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Undergraduate Admission Test 2023: Mathematics Honours

Full Marks : 75 **Time : 1 hour**

1. If $f(x) + 2f\left(\frac{1}{x}\right) = 3x$ for $x \neq 0$, and $S = \{x \in \mathbb{R} : f(x) = f(-x)\}$. Then S
   a) is empty set.
   b) contains exactly one element.
   c) contains exactly two elements.
   d) contains more than two elements.

2. The number of solution/s of the following equation is
   $$e^{\sin x} - e^{-\sin x} = 4$$
   a) 0
   b) 1
   c) 2
   d) infinitely many

3. The sides of a triangle are in the ratio $1:\sqrt{3}:2$, then the angles of the triangle are in the ratio
   a) $1:3:5$
   b) $2:3:4$
   c) $3:2:1$
   d) $1:2:3$

4. The coefficient of x^{18} in the product $(1+x)(1-x)^{10}(1+x+x^2)^9$ is
   a) 84
   b) -126
   c) 126
   d) -84

5. If z is any complex number satisfying $|z-3-2i| \leq 2$, then the minimum value of $|2z-6+5i|$ is
   a) 5
   b) 7
   c) 3
   d) 0

6. The number of seven-digit integers, with sum of the digits equal to 10 and formed by using the digits 1,2 and 3 only, is
   a) 55
   b) 66
   c) 77
   d) 88

7. The sum of the series $\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\cdots$ up to infinity is
   a) $e^{-\frac{1}{2}}$
   b) $e^{\frac{1}{2}}$
   c) $\frac{1}{e^2}$
   d) $\frac{1}{e}$

8. The points $(0,\frac{8}{3})$, (1,3), (82,30) are vertices of
   a) an obtuse angled triangle.
   b) an acute angled triangle.
   c) a right-angled triangle.
   d) none of these.

9. The common tangent to the circles $x^2+y^2=4$ and $x^2+y^2+6x+8y-24=0$ also passes through the point
   a) (6, -2).
   b) (4, -2).
   c) (-6,4).
   d) (4, -6).

10. $\lim_{x\to 0}\frac{\log x^n-[x]}{[x]}$ =?, where n is a natural number and $[x]$ denotes the greatest integer less than or equal to x.
    a) 1
    b) -1
    c) 0
    d) does not exist

11. Let A, B, C and D be four non-empty sets. The contrapositive statement of "If $A\subseteq B$ and $B\subseteq D$, then $A\subseteq D$" is
    a) If $A\nsubseteq C$, then $A\nsubseteq B$ and $B\subseteq D$.
    b) If $A\nsubseteq C$, then $A\nsubseteq B$ or $B\nsubseteq D$.
    c) If $A\nsubseteq C$, then $A\subseteq B$ and $B\nsubseteq D$.
    d) If $A\nsubseteq C$, then $A\nsubseteq B$ and $B\nsubseteq D$.

12. Let $x_1, x_2, \ldots, x_n$ be n observations such that $\sum {x_i}^2=400$ and $\sum x_i=80$. Then a possible value of n among the following is
    a) 18
    b) 15
    c) 12
    d) 9

13. The solution of the equation $\log_7\log_5(\sqrt{x+5}+\sqrt{x})=0$ is
    a) 5
    b) 0
    c) 4
    d) None of these

14. Let $R = \{(1,3), (4,2), (2,4), (2,3), (3,1)\}$ be a relation on the set $A = \{1,2,3\}$. The relation is
    a) not transitive
    b) not symmetric
    c) not reflexive
    d) function

15. A function f from the set of natural numbers to integers defined by
$$f(n) = \begin{cases} \frac{n-1}{2}, when\ n\ is\ odd \\ \frac{-n}{2}, when\ n\ is\ even \end{cases}$$ is
    a) onto but not one-one.
    b) one-one and onto both.
    c) neither one-one nor onto.
    d) one-one but not onto.

16. The value of $\tan(\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3})$ is
    a) $\frac{6}{17}$
    b) $\frac{17}{6}$
    c) $\frac{16}{7}$
    d) None of these

17. The maximum value of
$$f(x) = \begin{vmatrix} (\sin x)^2 & 1+(\cos x)^2 & \cos 2x \\ 1+(\sin x)^2 & (\cos x)^2 & cos2x \\ (\sin x)^2 & (\cos x)^2 & \sin 2x \end{vmatrix},$$
    where x is a real number, is
    a) $\sqrt{5}$
    b) 5
    c) $\sqrt{7}$
    d) $\frac{3}{4}$

18. Let $f: \mathbb{R} \to \mathbb{R}$ be a continuous function. Then $\lim_{x\to\frac{\pi}{4}} \frac{\frac{\pi}{4}\int_2^{(sec\,x)^2} f(x)\,dx}{x^2 - \frac{\pi^2}{16}}$ is equal to
    a) f(2)
    b) $2f(\sqrt{2})$
    c) 2f(2)
    d) 4f(2)

19. If the line $ax + by + c = 0$ is normal to the curve $xy = 1$, then
    a) ab>0.
    b) ab<0.
    c) a=0, b≠0.
    d) a≠0, b=0.

20. $\int_0^{\pi} \frac{e^{\cos x} \sin x}{(1+(\cos x)^2)(e^{\cos x}+e^{-\cos x})} dx =$
    a) $\frac{\pi^2}{4}$
    b) $\frac{\pi^2}{2}$
    c) $\frac{\pi}{4}$
    d) $\frac{\pi}{2}$

21. For which of the following values of m, is the area of the region bounded by the curve $y = x - x^2$ and the line $y = mx$ is equals $\frac{9}{2}$?
    a) 2, -4
    b) -2, -4
    c) 2, 4
    d) -2, 4

22. The differential equation whose solution is $Ax^2 + By^2 = 1$ where A and B are arbitrary constants is of
    a) second order and second degree.
    b) first order and second degree.
    c) first order and first degree.
    d) second order and first degree.

23. The non-zero vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ are related by $\vec{a} = 2\vec{b}$ and $\vec{c} = -7\vec{b}$. Then the angle between $\vec{a}$ and $\vec{c}$ is
    a) π
    b) 0
    c) $\frac{\pi}{4}$
    d) $\frac{\pi}{2}$

24. The equation of a plane containing the line of intersection of the planes 2x-y-4=0 and y+2z-4=0 and passing through the points (1,1,0) is
    a) x-3y-2z=-2.
    b) 2x-z=2.
    c) x-y-z=0.
    d) x+3y+z=4.

25. Let S= {1, 2, 3, 4, 5, 6}. Then the probability that a randomly chosen onto function $g: S \to S$ satisfies $g(3) = 2g(1)$ is
    a) $\frac{1}{15}$.
    b) $\frac{1}{5}$.
    c) $\frac{1}{30}$.
    d) $\frac{1}{10}$.