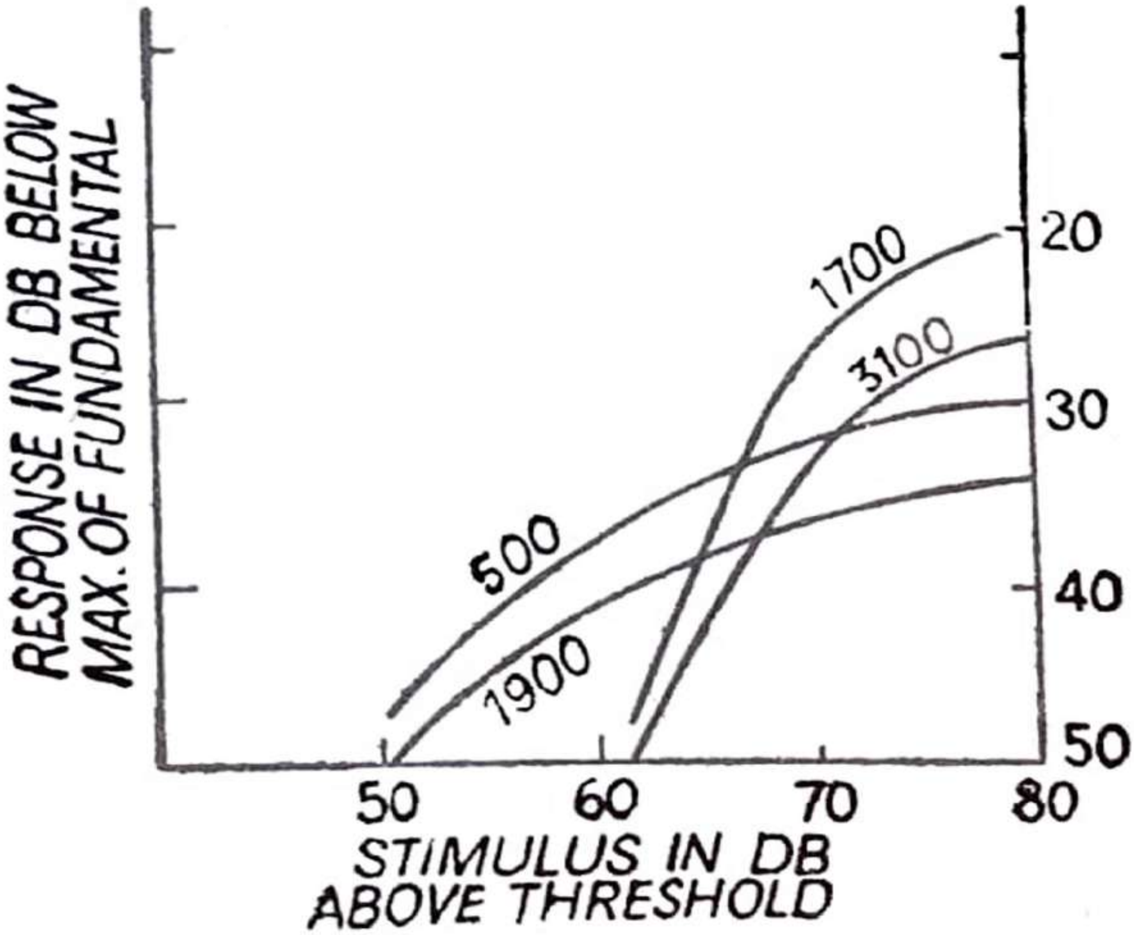


Figure : 1.2

Derivation : Let, two pure,strong tones are applied to a system from the outside and their frequencies are different from each other and also different from normal frequencies. The form of equation of motion will be:

$$\ddot{x} + p^2 x + ax^2 = A_1 \sin f_1 t + A_2 \sin f_2 t$$

(1.8)

Where f_1 and f_2 are the angular frequencies of two tones respectively.
Ignoring the small term 'ax²', the solution of equation (1.8) will be -

$$\mathbf{x} = \frac{\mathbf{A_1 \sin f_1 t}}{\mathbf{p^2 - f_1^2}} + \frac{\mathbf{A_2 \sin f_2 t}}{\mathbf{p^2 - f_2^2}} \qquad (1.9)$$

The nature of the vibration with the p/2π frequency is not captured here - it can be assumed that the damped vibration has subsided after a considerable time but the effect of the damped vibration on forced vibration is very small..
Determining the value of 'ax²' from equation (1.9) and putting it at equation (1.8) we can get,

$$\ddot{x} + p^2 x = A_1 \sin f_1 t + A_2 \sin f_2 t - \frac{aA_1^2 \sin^2 f_1 t}{\left(p^2 - f_1^2\right)^2} - \frac{aA_2^2 \sin^2 f_2 t}{\left(p^2 - f_2^2\right)^2} - \frac{2aA_1 A_2 \sin f_1 t \cdot \sin f_2 t}{\left(p^2 - f_1^2\right)\left(p^2 - f_2^2\right)} \qquad (1.10)$$

We know from trigonometry,

$$\mathbf{\sin^2 f_1 t = \frac{1}{2}\left(1 - \cos 2f_1 t\right)}$$

$$\mathbf{\sin^2 f_1 t \cdot \sin^2 f_2 t = \frac{1}{2}\left[\cos\left(f_1 - f_2\right)t + \cos\left(f_1 + f_2\right)t\right]}$$

So, equation (1.10) will be

$$\begin{aligned}\ddot{x} + p^2 x = A_1 \sin f_1 t + A_2 \sin f_2 t + \frac{aA_1^2 \cos 2f_1 t}{2\left(p^2 - f_1^2\right)^2} \\ + \frac{aA_2^2 \cos 2f_2 t}{2\left(p^2 - f_2^2\right)^2} - \frac{aA_1 A_2 \cos\left(f_1 - f_2\right)t}{\left(p^2 - f_1^2\right)\left(p^2 - f_2^2\right)} \\ + \frac{aA_1 A_2 \cos\left(f_1 + f_2\right)t}{\left(p^2 - f_1^2\right)\left(p^2 - f_2^2\right)} - \frac{aA_1^2}{2\left(p^2 - f_1^2\right)^2} - \frac{aA_2^2}{2\left(p^2 - f_2^2\right)^2}\end{aligned} \tag{1.11}$$

Each of the sine and cosine terms will generate an forced vibration and the constant number will change the position of the midpoint of the two oscillations. So the complete solution would be:

$$\begin{aligned}x = -\frac{aA_1^2}{2\left(p^2 - f_1^2\right)^2 p^2} - \frac{aA_2^2}{2\left(p^2 - f_2^2\right)^2 p^2} + \frac{A_1 \sin f_1 t}{\left(p^2 - f_1^2\right)} \\ + \frac{A_1 \sin f_2 t}{\left(p^2 - f_2^2\right)} + \frac{aA_1^2 \cos 2f_1 t}{2\left(p^2 - f_1^2\right)\left(p^2 - 4f_1^2\right)} \\ + \frac{aA_2^2 \cos 2f_2 t}{2\left(p^2 - f_2^2\right)\left(p^2 - 4f_2^2\right)} - \frac{aA_1 A_2 \cos\left(f_1 - f_2\right)t}{\left(p^2 - f_1^2\right)\left(p^2 - f_2^2\right)\left[p^2 - \left(f_1 - f_2\right)^2\right]} \\ - \frac{aA_1 A_2 \cos\left(f_1 + f_2\right)t}{\left(p^2 - f_1^2\right)\left(p^2 - f_2^2\right)\left[p^2 - \left(f_1 + f_2\right)^2\right]}\end{aligned} \tag{1.12}$$

The presence of oscillations of double the frequencies ($f_1/2π$ and $f_2/2π$) of the force applied in the above solution is observed. The tune of the multiplicative frequency of a tune is called the harmonic or full-fledged tune of the low-frequency tune.It is seen that if there is a higher power position of the displacement without the proportional force of displacement in restoring force, then a multiplicative tune arises. Again oscillations of frequencies $(f_1+f_2)/2π$ and $(f_1-f_2)/2π$ also arises. These tones are generally called combination tones. Tone of frequency $(f_1+f_2)/2π$ is called summation tone and the tone of frequency $(f_1-f_2)/2π$ is called difference tone. It is clear from Equation (1.12) that the intensity of difference tone will be generally greater than the strength of a summation tone.

Example :

The difference tone is most readily heard when a police whistle or a referee's whistle is blown. These whistles consist of two slightly unequal short barrels, each giving a fairly high frequency note. But when the whistle is blown it gives a strong low-pitched note. of the frequencies emitted by the individual barrels. If, when the whistle is sounding. one of the barrels is closed by a finger, the strong difference tone disappears and a feeble high frequency note is heard in its place.

Conclusion : At one time there was some controversy over the question whether combination tones were subjective or objective in nature. This controversy no longer exists as it has been conclusively proved that combination tones may have both subjective and objective existence. The essential condition for their production is that the intensity of the two superposed tones must be high enough to make the responding system behave in a non-linear way. The ear has an intrinsic non-linearity of response to intense tones ; this accounts for the subjective nature of combination tones. A mechanical system may be non linear by construction for large amplitude. When such a system is subjected double forcing, its vibrations contain combination frequencies. This accounts for the objective existence.

References

Sites :

- https://en.wikipedia.org/wiki/Combination_tone
- https://www.britannica.com/science/combination-tone
- https://en.wikipedia.org/wiki/Musical_tone

Books :

- Advanced Acoustics by Dr. D. P. Raychaudhuri

Acknowledgement

Primarily I would like to profound gratitude to Ramakrishna Mission Vivekananda Centenary College, Rahara for including the project as part of our academic curriculum. This has provided me an opportunity to learn something beyond the text book. I wish to express my deep gratitude and sincere thanks to revered Principal Maharaj Swami Kamalasthananda, Ramakrishna Mission Vivekananda Centenary College, Rahara, for his encouragement and the facilities provided for this project work. I extend my sincere thanks to Prof. Atisdipankar Chakraborty,Ramakrishna Mission Vivekananda Centenary College,Rahara and my mentor who guided me to successful completion of the project. I take this opportunity to express my deep sense of gratitude for his valuable support and constructive comments.